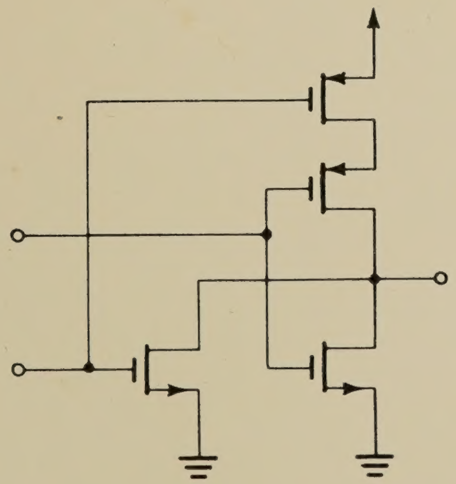
SOLUTIONS MANUAL
for

MICRO-ELECTRONIC CIRCUITS

Adel S. Sedra
Kenneth C. Smith



SOLUTIONS MANUAL
FOR
MICROELECTRONIC CIRCUITS

BY

ADEL S. SEDRA
and
KENNETH C. SMITH

HOLT, RINEHART & WINSTON

New York Chicago San Franciso Philadelphia
Montreal Toronto London Sydney Tokyo
Mexico City Rio de Janeiro Madrid

ISBN 0-03-057698-9

Address correspondence to:
383 Madison Avenue
New York, N.Y. 10017

Printed in the United States of America
Published simultaneously in Canada
2 3 4 5 066 9 8 7 6 5 4 3 2 1

CBS COLLEGE PUBLISHING
Holt, Rinehart and Winston
The Dryden Press
Saunders College Publishing

PREFACE

This manual contains complete solutions for all 222 exercises and 572 problems included in the book MICRO-ELECTRONIC CIRCUITS. For each chapter, the exercises' solutions are presented first and are followed by the problems' solutions.

Communications concerning detected errors should be sent to A. Sedra, Department of Electrical Engineering, University of Toronto, Toronto, Ontario, Canada, M5S 1A4 and, needless to say, will be greatly appreciated.

CHAPTER 1 – EXERCISES

(1.1) Power $P = \frac{1}{T}\int_0^T (v^2/R)\,dt$ can be found directly by integrating the square waveform over the interval T, one period of the wave of frequency $\omega = 2\pi/T$ or, alternatively, the series representation can be integrated over the same interval, that is:

$$P = \frac{1}{T}\int_0^T (v_1 + v_3 + v_5 + \cdots)^2/R\;dt$$

or
$$P = \frac{1}{T}\int_0^T \left(\frac{4V}{\pi R}(\sin\omega t + \tfrac{1}{3}\sin 3\omega t + \tfrac{1}{5}\sin 5\omega t + \cdots)\right)^2 dt$$

$$= \left(\frac{4V}{\pi}\right)^2 \frac{1}{TR}\int_0^T (\sin^2\omega t + \tfrac{1}{3}\sin\omega t\sin 3\omega t + \tfrac{1}{5}\sin\omega t\sin 5\omega t + \cdots$$
$$+ \tfrac{1}{3}\sin\omega t\sin 3\omega t + \tfrac{1}{9}\sin^2 3\omega t$$
$$+ \tfrac{1}{15}\sin 3\omega t\sin 5\omega t + \cdots$$
$$+ \tfrac{1}{5}\sin\omega t\sin 5\omega t + \cdots$$
$$+ \cdots)\,dt$$

Now since $\sin A\sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$$P = \left(\frac{4V}{\pi}\right)^2 \frac{1}{TR}\int_0^T (\sin^2\omega t + \tfrac{1}{9}\sin^2 3\omega t + \tfrac{1}{25}\sin^2 5\omega t + \cdots$$
$$+ \tfrac{1}{3}\cos 2\omega t + \tfrac{1}{3}\cos 4\omega t$$
$$+ \tfrac{1}{15}\cos 2\omega t + \tfrac{1}{15}\cos 8\omega t + \cdots)\,dt$$

Now since $\int_0^T \cos n\omega t\,dt = 0$

$$P = \left(\frac{4V}{\pi}\right)^2 \frac{1}{TR}\int_0^T (\sin^2\omega t + \tfrac{1}{9}\sin^2 3\omega t + \tfrac{1}{25}\sin^2 5\omega t + \cdots)\,dt$$
$$= \frac{1}{T}\int_0^T (v_1^2/R + v_3^2/R + v_5^2/R + \cdots)\,dt$$
$$= P_1 + P_3 + P_5 + \cdots \qquad \text{QED}$$

Note that the energy available in a waveform in an interval T to a load R is proportional to P_T.

For the square waveform
$$P_S = \frac{1}{T}\int_0^T v^2/R\,dt = \frac{1}{RT}\left(\int_0^{T/2} V^2 dt + \int_{T/2}^T (-V)^2 dt\right) = \frac{1}{RT}(V^2 T/2 + V^2 T/2) = V^2/R$$

For each sine wave component (eg P_n) having a peak V_{Pn} over the same interval
$$P_n = \frac{1}{T}\int_0^T (V_{Pn}\sin\omega t)^2/R\,dt = \left(\frac{V_P}{\sqrt{2}}\right)^2/R$$

For the fundamental (harm. #1) $P_1 = \left(\frac{4V}{\pi\sqrt{2}}\right)^2/R = 0.81\,V^2/R = 0.81 P_S$

For the first 3 harmonics $P = P_1 + P_2 + P_3 = P_1(1 + 0 + (1/3)^2) = 0.90 P_S$

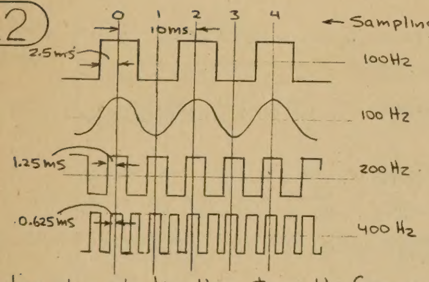
For the first 5 harmonics $P = P_1 + P_3 + P_5 = P_1(1 + 1/9 + 1/25) = 0.932 P_S$

For the first 7 harmonics $P = P_1 + \cdots + P_7 = P_1(1 + 1/9 + 1/25 + 1/49) = 0.949 P_S$

For the first 9 harmonics $P = P_1 + \cdots + P_9 = P_1(1 + 1/9 + 1/25 + 1/49 + 1/81) = 0.959 P_S$

Note that 90% of the square wave's energy is in the first 3 harmonics, that is in the fundamental and the third.

(1.2)



	samples
100Hz	+1 −1 +1 −1 +1
100 Hz	+1 −1 +1 −1 +1
200 Hz	+1 +1 +1 +1 +1
400 Hz	+1 +1 +1 +1 +1

Sampling at a rate less than twice the frequency of the information of interest (f) is called undersampling and the ambiguity to which it leads is called aliassing. The critical sampling frequency ($2f$) is called the Nyquist rate (or frequency).

(1.3) The amplifier operates linearly with a gain of 1000 until a level of ±10 is reached. Operation is summarized in the table:

Peak to Peak Input	1μV	100μV	1mV	100mV	1V
Peak input	±0.5μV	±50μV	±0.5mV	±50mV	±0.5V
Peak output (potential)	±0.5mV	±50mV	±0.5 V	±50 V	±500V
Peak output (actual)	±0.5mV	±50mV	±0.5 V	±10 V	±10 V

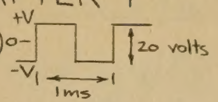
limiting

(1.4) The minimum sampling frequency necessary for the high end of the band is 2X3400 = 6800Hz. The maximum adequate sampling interval is 1/6800Hz or 147 μs.

If in practice each sample requires 10μs, then the number of channels which can be handled must not exceed 147/10 or 14.7. Thus the number of channels which can be implimented is 14.

CHAPTER 1 – PROBLEMS

(1.1)



The square wave can be represented by the series $\frac{4V}{\pi}(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \cdots)$ where $V = 10$ volts and $\omega = 2\pi f$ and $f = 1$ kHz.

The angular frequency of the wave is $\omega = 2\pi f = 6.28$ krad/s

The angular frequency of the fundamental is also 6.28 krad/s

The amplitude of the fundamental is $4V/\pi = \frac{4(20/2)}{\pi} = 12.73$V

The amplitude of the second harmonic is 0.0V

The amplitude of the third harmonic is 1/3 that of the fund.

The third harmonic has the same phase as the fundamental.

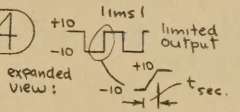
(1.2) A 10kHz square wave of amplitude V when passed through a channel which cuts off at 16 kHz is reduced to its fundamental component at 10kHz whose amplitude is $4V/\pi$ volts.

The energy available in a square wave of amplitude V is proportional to V^2 while the energy of a sine wave of amplitude $4V/\pi$ is proportional to $\left(\frac{1}{\sqrt{2}}\frac{4V}{\pi}\right)^2$ or $0.81V^2$. Thus the adult described perceives 0.81 or 81% of the available signal energy.

(1.3) For this system the level of uncertainty is ±0.5 which as a fraction of full scale (ie 9) is 0.5/9 x100 or 5.6%, and as a fraction of the smallest non-zero signal (ie 1) is 0.5/1 x100 or 50%

9 —
8 —
7 —
2 —
1 —
0 —

(1.4)



The maximum output is ±10V which for a gain of 1000 occurs for inputs above ±10mV. For a sine wave input of 1volt (peak), a level of 10mV is reached at an angle of $\sin^{-1}\frac{10\text{mV}}{1\text{V}} = 0.573°$ corresponding to a time t of 0.573/360 × 1ms or 1.6 μs.

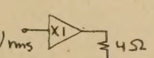
Since for a high-speed amplifier, the resulting wave rises in only 3.2μs, it can be seen as nearly square with a fundamental of peak amplitude $4/\pi(10)$ or 12.73 V.

Thus the "effective gain" is 12.73/1 or 12.73

(1.5) a) The RMS value of the 1kHz component with amplitude 12.73 volts is $12.73/\sqrt{2} = 9.00$V

b) Up to 10kHz, there are components at 3, 5, 7, 9 kHz whose RMS values are 1/3, 1/5, 1/7, 1/9 as large resp. The RMS of the available harmonics is
$$\sqrt{(1/3)^2 + (1/5)^2 + (1/7)^2 + (1/9)^2} \text{ or } 0.429 \text{ or } 42.9\%$$
of the fundamental. This distortion accordingly has an RMS value of 0.429(9.00) or 3.86V

(1.6)



Power at the speaker $= V^2/R = 1^2/4 = 0.25$W

(1.7) The signal passing through the channel consists of the fundamental at 1 kHz and the 3rd harmonic at 3kHz and 1/3 of the fundamental amplitude:

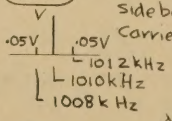
Energy in a square wave of amplitude $V \propto V^2$

Energy in its fundamental component $\propto \left(\frac{4V}{\pi\sqrt{2}}\right)^2 = 0.81V^2$

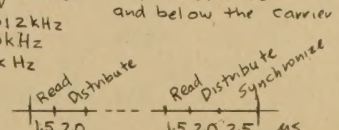
Energy in its third harmonic $\propto \left(\frac{1}{3}\frac{4V}{\pi\sqrt{2}}\right)^2 = 0.09V^2$

Total received energy $\propto 0.81V^2 + 0.09V^2$ or $0.90V^2$ that is 90% of that transmitted

(1.8) The modulation index m is 0.1. Thus the sideband amplitude is ½m or 0.05 of the carrier. The sidebands are at 2kHz above and below the carrier at 1010 kHz



(1.9)



$10(1.5 + 2.0) + 2.5 = 37.5\,\mu s$

The minimum cycle time for a 10 channel system is 37.5μs

Thus the maximum channel sample rate is $\frac{1}{37.5\mu s} = 26.7$kHz

Chapter 2 – Exercises

2.1 $r_D = 1/g_D$ where $g_D = \left.\frac{\partial i_D}{\partial v_D}\right|_{i_D=I_D} = \left[I_S\left(\frac{1}{V_T}\right)e^{v_D/V_T}\right]_{i_D=I_D}$

But $I_D = I_S e^{V_D/V_T}$; Thus $g_D = I_D/V_T$

and $r_D = V_T/I_D = \frac{25\,mV}{1\,mA} = 25\ \Omega$

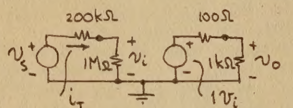
and $V_D = V_T \ln(I_D/I_S) = 25 \ln\left(\frac{1\times10^{-3}}{10^{-15}}\right) = 690.8\ mV$

2.2 Power delivered to load $= 10^2/1k\Omega = 100\ mW$

Power drawn from dc supply = Power delivered to the load + Power lost in the amplifier $= 100 + 20 = 120\ mW$

DC current drawn from the supply $= \frac{120\,mW}{15V} = 8\,mA$

2.3



$v_i = v_s \frac{1}{1+0.2} = \frac{v_s}{1.2}$

$v_o = v_i \frac{1}{1+0.1} = \frac{v_i}{1.1}$

$v_o = \frac{1}{1.1}\cdot\frac{v_s}{1.2}$ or $v_o/v_s = \frac{1}{1.1\times1.2} = 0.76$

$20 \log v_o/v_s = -2.4\ dB$

$i_I = v_s/1.2M\Omega$; $i_o = v_o/1k\Omega$;

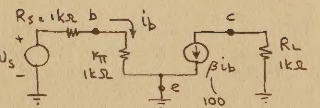
$i_o/i_I = v_o/v_s \times 1200 = 1200 \times 0.76 = 912$

$20 \log i_o/i_I = 59.2\ dB$

Power gain $= \frac{v_o i_o}{v_s i_I} = 0.76 \times 912 = 693 = P_o/P_I$

$10 \log P_o/P_I = 10 \log 693 = 28.4\ dB$

2.4



$v_o = -\beta i_b \times R_L = -100 \times 1k\Omega \times i_b$

$v_s = i_b(R_s + r_\pi) = i_b \times 2k\Omega$

$v_o/v_s = -50\ V/V$

$20\log|v_o/v_s| = 34\ dB$

$i_o/i_I = -\beta i_b/i_b = -100$

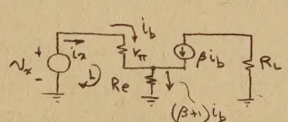
$20\log|i_o/i_I| = 40\ dB$

$P_o = v_o i_o$; $P_i = v_s i_b$

$P_o/P_i = \left(\frac{v_o}{v_s}\right)\times\left(\frac{i_o}{i_b}\right) = 50 \times 100 = 5000$

$10 \log P_o/P_i = 37\ dB$

2.5



$R_{in} = v_x/i_x = v_x/i_b$

Writing a loop equation for Loop L : $v_x = i_b r_\pi + (\beta+1) i_b R_e$

Thus $v_x/i_x = r_\pi + (\beta+1)R_e$ from which

$R_{in} = r_\pi + (\beta+1)R_e$

2.6 $T = 1ms$; $f = 1/T = 1000\ Hz$; $\omega = 2\pi f = 2\pi\times10^3\ rad/s$

2.7 $\omega = 10^3\ rad/s$; $\omega_o = 1/CR = 1/(10^{-6}\times10^3) = 10^3\ rad/s$

Thus $|T| = 1/\sqrt{2}$; $\phi = -45°$ and

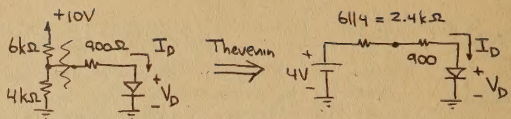
$v_o(t) = 10/\sqrt{2}\ \sin(10^3 t - \pi/4)$

2.8 Using the voltage divider rule:

$V_o/V_i = \frac{R}{R + 1/j\omega C} = \frac{j\omega CR}{1 + j\omega CR}$

$|V_o/V_i| = \frac{\omega CR}{\sqrt{1 + (\omega CR)^2}}$

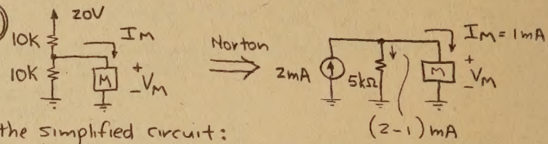
2.9



From the simplified circuit:

$I_D = \frac{4 - V_D}{2.4 + 0.9} = \frac{4 - 0.7}{3.3} = \frac{3.3}{3.3} = 1\,mA$

2.10

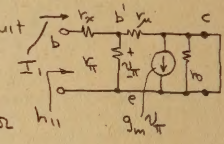


From the simplified circuit:

$V_M = (2-1)5k\Omega = 5V$

2.11 $R_{in} = R/(1-\mu) = \frac{100\,k\Omega}{1 - 0.95} = 2{,}000\,k\Omega = 2M\Omega$

2.12 To find h_{11} and h_{21} short circuit port 2:



$h_{11} = r_x + (r_\pi \| r_\mu)$
$= 100 + (2.5K \| 10M) = 2.6k\Omega$

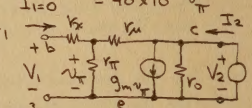
$v_\pi = I_1(r_\pi \| r_\mu) = I_1(2.5k \| 10M) = I_1 \times 2.5k\Omega$

and $I_1 = v_\pi / 2.5k\Omega$

From a node equation at c : $I_2 = g_m v_\pi - (v_\pi/r_\mu) = 40\times10^{-3}v_\pi - \frac{v_\pi}{10^7}$
$= 40\times10^{-3} v_\pi$

Thus $h_{21} = \frac{I_2}{I_1} = \frac{40\times10^{-3}v_\pi}{v_\pi/2.5\times10^3} = 100$

To find h_{12} and h_{22}, open circuit port 1



Since $I_1 = 0$, $V_1 = v_\pi$; Using the voltage divider rule: $v_\pi = V_2 \frac{r_\pi}{r_\pi + r_\mu}$

Thus $h_{12} = \frac{V_1}{V_2} = \frac{v_\pi}{V_2} = \frac{r_\pi}{r_\pi + r_\mu} = \frac{2.5\times10^3}{2.5\times10^3 + 10^7} \approx 2.5\times10^{-4}\ V/V$

From a node equation at c: $I_2 = V_2/r_o + g_m v_\pi + V_2/(r_\pi + r_\mu)$

But $v_\pi = V_2 r_\pi/(r_\pi + r_\mu)$, thus $I_2 = V_2/r_o + (g_m r_\pi + 1)V_2/(r_\pi + r_\mu)$

Thus $h_{22} = \frac{I_2}{V_2} = \frac{1}{r_o} + \frac{g_m r_\pi + 1}{r_\pi + r_\mu} = \frac{1}{10^5} + \frac{40\times10^{-3}\times2.5\times10^3 + 1}{2.5\times10^3 + 10^7}$
$= 10^{-5} + 101\times10^{-7} = 2.01\times10^{-5}\ ℧$

2.13 dc transmission $= \frac{10k\Omega}{10k\Omega + 10k\Omega} = 0.5 \rightarrow -6dB$

$f_o = \omega_o/2\pi = \frac{1}{2\pi\tau} = \frac{1}{2\pi\times100\times10^{-12}(10k\Omega \| 10k\Omega)} = 318\ kHz$

$|T| = \frac{0.5}{\sqrt{1 + (f/f_o)^2}}$; Thus $|T(2MHz)| = \frac{0.5}{\sqrt{1 + (2000/318)^2}} = .0785 \rightarrow -22dB$

2.14 High frequency gain $= 100 \rightarrow 40\,dB$

$f_o = \omega_o/2\pi = \frac{1}{2\pi\tau} = \frac{1}{2\pi\times0.1\times10^{-6}\times100\times10^3} = 15.9\ Hz$

$|T| = \frac{100}{\sqrt{1 + (f_o/f)^2}}$; Thus $|T(1Hz)| = \frac{100}{\sqrt{1 + (15.9)^2}} = 6.28 \rightarrow 16dB$

2.15 $v_o(\infty) = 3\times10^{-3}\times1\times10^3 = 3V$; $v_o(0+) = 0$

$v_o(t) = 3 - (3 - 0)e^{-t/\tau}$ where $\tau = 100\times10^{-12}\times1\times10^{-3} = 10^{-7}s$

That is $v_o(t) = 3(1 - e^{-10^7 t})$

2.16 $v_o(0+) = 2\times10^{-3}\times2\times10^3 = 4V$; $v_o(\infty) = 0$

$v_o(t) = 0 - (0 - 4)e^{-t/\tau} = 4e^{-t/\tau}$ where $\tau = L/R = 10^{-5}/2\times10^3 = 5\times10^{-9}s$

That is $v_o(t) = 4e^{-2\times10^8 t}$

2.17 $t_r = t_f = 2.2\tau = 2.2/\omega_o = \frac{2.2}{2\pi f_o} = \frac{2.2}{2\pi\times10^7} = 35\,ns$

2.18 Percent Sag $= T/\tau \times 100$; Thus $1 = \frac{10\mu s}{\tau}\times100$ or $\tau = 1ms$

But $\tau = C(1k\Omega + 4k\Omega) = C\times5\times10^3$

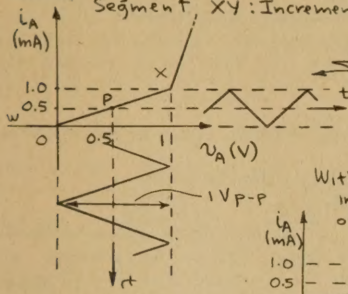
Thus $C = 10^{-3}/5\times10^3 = 0.2\,\mu F$

2.19 Undershoot = Decay in pulse amplitude

$= 10 - 10e^{-T/\tau} = 10(1 - e^{-1}) = 6.32V$

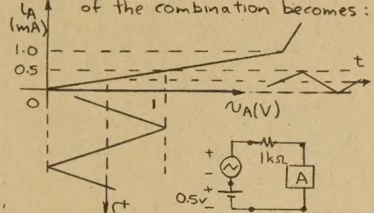
Chapter 2 - Problems

2.1 Segment WO: Incremental conductance = 0
Segment OX: Incremental conductance = $\frac{1mA}{1V}$ = 1mA/V
Segment XY: Incremental cond. = $\frac{6-1}{1.5-1}$ = 10mA/V



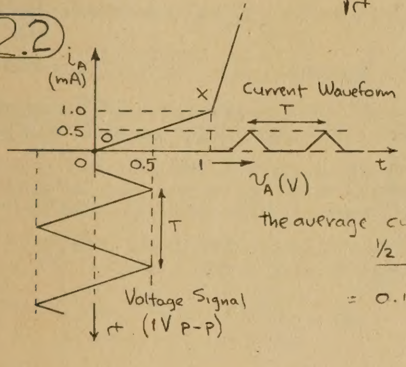
The current has a triangular form with 1mA peak to peak amplitude and an average value of 0.5mA

With a 1 kΩ resistance inserted in series the characteristic of the combination becomes:



For which the average current is 0.25mA and the signal current is 0.5 mA p-p

2.2

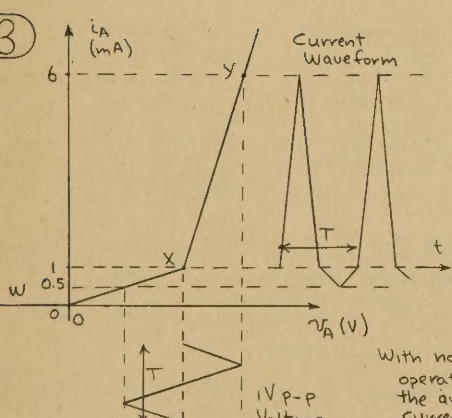


With no signal, the average current is zero.

With a triangular 1 volt p-p signal, the peak current is 0.5mA and the average current is

$$\frac{\frac{1}{2} \times \frac{T}{2} \times 0.5}{T} = 0.125 \text{ mA}$$

2.3

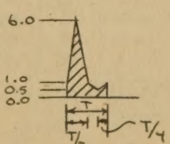


With no signal, and operation at X, the average bias current is 1.0 mA. With a 1V p-p signal applied, the positive peak current is 6mA and the negative peak current is 0.5mA. The average current is found by evaluating the hatched area and dividing by T:

$$\text{Area} = \frac{T}{2}\times 1 + \frac{1}{2}\times\frac{T}{2}\times 5 + \frac{T}{2}\times 1 - \frac{1}{2}\times\frac{T}{2}\times 0.5$$

$$= T\left[\frac{1}{2} + \frac{5}{4} + \frac{1}{2} - \frac{0.5}{2}\right]$$

$$= 2.125\,T$$



Average current = 2.125 mA

2.4 For D, $i_D = I_S e^{v_D/V_T}$ for $I_S = 10^{-15}$A, $V_T = 25$mV

For I_D = 10mA, $10\times10^{-3} = 10^{-15} e^{V_D/25}$

for which $V_D = 25 \ln 10^{13} = 748.3$ mV

The incremental conductance is obtained as follows:

$$g_d = \left.\frac{\partial i_D}{\partial v_D}\right|_{i_D = I_D} = I_S \times \frac{1}{V_T} e^{V_D/V_T} = I_D/V_T$$

Thus $r_d = 1/g_D = V_T/I_D = 25\text{mV}/10\text{mA} = 2.5\,\Omega$

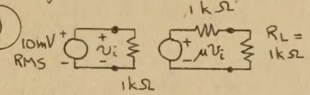
2.5 Input peak-to-peak voltage $= 10\text{mV} \times \frac{1k\Omega}{1k\Omega + 9k\Omega}$
= 1 mV

2.6 Power delivered to load = $10^2/1$ = 100mW
Power drawn from dc supply = 15V × 8mA = 120 mW
Power lost in amplifier = 120 − 100 = 20 mW

2.7 For the unloaded condition:
Power dissipated in amplifier equals the power drawn from the supplies = 15×1 + 15×1 = 30mW
For the loaded condition:
Power drawn from the supplies = 15×10 + 15×10 = 300mW
Power delivered to the load = $10^2/1$ = 100mW
Power dissipated in the amplifier = 300 − 100 = 200 mW

2.8 RMS load voltage = 10mV × 100 = 1V
Power delivered to the load = 1^2/1kΩ = 1mW
Since the amplifier input current is zero, its power gain is infinite

2.9



The load voltage = $10\text{mV}\times 100 \times \frac{1}{1+1} = 0.5$V RMS
The load power = $0.5^2/1k\Omega = 0.25$ mW

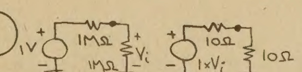
The power lost in the amplifier = Power dissipated in the amplifier input resistance + power dissipated in amp. output resistance $= \frac{(10^{-2})^2}{1k} + \frac{(0.5)^2}{1k} \simeq 0.25$ mW

Power gain $= \frac{\text{Power to load}}{\text{Power from Source}} = \frac{0.25}{10^{-4}} = 2500$

The dc supply must provide at least the power lost in the output and load resistors or 0.25 + 0.25 = 0.50mW

2.10 For the direct connection:
Loudspeaker voltage $= 1\times\frac{10}{10+10^6} \simeq 10^{-5}$V
Loudspeaker power $= (10^{-5})^2/10 = 10^{-11}$W
Source power $\simeq 1^2/10^6 = 10^{-6}$W ; Power gain $= 10^{-11}/10^{-6} = 10^{-5}$
For an ideal unity gain buffer:
Loudspeaker voltage = 1V ; Loudspeaker power = $1^2/10$ = 0.1W
Voltage gain = 1 ; Power gain is infinite since the current drawn from the source by an ideal buffer is zero.

2.11

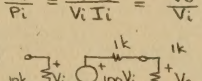


Output voltage $= 1\times\frac{1}{1+1}\times 1 \times \frac{10}{10+10} = 0.25$V

Output power = $(0.25)^2/10$ = 6.25mW
Voltage gain = 0.25
Power from source = $1^2/2\times10^6 = 0.5\times10^{-6}$W
Power gain $= \frac{6.25\times10^{-3}}{0.5\times10^{-6}} = 12{,}500$

2.12 Voltage gain = 20 log 100 = 40dB
Current gain = 20 log 1000 = 60 dB
Power gain = 10 log(100×1000) = 50dB
Since $\frac{P_o}{P_i} = \frac{V_o I_o}{V_i I_i} = \frac{V_o}{V_i}\times\frac{I_o}{I_i}$

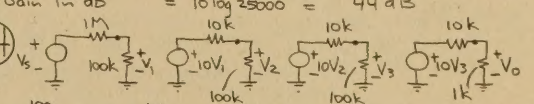
2.13



$P_{in} = V_i^2/10$ mW
$P_{out} = V_o^2/1 = (50V_i)^2/1 = 2500V_i^2$

Power Gain $= P_{out}/P_{in} = 2500V_i^2/(V_i^2/10) = 25{,}000$
Power Gain in dB = 10 log 25000 = 44 dB

2.14



$$V_o/V_s = \frac{100}{100+1000}\times 10 \times \frac{100}{100+10}\times 10\times\frac{100}{100+10}\times 10\times\frac{1}{1+10} = 6.83 \text{ V/V}$$

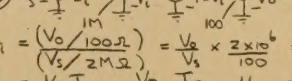
20 log V_o/V_s = 16.7 dB

$V_o/V_1 = 10\times\frac{100}{110}\times 10\times\frac{100}{110}\times 10\times\frac{1}{11} = 75.13$

20 log V_o/V_1 = 37.5 dB

These values differ because of the loss incurred in coupling the source to the amplifier.
This coupling loss is 37.5 − 16.7 = 20.8 dB

2.15

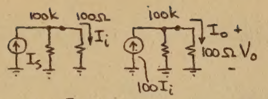


$V_o/V_s = \frac{1}{1+1}\times 1\times\frac{100}{100+100} = 1/4$
or in dB 20log 1/4 = −12 dB

$$I_o/I_i = \frac{(V_o/100\Omega)}{(V_s/2M\Omega)} = \frac{V_o}{V_s}\times\frac{2\times10^6}{100} = 1/4 \times 2\times10^4 = 5000 \text{ or } 74\text{ dB}$$

$P_o/P_s = \frac{V_o I_o}{V_s I_i} = \frac{V_o}{V_s}\cdot\frac{I_o}{I_i} = 1/4 \times 5000 = 1250$ which in dB is 10 log 1250 ≃ 31 dB

(2.16)

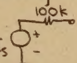


$I_i = I_s \frac{10^5}{10^5+10^2}$

$I_o = 100 I_i \frac{10^5}{10^5+10^2}$

Thus $I_o = 100\left(\frac{10^5}{10^5+10^2}\right)^2 I_s$ whence $I_o/I_s = 99.8$ or nearly 40dB

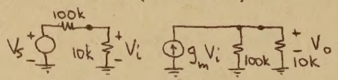
To find the voltage gain, transform the source to its Thevenin equivalent: $V_s = 100k I_s$



Voltage gain of the loaded amplifier is $V_o/V_s = \frac{I_o \times 100\Omega}{I_s \times 100k\Omega} = \frac{99.8}{1000} = 0.998 \approx 0.1 \equiv -20dB$

Voltage gain of the unloaded amplifier is $V_o/V_s = \frac{100 I_i \times 100k}{I_i \times (100k+100)} \approx 100$ or 40dB

(2.17)



$V_i = V_s \frac{10}{100+10}$

$V_o = g_m V_i (100k \| 10k)$

or $V_o = 10 \times \frac{10}{110} V_s \times \frac{100\times10}{110}$

Thus $V_o/V_s = 1000/121 = 8.26 \equiv 18.3$ dB

(2.18) Review Example 2.2: $i_b = 1\mu A$; $i_e = 100\mu A$

From Fig 2.11b we see that $i_e = (\beta+1) i_b$

Thus $\beta+1 = 100$ and $\beta = 99$.

Using the relationship for α derived in Ex 2.2, ie $\alpha = \frac{\beta}{\beta+1}$, or from Fig 2.11d, ie $i_c = \alpha i_e = \alpha\times100 = i_e - i_b = 99$, we see that $\alpha = 0.99$

For $v_{be} = 2mV$, from Fig 2.11b, $i_b = \frac{v_{be}}{r_\pi}$ from which

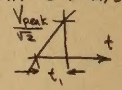
$r_\pi = \frac{v_{be}}{i_b} = \frac{2mV}{1\mu A} = 2k\Omega$ and from Fig 2.11d, $i_e = \frac{v_{be}}{r_e}$, so

$r_e = \frac{v_{be}}{i_e} = \frac{2mV}{100\mu A} = 20\Omega$ and from Fig 2.11c, $i_c = g_m v_{be}$,

$g_m = \frac{i_c}{v_{be}} = \frac{99}{2} = 49.5$ mA/V

(2.22) $f = 60$ Hz; $T = 1/f = 16.67$ ms; $\omega = 2\pi f = 377$ rad/s

Since RMS value = Peak value$/\sqrt{2}$ the value t_1 is found from:



$V_p \sin\omega t_1 = V_p/\sqrt{2}$

or $\omega t_1 = \pi/4$ rad from which $t_1 = 2.08$ ms

Note that since $\omega t_1 = \pi/4$, we see that the absolute value of a sine wave exceeds its RMS value for half the cycle

(2.23) $T = 1\mu s$; $f = 1/T = 1MHz$; $\omega = 2\pi f = 6.28$ Mrad/s

(2.24) Using the voltage divider rule: $V_o/V_i = \frac{R}{R + 1/j\omega C} = \frac{1}{1 - j\frac{1}{\omega CR}}$

This transfer function is of the form given in Eqn 2.25 with $K = 1$ and $\omega_o = 1/CR$. Thus the network is a high-pass filter with magnitude and phase responses as in Fig 2-39

(2.25) $|T(\omega)| = 1/\sqrt{1+(\omega/\omega_o)^2}$ and $\phi(\omega) = -\tan^{-1}(\omega/\omega_o)$

where ω_o is the corner or 3dB frequency equal to $1/CR$

At $\omega = 0.1\omega_o$, $|T| = 0.995$; $20\log|T| = -0.04dB$; $\phi = -5.7°$

At $\omega = 10\omega_o$, $|T| = 0.0995$; $20\log|T| = -20.04dB$; $\phi = -84.3°$

(2.26) $C = 0.1\mu F$, $R = 1.59k$, $\omega_o = 1/CR = \frac{1}{10^{-7}\times1.59k} = 2000\pi$ rad/s

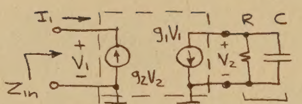
$v_I = 10\sqrt{2}\sin 2\pi\times100t + 10\sqrt{2}\sin 2\pi\times1000t + 10\sqrt{2}\sin 2\pi\times10000t$

For the network: $|T| = 1/\sqrt{1+(\omega/\omega_o)^2}$ and $\phi = -\tan^{-1}(\omega/\omega_o)$

Thus
for the first component $|T| = 0.995$, $\phi = -5.7°$ or -0.099 r
for the second component $|T| = 0.707$ $\phi = -45°$ or -0.785 r
for the third component $|T| = 0.0995$ $\phi = -84.3°$ or -1.47 r

Thus
$v_O = 14.07\sin(200\pi t - 0.099) + 10\sin(2000\pi t - 0.785) + 1.407\sin(20000\pi t - 1.47)$

(2.19) Frequency domain analysis for Z_{in}:



$Z_{in} = V_1/I_1 = V_1/{-g_2V_2}$

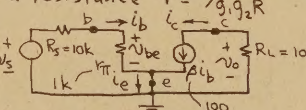
But $V_2 = -g_1V_1/Y$

Thus $Z_{in} = \frac{1}{g_1g_2/Y} = \frac{Y}{g_1g_2}$; but $Y = 1/R + j\omega C$

Thus $Z_{in} = \frac{1}{g_1g_2R} + j\omega(C/g_1g_2)$

Therefore the equivalent circuit seen looking into port 1 consists of an inductance $L = C/g_1g_2$ in series with a resistance $r = 1/g_1g_2R$

(2.20)



$i_b = \frac{v_s}{R_s + r_\pi} = \frac{v_s}{11k}$

$v_o = -\beta i_b R_L = -100\times10k\times i_b$

Thus $v_o/v_s = \frac{-100\times10\times i_b}{i_b\times11} = -90.9$ V/V for which

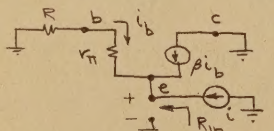
$20\log|v_o/v_s| = 39.2$ dB

The current gain $i_c/i_b = \beta = 100 \equiv 40$ dB

The power gain $\frac{v_o(-i_c)}{v_s(i_b)} = 90.9\times100 = 9090$ for which

$10\log 9090 = 39.6$ dB

(2.21)

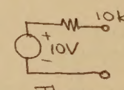


At node e we have: $i + \beta i_b + i_b = 0$ from which $i = -(\beta+1)i_b$

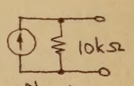
Also $v = -i_b(R + r_\pi)$

Thus $R_{in} \equiv \frac{v}{i} = \frac{-i_b(R+r_\pi)}{-i_b(\beta+1)} = (R+r_\pi)/(\beta+1)$

(2.27)

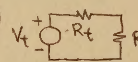


Thevenin



Norton

(2.28)

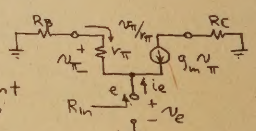


$10 = V_t\frac{1}{R_t+1}$

$11 = V_t\frac{2}{R_t+2}$

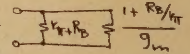
from which $R_t = 2/9$ kΩ, $V_t = 110/9$ V

(2.29) $v_e = -v_\pi - \frac{v_\pi}{r_\pi}R_B = v_\pi\left(1+\frac{R_B}{r_\pi}\right)$



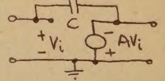
Thus we may use the Source Absorption Theorem to replace the dependent source $g_m v_\pi$ by a resistance $v_e/{-g_m v_\pi} = \frac{(1+R_B/r_\pi)}{g_m}$

Thus R_{in} becomes $(r_\pi+R_B) \| \left(\frac{1+R_B/r_\pi}{g_m}\right)$

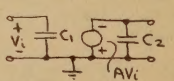


(2.30) $C_1 = C(1+A)$

$C_2 = C(1+1/A)$



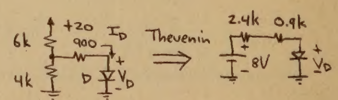
A	C_1	C_2
−2	−C	0.5C
−1	0	0
0	C	Undefined (amp. out, shorted)
+1	2C	2C
+2	3C	1.5C



(2.31) $\frac{4}{6+4}\times20 = 8V$

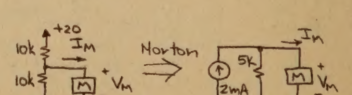
$4\|6 + 0.9$

$I_D = \frac{8-0.7}{2.4+0.9} = 2.21$ mA



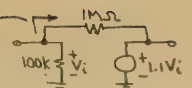
(2.32) $20/10k = 2mA$

$10\|10 = 5k$



Since I_M mA $= 0.1V_M$ V replace M with a resistor of value $V_M/I_M = 10k\Omega$. Thus $V_M = 2\times\frac{5\times10}{5+10} = 6.67V$

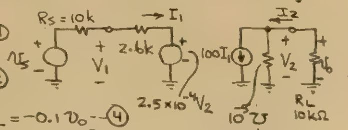
(2.33) Due to the 1MΩ feedback resistance, $I = \frac{V_i - 1.1V_i}{1M}$



an equivalent resistance of $V_i/I = -10M$ results. This, in parallel with 100k produces an equivalent input resistance of $\frac{0.1\times(-10)}{0.1-10} = 101k$
Note that if the amplifier gain is raised to 11, the net input resistance is raised to infinity!

(2.34) $V_1 = 2.6I_1 + 2.5\times10^{-4}V_2$ --①

$I_2 = 100I_1 + 0.01\,V_2$ --②



But $V_2 = v_o$ --③ ; $I_2 = -v_o/R_L = -0.1v_o$ --④

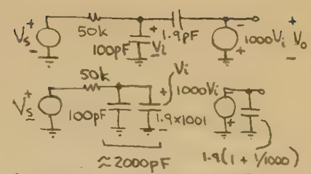
$V_1 = v_s - I_1R_s = v_s - 10I_1$ --⑤

Substituting ③ and ④ into ② :
$-0.1v_o = 100I_1 + 0.01v_o$; $-0.11v_o = 100I_1$;
or $I_1 = -0.11\times10^{-2}v_o$ --⑥
Substituting ⑥ into ⑤ : $V_1 = v_s + 0.011v_o$ --⑦
Substituting ③, ⑥, ⑦ into ① :
$v_s + 0.011v_o = -2.6\times0.11\times10^{-2}v_o + 2.5\times10^{-4}v_o$
ie $v_s = v_o[-0.011 - 2.6\times0.11\times10^{-2} + 2.5\times10^{-4}]$
or $v_o/v_s = -73.5$

(2.35) h parameters : $V_1 = h_{11}I_1 + h_{12}I_2$ --①
$I_2 = h_{21}I_1 + h_{22}V_2$ --②
y parameters : $I_1 = y_{11}V_1 + y_{12}V_2$ --③
$I_2 = y_{21}V_1 + y_{22}V_2$ --④

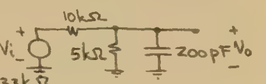
See $y_{11} = I_1/V_1|_{V_2=0}$: Setting $V_2 = 0$ in ① gives $V_1 = h_{11}I_1$, thus $\boxed{y_{11} = 1/h_{11}}$ --⑤. Now $y_{12} = I_1/V_2|_{V_1=0}$: Setting $V_1 = 0$ in ① gives $0 = h_{11}I_1 + h_{12}V_2$; Thus $\boxed{y_{12} = -h_{12}/h_{11}}$ --⑥
See $y_{21} = I_2/V_1|_{V_2=0}$: Setting $V_2 = 0$ in ① and ② gives $V_1 = h_{11}I_1$ and $I_2 = h_{21}I_1$; Thus $\boxed{y_{21} = h_{21}/h_{11}}$ --⑦
See $y_{22} = I_2/V_2|_{V_1=0}$: Setting $V_1 = 0$ in ① gives $I_1 = \frac{-h_{12}}{h_{11}}V_2$
Substituting for I_1 in ② : $I_2 = -\frac{h_{21}h_{12}}{h_{11}}V_2 + h_{22}V_2$
Thus $\boxed{y_{22} = h_{22} - \frac{h_{12}h_{21}}{h_{11}}}$ --⑧

(2.40) Use Miller's Theorem to obtain the equivalent circuit :



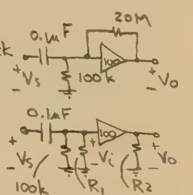
Since $V_o = -1000V_i$, the response is determined by V_i/V_s, the transfer function of the low-pass STC network at the input.
The time constant is $2000\times10^{-12}\times50\times10^{3} = 10^{-4}$s
Thus $w_o = 10^4$ rad/s and
$T(w) = V_o(w)/V_i(w) = \frac{-1000}{1+j\,w/w_o} = \frac{-1000}{1+j\,w/10^4}$

(2.41) The resulting circuit is



DC transmission $= \frac{5}{5+10} = 1/3$
Equivalent resistance $= \frac{5\times10}{5+10} = 3.33k\Omega$
$\tau = 3.33\times10^{3}\times200\times10^{-12} = 0.666\mu s$
$f_o = 1/2\pi\tau = \frac{10^6}{2\pi\times0.666} \approx 239$ kHz
$|T| = \frac{1/3}{\sqrt{1+(f/f_o)^2}}$; Thus at $f = 2MHz$, we have
$|T| = \frac{1/3}{\sqrt{1+\left(\frac{2000}{239}\right)^2}} \approx 0.04 \equiv -28dB$

(2.42) Use Miller's theorem to model the feedback

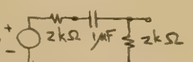


$R_1 = \frac{20M}{1-100} \approx -200k\Omega$
$R_2 = \frac{20M}{1-1/100} \approx 20M\Omega$
Now since $V_o = 100V_i$, the response of the circuit is determined by the STC high pass input network for which
$\tau = 0.1\times10^{-6}R_{eq}$ where $R_{eq} = 100k\,||\,(-200k) = \frac{100(-200)}{100-200} = 200k$
Thus $\tau = 0.1\times10^{-6}\times200\times10^{3} = 20$ ms.
High frequency gain $= 100 \equiv 40$dB
3dB frequency $f_o = 1/2\pi\tau = 1/2\pi(20\times10^{-3}) \approx 8$Hz below which the gain drops by 6dB/octave. Since 1Hz is separated by 3 octaves, the gain at 1Hz is
$40 - 3(6) = 22$ dB.

(2.35) cont'd.
From ⑤ we obtain $\boxed{h_{11} = 1/y_{11}}$ --⑨
From ⑥ and ⑨ we obtain $\boxed{h_{12} = -y_{12}/y_{11}}$ --⑩ and
From ⑨ and ⑦ $\boxed{h_{21} = y_{21}/y_{11}}$ --⑪ and from ⑨, ⑩, ⑪, ⑧
$h_{22} = y_{22} + (-y_{12}/y_{11})(y_{22}/y_{11})(y_{11})$ from which
$\boxed{h_{22} = y_{22} - y_{12}y_{21}/y_{11}}$ --⑫

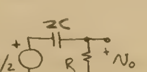
(2.36)
$h_{11} = V_1/I_1|_{V_2=0} = \frac{26mV}{0.01mA} = 2600\,\Omega$
$h_{12} = V_1/V_2|_{I_1=0} = \frac{2.5\times10^{-3}}{10} = 2.5\times10^{-4}$
$h_{21} = I_2/I_1|_{V_2=0} = \frac{1.0\,mA}{0.01\,mA} = 100$
$h_{22} = I_2/V_2|_{I_1=0} = \frac{0.1\,mA}{10\,V} = 10^{-5}$℧

(2.37) $\tau = 1\mu F\times(2k+2k) = 10^{-6}\times4\times10^{3} = 4$ms



At $t = 0$, the capacitor acts as a short circuit; thus a 5V step appears at the output. For $t > 0$, the capacitor charges and the output voltage decays exponentially to zero with a time constant of 4ms. Thus
$v_o = 5e^{-t/4}$ for t in ms.
If the time to recover from all but 5% of the initial change is denoted T, then $0.05\times5 = 5e^{-T/4}$ for which $T \simeq 12$ ms

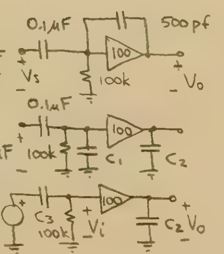
(2.38) Applying Thevenins theorem on the capacitance divider, get →



The circuit is high pass with a time constant of (2CR).

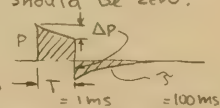
(2.39) $\phi = -\tan^{-1}(w/w_o)$
At $w = 0.1\,w_o$, $\phi = -\tan^{-1}0.1 = -5.7°$
At $w = 10\,w_o$, $\phi = -\tan^{-1}10 = -84.3°$

(2.43) Using Miller's theorem :



$C_1 = 500(1-100) \approx -0.05\mu F$
$C_2 = 500(1-1/100) \approx 500$ pF
Now use Thevenin's theorem to replace the capacitive divider : $C_3 = 0.1 - 0.05 = 0.05\mu F$
$V_s' = \frac{0.1}{0.1-0.05}V_s = 2V_s$
Now since $V_o = 100\,V_i$, the response is that of the input STC high pass
High frequency gain $= V_o/V_s|_{w\to\infty} = 2\times100 = 200 \equiv 46$ dB
$\tau = .05\times10^{-6}\times100\times10^{3} = 5$ms
$w_o = 1/\tau = 200$ rad/s
3dB frequency $f_o = \frac{200}{2\pi} = 31.83$ Hz ≈ 32 Hz
Since 1Hz lies 5 octaves below, the gain at 1Hz will be $46 - 5(6) = 16$dB

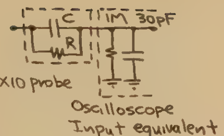
(2.44) $\Delta P \simeq P(T/\tau) = 10\times1/100 = 0.1$mA for $P = 10$mA
Since the network is of the high pass type the average value of the output should be zero: Thus the hatched positive and negative areas should be equal.



The positive area (for small droop) is approximately $= P\times T = 10\times10^{-3}\times10^{-3}$, a charge of 10^{-5}coulombs. Thus the charge that flows backward must also be 10^{-5}C.
Alternatively, its value may be calculated from the fact that the recovery is exponential and initially ΔP. Thus its area is $\Delta P\times\tau = 0.1\times10^{-3}\times100\times10^{-3} = 10^{-5}$coulombs

(2.45) At very low frequencies ($\omega \to 0$) a resistive divider is formed from R and $1M\Omega$.

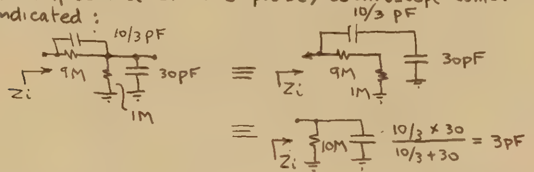
Thus $\frac{1M}{R+1M} = 0.1 \rightarrow R = 9M\Omega$



At very high frequencies the circuit reduces to a capacitance divider incorporating C and 30pF

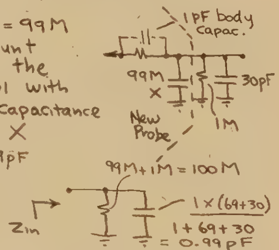
Thus $\frac{C}{C+30} = 0.1 \rightarrow C = 10/3\ pF$;

From the results of Examples 2.9, 2.10, 2.12, we can see that the circuit will provide a transfer function whose magnitude is 0.1 at all frequencies. Using the techniques of these examples one can find the input impedance of the probe/oscilloscope comb. as indicated :



(2.46) Design: $\frac{1M}{R+1M} = 0.01 \rightarrow R = 99M$ which brings with it a shunt capacitance of 1pF. To make the capacitance divider also 0.01 with 1pF in the probe, the scope capacitance must be increase by a shunt X

Thus $\frac{1}{1+30+X} = 0.01 \rightarrow X = 69pF$



(2.47) $i_o = 3e^{-t/\tau}$ mA for $\tau = CR = 100 \times 10^{-12} \times 10^{3} = 100$ ns.

(2.48) $i_o = 2e^{-t/\tau}$ mA for $\tau = L/R = \frac{10 \times 10^{-6}}{2 \times 10^{3}} = 5$ ns.

(2.49) $v_o = -20mA \times 2k\Omega\ e^{-t/\tau} = -40e^{-t/\tau}$ volts

for $\tau = L/R = \frac{10 \times 10^{-6}}{2 \times 10^{3}} = 5$ ns.

(2.50) The fastest rise time that can be observed is that characteristic of the oscilloscope amplifier acting as a low pass STC network with cutoff at 50 MHz, for which the rise time

$t_r \simeq 2.2\tau = 2.2/\omega_o = \frac{2.2}{2\pi \times 50 \times 10^{6}} = 7$ ns.

(2.51) $t_r = \sqrt{t_p^2 + t_n^2}$: Let t_p be the longer, $t_p = kt_n$

Thus $t_r = t_p\sqrt{1 + 1/k^2}$

For t_r to be dominated to within 90% by t_p, $t_p = 0.9t_r$ in which case $k \approx 2$. Thus if one of the rise times is at least twice the other, the shorter will contribute at most 10% of the total.

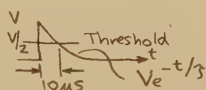
Now to reproduce pulses with rise times of 100 ns, the amplifier rise time must be at least twice as fast, namely 50ns (or less), for which the rise time observed will be 111.8 ns, an error of about 12%.

For a rise time of 50ns or less, the amplifier bandwidth must be at least f_o where :

$f_o = 0.35/t_r = \frac{0.35}{50 \times 10^{-9}} = 7$ MHz

(2.52) $V/2 = Ve^{-10/\tau}$

$\tau = 14.43\ \mu s$



(2.53) The greatest sag occurs with the longest pulse, ie the 10 ms wide pulse. The decay of this pulse is described by $v = Pe^{-t/\tau}$

For maximum sag of P/4 the time constant should satisfy

$\frac{3}{4}P = Pe^{-10/\tau}$ for which $\tau = 34.8$ ms

This corresponds to a lower 3dB frequency f_o of

$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\tau} = 4.6$ Hz

If the approximate sag formula were used, we would have chosen τ from $\frac{P/4}{P} = \frac{10ms}{\tau}$ for which $\tau = 40$ ms.

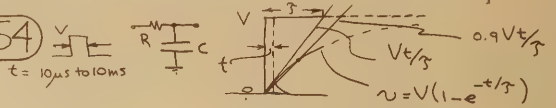
(Note that this would have been a more conservative design)

The resulting sag, ΔP, can be calculated from

$P - \Delta P = Pe^{-10/40}$ for which $\Delta P/P = 1 - e^{-10/40} = 0.22$

(which is less than the 0.25 specified)

(2.54)



The greatest error occurs for the longest pulse t and reaches 10% when $0.9Vt/\tau = V(1-e^{-t/\tau})$ or for $x = t/\tau$

when $0.9x = (1-e^{-x})$ or using the first four terms of the exponential series, when $0.9x = (1-(1-x+x^2/2-x^3/6))$

or $0.1x = x^2/2 - x^3/6$

or $x^2 - 3x + 0.6 = 0$ from which x is 0.2154 or 2.78 (not possible)

Now for $x = .215$ at $t = 10$ms, $\tau = 46.5$ ms

At 10 μs, the output voltage is $V(1-e^{-10^{-3}/46.5}) = 0.215 \times 10^{-3}$ V

At 10 ms, the output voltage is $V(1-e^{-10/46.5}) = 0.1935$ V

(Note that the larger output is 0.9×10^{3} times the smaller)

(2.55) Percent sag $= T/\tau \times 100$

Thus $1 = (1ms/\tau)100$ or $\tau = 100$ ms

But $\tau = C(2k\Omega + 3k\Omega)$

Thus $C = \frac{100 \times 10^{-3}}{5 \times 10^{3}} = 20\mu F$

and $f_o = \omega_o/2\pi = \frac{1}{2\pi\tau} = \frac{1}{2\pi \times 0.1} = 1.6$ Hz

(2.56) Per unit droop is $\frac{T/2}{\tau}$ where T is the period of the square wave and $\tau = 1/\omega_o = \frac{1}{2\pi f_o} = \frac{1}{2\pi \times 20} \approx 8$ms

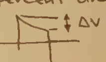
For 10% droop, $T = 2 \times 8 \times 0.1 = 1.6$ ms

Thus the frequency of the test square wave is

$f = 1/T = \frac{1}{1.6 \times 10^{-3}} = 625$ Hz

If a square wave of 20 Hz is used the percent droop observed is calculated as:

$\Delta V/V = 1 - e^{-25/8} = 95.6\%$



(2.57) At $t = t_p = 1$ms, the output is $10e^{-1/1} = 3.68$ V

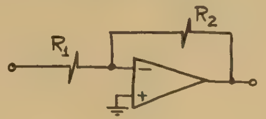
Thus the undershoot must be $10 - 3.68 = 6.32$ V

For the undershoot to be ≤ 1V, then the time constant should be as obtained from

$9 = 10e^{-1/\tau}$ for which $\tau = 9.49$ ms

CHAPTER 3—EXERCISES

3.1 $R_{in} = R_1$

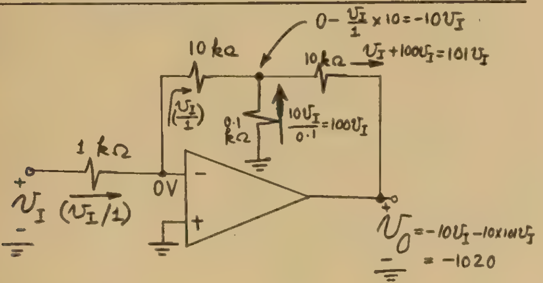


Thus $R_1 = \underline{1\ k\Omega}$

$\text{Gain} = -\frac{R_2}{R_1} = -100$

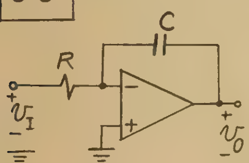
Thus $R_2 = \underline{100\ k\Omega}$

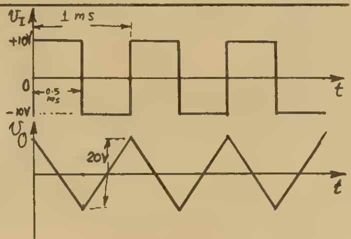
3.2



$\frac{V_O}{V_I} = \underline{-1020}$

3.3





Consider a half period during which the input is at +10V. The capacitor will be supplied with a constant current of (10/R) Amps. Thus the charge transferred to the capacitor will be (10/R)×(T/2), where T is the period of the square wave. This charge causes the capacitor voltage, and hence the output voltage, to change by 20 V. Therefore we can write:

$$\frac{10}{R} \times \frac{1\ \text{ms}}{2} = C \times 20\text{V}$$

and obtain

$CR = \underline{250\ \mu s}$

3.4 $\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2}$

where $Y_1 = (1/1\ k\Omega)$,

and $Y_2 = \frac{1}{100\ k\Omega} + j\omega \times 100 \times 10^{-12}$

Thus, $\frac{V_o}{V_i} = \frac{-10^{-3}}{10^{-5} + j\omega \times 10^{-10}} = \frac{-100}{1 + j(\omega/10^5)}$

which is of the form of the transfer function of an STC low-pass network having a dc gain of $\underline{-100}$ and a corner (3-dB) frequency of $\underline{10^5\ \text{rad/s}}$.

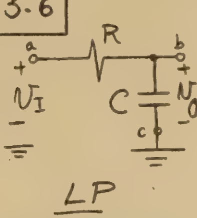
3.5 Superposition:

(a) Set $V_2 = 0$; $V_{O1} = V_1 \frac{3}{3+2}\left(1+\frac{9}{1}\right) = 6V_1$

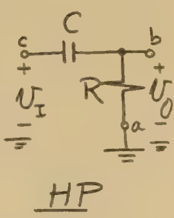
(b) Set $V_1 = 0$; $V_{O2} = V_2 \frac{2}{3+2}\left(1+\frac{9}{1}\right) = 4V_2$

Thus $V_O = \underline{6V_1 + 4V_2}$

3.6



By interchanging the input terminal with ground (which is the complementary transformation) we obtain ⟹



3.7 Choose $R_1 = R_3$ and $R_2 = R_4$

$R_{in} = 2R_1 = 4\ k\Omega \longrightarrow R_1 = \underline{2\ k\Omega}$

$\text{Differential Gain} = \frac{R_2}{R_1} = 100 \longrightarrow R_2 = \underline{200\ k\Omega}$

3.8 The voltage at the positive input terminal, V_+, is obtained using the voltage divider rule as follows:

$$V_+ = V_i \frac{20 \times 10^3}{20 \times 10^3 + \frac{1}{j\omega \times 0.01 \times 10^{-6}}}$$

Since the op amp is ideal, $V_- = V_+$. Now the current I through the pair of 10 kΩ resistors can be found from

$$I = \frac{V_i - V_-}{10^4} = -V_i \frac{j(10^4/\omega)}{20 \times 10^3 - j(10^8/\omega)}$$

Finally, V_o can be obtained from

$$V_o = V_- - 10 \times 10^3\, I$$

$$= V_i \frac{20 \times 10^3}{20 \times 10^3 - j(10^8/\omega)} + V_i \frac{j(10^8/\omega)}{20 \times 10^3 - j(10^8/\omega)}$$

Thus the transfer function is given by

$$\frac{V_o}{V_i} = \frac{20 \times 10^3 + j(10^8/\omega)}{20 \times 10^3 - j(10^8/\omega)}$$

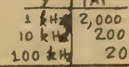
For any ω, including $\omega = 5{,}000$ rad/s, we see that $\underline{\left|\frac{V_o}{V_i}\right| = 1}$. The phase is given by

$$\phi(\omega) = 2\tan^{-1}\left(10^8 / 20 \times 10^3 \times \omega\right)$$

$$\phi(5{,}000\ \text{rad/s}) = 2\tan^{-1} = +90^\circ$$

3.9 $|A| \simeq f_t / f = 2\ \text{MHz}/f$

Thus:



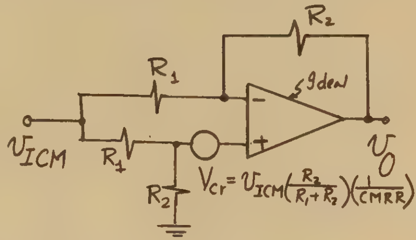
3.10 $f_{3dB} = f_t / \left(1 + \frac{R_2}{R_1}\right) = \frac{2\ \text{MHz}}{1+99} = \underline{20\ \text{kHz}}$

$$t_r \simeq 2.2\tau = \frac{2.2}{\omega_{3dB}} = \frac{2.2}{2\pi f_{3dB}} = \underline{17.5\ \mu s}$$

3.11

Using Superposition:

* v_{ICM} gives rise to $v_{O1} = 0$



* V_{cr} gives rise to

$$v_{O2} = V_{cr}\left(1 + \frac{R_2}{R_1}\right) = v_{ICM}\left(\frac{R_2}{R_1}\right)\left(\frac{1}{CMRR}\right)$$

Thus, $v_O = v_{O1} + v_{O2} = v_{ICM}\left(\frac{R_2}{R_1}\right)\left(\frac{1}{CMRR}\right)$

Common-Mode Gain $= \left(\frac{R_2}{R_1}\right)\left(\frac{1}{CMRR}\right)$

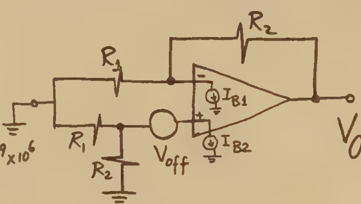
$$= 10^3 \times \frac{1}{10^4} = \underline{0.1\ V/V}$$

3.12

$V_O = V_{off}\left(1 + \frac{R_2}{R_1}\right) + I_{off} R_2$

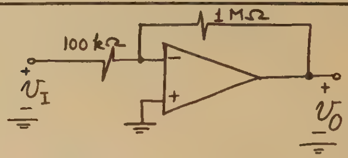
$= 3\times10^{-3} \times 101 + 50\times10^{-9}\times10^6$

$= \underline{353\ mV}$

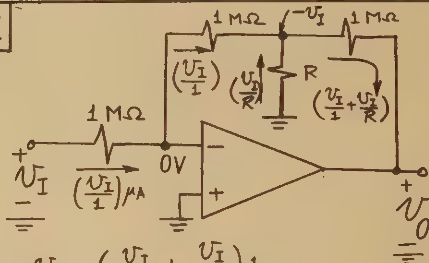


CHAPTER 3—PROBLEMS

3.1



3.2



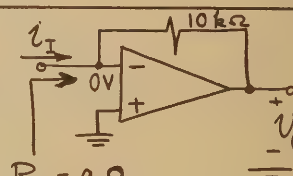
$$v_O = -v_I - \left(\frac{v_I}{1} + \frac{v_I}{R}\right)1$$

$$= -2v_I - \frac{v_I}{R}$$

$$\frac{v_O}{v_I} = -2 - \frac{1}{R} = 100 \longrightarrow \underline{\underline{R = 10.2\ k\Omega}}$$

3.3

$$v_O = -i_I \times 10\,k\Omega$$



$R_{in} = 0\ \Omega$

3.13 $\omega_0 = 2\pi f_0 = 2\pi \times 10 = \frac{1}{C R_1}$ (1)

For $f \gg f_0$, Gain $\simeq -R_2/R_1 = -100$ (2)

For $f \gg f_0$, Input Resistance $\simeq \underline{R_1 = 1\ k\Omega}$ (3)

Using Eqs. (1), (2) & (3) we obtain

$\underline{\underline{R_2 = 100\ k\Omega}}$ and $\underline{\underline{C = 15.9\ \mu F}}$

The response is that of a high-pass STC network with a high-frequency gain of 100 and a corner (3-dB) frequency of 10 Hz. Thus the gain $G(\omega)$ is given by

$$G(\omega) = \frac{-100}{1 - j(f_0/f)}$$

$$|G| = \frac{100}{\sqrt{1 + (f_0/f)^2}}$$

and $\phi = 180° + \tan^{-1}(f_0/f)$

At $f = 100$ Hz we obtain

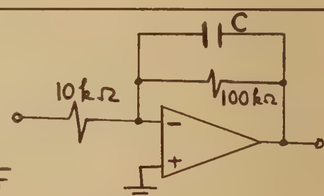
$|G| = 99.5$

& $\phi = 180° + 5.7°$

3.4

$C \times 100\times10^3 = \frac{1}{10^5}$

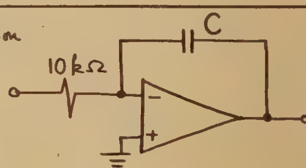
$C = 10^{-10}\ F = 100\ pF$



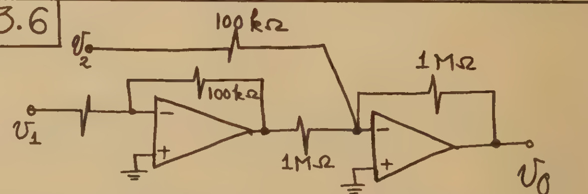
3.5 Refer to the solution of Exercise 3.3.

$\frac{1V}{10k\Omega} \times \frac{1\ ms}{2} = C \times 2V$

$C = 25\times10^{-9} = \underline{25\ nF}$



3.6



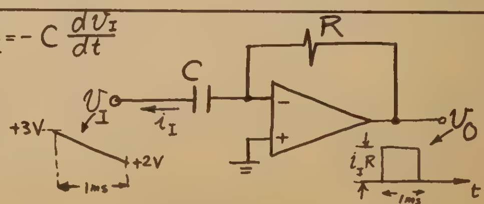
3.7 $i_I = -C\frac{dv_I}{dt}$

Thus

$10^{-6} = -C \times \frac{-1}{10^{-3}}$

$C = 10^{-9}\ F$

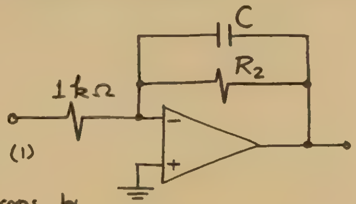
$= \underline{\underline{1\ nF}}$



Amplitude of output pulse = 1 V = $i_I R = 10^{-6} \times R$

Thus, $\underline{\underline{R = 1\,M\Omega}}$

3.8



$$CR_2 = \frac{1}{\omega_{3dB}} = \frac{1}{100} = 0.01\ \text{s} \quad (1)$$

Since the gain drops by 20 dB/decade and it reaches unity in two decades ($\omega_{3dB} = 100$ rad/s, $\omega_{unity\text{-}gain} = 10^4$ rad/s) then the low-frequency gain must be 40 dB, i.e. 100 V/V. Thus

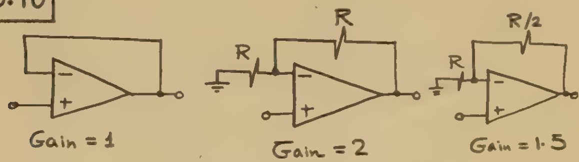
$$\frac{R_2}{R_1} = 100 \longrightarrow \underline{R_2 = 100\ k\Omega}$$

Now using Eq. (1) yields

$$C = \frac{0.01}{10^5} = \underline{\underline{0.1\ \mu F}}$$

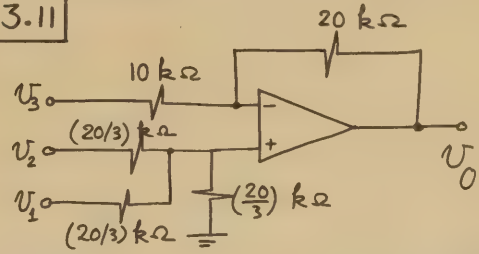
At 10 rad/s the gain is nearly $\underline{\underline{40\,dB}}$

3.10

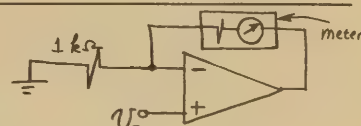


$R = 10\ k\Omega$

3.11

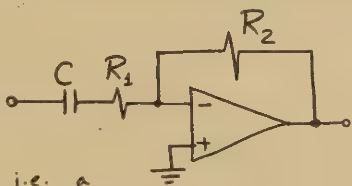


3.12



Since the current through the meter movement is independent of its resistance, nothing need to be done when the meter movement is changed to one with a 1-kΩ resistance.

3.9



$$CR_1 = \frac{1}{\omega_0} = 10^{-2} \quad (1)$$

For unity gain to be at 10 rad/s, i.e. a decade lower than ω_0, the high-frequency gain must be 20 dB (Because the gain drops at 20 dB/decade). Since at high frequency the gain $= -\frac{R_2}{R_1}$ it follows that

$$R_2 = 10\ R_1 \quad (2)$$

Resistive component of input impedance $= R_1$

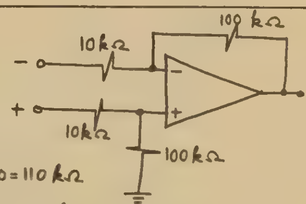
Thus, $\underline{\underline{R_1 = 10\,k\Omega}}$

From (2), $\underline{\underline{R_2 = 100\ k\Omega}}$

& From (1), $C = \frac{10^{-2}}{10^4} = \underline{\underline{1\ \mu F}}$

Since $\omega = 10^3$ rad/s is a decade higher than ω_{3dB} (ω_0) then the gain at $\omega = 10^3$ rad/s is very nearly equal to $\underline{\underline{20\,dB}}$.

3.13



At the positive input terminal of the closed-loop amplifier: $R_{in} = 10 + 100 = 110\ k\Omega$

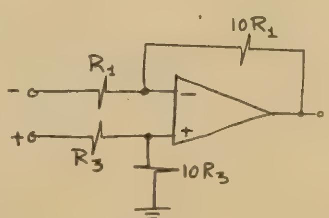
At the negative input terminal: $R_{in} = 10\ k\Omega$

Revised Design

$R_1 + R_3 = 20\ k\Omega \quad (1)$

$R_3 + 10R_3 = R_1 \quad (2)$

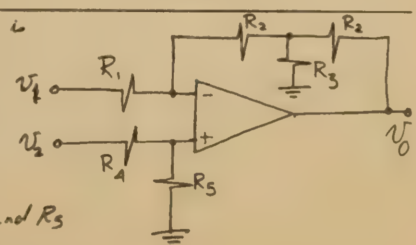
Solving (1) and (2) yields:



$R_1 = \underline{\underline{\frac{55}{3}}}\ k\Omega$ and $R_3 = \underline{\underline{\frac{5}{3}}}\ k\Omega$

3.14

The circuit is as shown. It is desired to find the values for R_1, R_2, R_3, R_4 and R_5 so that:



$$v_o = 10\,(v_2 - v_1)$$

and the input resistance R_{in},

$$R_{in} = R_1 + R_4 = 2\,M\Omega$$

and the largest resistance used is $1\,M\Omega$.

The gain from v_1 to v_o can be found by setting v_2 to zero to be,

$$\frac{v_o}{v_1} = -\left(\frac{R_2}{R_1}\right)\left(2 + \frac{R_2}{R_3}\right)$$

Thus, our first design constraint is

$$\left(\frac{R_2}{R_1}\right)\left(2 + \frac{R_2}{R_3}\right) = 10 \qquad (1)$$

The gain from v_2 to v_o can be found by setting $v_1 = 0$ to be,

$$\frac{v_o}{v_2} = \left(\frac{R_5}{R_4+R_5}\right)\left[1 + \frac{2R_2}{R_1} + \frac{R_2}{R_3}\left(1 + \frac{R_2}{R_1}\right)\right]$$

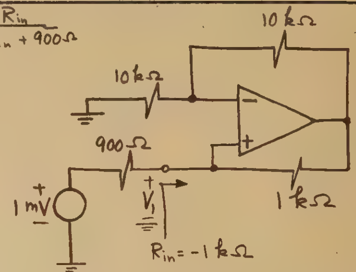
Thus, our second design constraint is

$$\frac{1}{1+\left(\frac{R_4}{R_5}\right)}\left[1 + 2\left(\frac{R_2}{R_1}\right) + \left(\frac{R_2}{R_3}\right) + \left(\frac{R_2}{R_1}\right)\left(\frac{R_2}{R_3}\right)\right] = 10 \qquad (2)$$

In order to obtain $R_{in} = R_1 + R_4 = 2\,M\Omega$ we shall select $R_1 = R_4 = 1\,M\Omega$. Furthermore to avoid any resistance greater than $1\,M\Omega$ we shall select $R_5 = 1\,M\Omega$. Substituting for $\frac{R_4}{R_5} = 1$ in equation (2) and solving the resulting equation together with equation (1) results in $R_2 = \frac{10}{11}\,M\Omega = 0.909\,M\Omega$ and $R_3 = \frac{10}{99}\,M\Omega = 0.101\,M\Omega$. Thus the complete design is

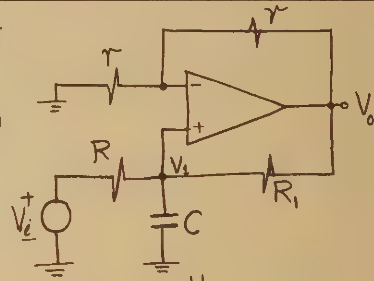
$R_1 = R_4 = R_5 = \underline{\underline{1\,M\Omega}}$, $R_2 = \underline{\underline{909\,k\Omega}}$, and $R_3 = \underline{\underline{101\,k\Omega}}$.

3.15 $V_1 = 1\,mV \times \dfrac{R_{in}}{R_{in} + 900\,\Omega}$



$$V_1 = 1 \times \frac{-1000}{-1000 + 900} = 10\,mV$$

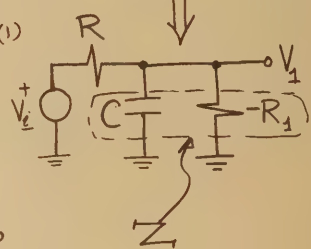
3.16 $\dfrac{V_1}{V_i} = \dfrac{Z}{Z + R}$



$$\frac{V_1}{V_i} = \frac{1}{1 + R\left(j\omega C - \frac{1}{R_1}\right)} = \frac{1}{j\omega CR + \left(1 - \frac{R}{R_1}\right)}$$

$$\frac{V_o}{V_1} = 2$$

$$\frac{V_o}{V_i} = \frac{2}{\left(1 - \frac{R}{R_1}\right) + j\omega CR} \qquad (1)$$



Low-Frequency Gain

$$= \frac{2}{1 - \frac{R}{R_1}}$$

For a gain of +20,

$$\frac{R}{R_1} = 0.9 \qquad (2)$$

High-Frequency input resistance $= \underline{\underline{R = 100\,k\Omega}}$

Substituting in (2) we obtain

$$R_1 = \frac{100}{0.9} = \underline{\underline{111.1\,k\Omega}}$$

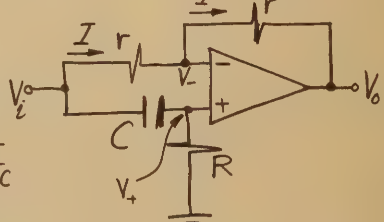
From Eq. (1)

$$\frac{V_o}{V_i} = \frac{20}{1 + j\omega CR \times 10}$$

$$\omega_{3dB} = \frac{1}{10\,CR} = 2\pi \times 1591$$

$$C = \frac{1}{10 \times 2\pi \times 1591 \times 100\times10^3} = \underline{\underline{100\,pF}}$$

3.17



$$V_+ = V_i\,\frac{R}{R + \frac{1}{j\omega C}}$$

$$V_- = V_+ = V_i\,\frac{R}{R + \frac{1}{j\omega C}}$$

$$I = (V_i - V_-)/r = \frac{V_i}{r}\,\frac{1/j\omega C}{R + \frac{1}{j\omega C}}$$

$$V_o = V_- - Ir = V_i\,\frac{R}{R + \frac{1}{j\omega C}} - V_i\,\frac{1/j\omega C}{R + \frac{1}{j\omega C}}$$

$$\frac{V_o}{V_i} = \frac{R - \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = -\frac{1 - j\omega CR}{1 + j\omega CR}$$

Thus $\left|\frac{V_o}{V_i}\right| = 1$ for all frequencies, and

$$\phi\Big|_{\omega \to 0} = 180^\circ$$

$$\phi\Big|_{\omega \to \infty} = 0^\circ$$

$$\phi = 180 - 2\tan^{-1}(\omega CR)$$

Thus $\phi = 90^\circ$ at $\omega = \frac{1}{CR}$

$$CR = \frac{1}{10^3} = \underline{\underline{10^{-3}\,s}}$$

At very low frequencies, C acts as open circuit and $R_{in} = r$. Thus $\underline{\underline{r = 50\,k\Omega}}$. At very high frequencies C acts as short circuit and $V_+ = V_i$; thus $R_{in} = R$. Thus $\underline{\underline{R = 50\,k\Omega}}$

Then $C = \frac{10^{-3}}{50 \times 10^3} = \underline{\underline{0.02\,\mu F}}$.

The input admittance Y_{in} is given by

$$Y_{in} = \frac{1}{R + \frac{1}{j\omega C}} + \frac{I}{V_i} = \frac{1}{R + \frac{1}{j\omega C}} + \frac{1/j\omega Cr}{R + \frac{1}{j\omega C}}$$

$$Y_{in} = \frac{1 + \frac{1}{j\omega Cr}}{R + \frac{1}{j\omega C}}$$

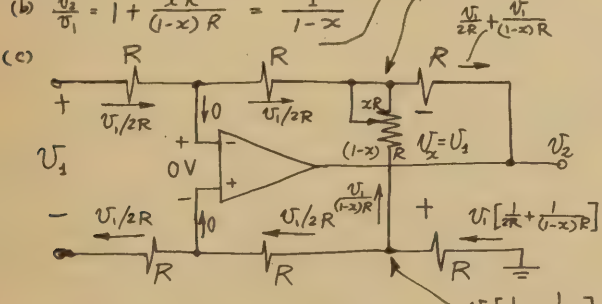
But $r = R$; thus $Y_{in} = \frac{1}{R}$.

Thus at all frequencies $Z_{in} = 50\,k\Omega$

$\boxed{3.18}$ (a) $\frac{v_2}{v_1} = \frac{-xR}{(1-x)R} = -\frac{x}{1-x}$

(b) $\frac{v_2}{v_1} = 1 + \frac{xR}{(1-x)R} = \frac{1}{1-x}$

(c)

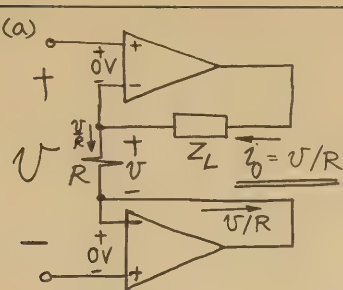


See analysis details on diagram.

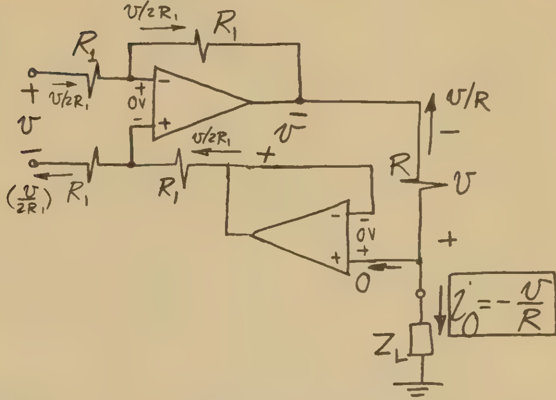
$$v_2 = -v_1\left(\frac{1}{2} + \frac{1}{1-x}\right) - v_1 - v_1\left(\frac{1}{2} + \frac{1}{1-x}\right)$$

$$\frac{v_2}{v_1} = -\left[2 + \frac{2}{1-x}\right] = \underline{-2\,\frac{2-x}{1-x}}$$

$\boxed{3.19}$ (a)



(b)



$\boxed{3.20}$ $f_b = \frac{f_t}{A_0} = \frac{10^6}{10^6} = \underline{\underline{1\text{ Hz}}}$

For an amplifier with gain of −100: $f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}}$

Thus, $f_{3dB} = \frac{10^6}{101} \simeq \underline{\underline{10^4\text{ Hz}}}$

$$\phi = -\tan^{-1}\left(\frac{f}{f_{3dB}}\right)$$

f, Hz	ϕ
10^3	-5.7°
10^4	-45°
10^5	-84.3°
10^6	-89.4°

$\boxed{3.21}$ Design #1

$$f_{3dB} = \frac{f_t}{100} = \frac{10^5}{10^2} = 10^3\text{ Hz}$$

Design #2

There are many possible circuits for using two op amps to achieve a gain of +100. We shall consider the case of two identical stages in cascade, each with a gain of +10. Each will have a 3-dB frequency of 10^4 Hz. The overall gain function will be

$$G = \left[\frac{10}{1 + j\frac{f}{10^4}}\right]^2$$

Thus

$$|G| = \frac{100}{1+\left(\frac{f}{10^4}\right)^2}$$

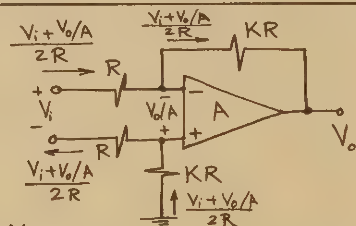
The 3-dB frequency of this function is obtained from

$$1+\left(\frac{f_{3dB}}{10^4}\right)^2 = \sqrt{2}$$

Thus, $f_{3dB} = 6.44 \times 10^3$ Hz

We conclude that Design #2 has a much wider (by a factor of 6.44) bandwidth than Design #1.

3.22



$$V_o = -\frac{V_i+V_o/A}{2R}\cdot KR - \frac{V_o}{A} - \frac{V_i+V_o/A}{2R}\cdot KR$$

$$= -K\left(V_i+\frac{V_o}{A}\right)-\frac{V_o}{A}$$

$$V_o\left(1+\frac{1+K}{A}\right) = -KV_i$$

$$\frac{V_o}{V_i} = \frac{-K}{1+(1+K)/A}$$

$$A \simeq f_t/jf$$

$$\frac{V_o}{V_i} = \frac{-K}{1+(1+K)j\frac{f}{f_t}} = \frac{-K}{1+j\frac{f}{f_t/(1+K)}}$$

Thus

$$\underline{\underline{f_{3dB} = f_t/(1+K)}}$$

3.23

$$G = \frac{-R_2/R_1}{1+\frac{1+R_2/R_1}{A}}$$

For, $A = \dfrac{A_0}{1+j\frac{\omega}{\omega_b}} \Rightarrow \dfrac{1}{A} = \dfrac{1}{A_0}+j\dfrac{\omega}{\omega_t}$

Thus, $$G(\omega) = \frac{-R_2/R_1}{1+\frac{1+\frac{R_2}{R_1}}{A_0}+j\frac{\omega}{\omega_t/(1+\frac{R_2}{R_1})}}$$

$\frac{R_2}{R_1} = 10 \qquad G(0) = \dfrac{-10}{1+\frac{11}{A_0}}$

Thus, $\underline{A_0 = 209}$ V/V

$$t_r \simeq 2.2\tau = \frac{2.2}{\omega_{3dB}} = \frac{2.2}{\omega_t/11} = \frac{2.2\times 11}{\omega_t}$$

For $t_r = 10\,\mu s$, $\omega_t \doteq (2.2\times 11)/10^{-5} = 2.42\times 10^6$ rad/s

$$\omega_b = \frac{\omega_t}{A_0} = \frac{2.42\times 10^6}{209} = 1.16\times 10^4 \text{ rad/s}$$

$$f_b = \omega_b/2\pi = \underline{1.84 \text{ kHz}}$$

3.24 The amplifier output begins to limit when the amplitude of the input sine-wave signal is $\frac{10\,V}{10^5} = 10^{-4}\,V = 0.1$ mV.

When the amplitude of the input sine wave is much greater than 10^{-4} V the output will be limited to ±10V. The slope of the rising and falling edges of the output waveform will be the smaller of the slew rate (10V/μs) and the slope of the tangent to the sine waveform at its zero crossing. If the input amplitude is too large then the slew rate of the op amp will detremine the rise and fall times of the output. Thus the time for the output to go from −10V to +10V will be 2μs. This is the "best" possible (i.e. has fastest rising and falling edges) square wave. It can be obtained for input sine waves whose amplitudes are greater than a minimum value V_p determined from:

$$\omega \times 10^5 \times V_p = 10^7 \text{ V/s}$$

$$V_p = \frac{10^7}{2\pi\times 10^4\times 10^5} = \underline{\underline{1.6 \text{ mV}}}$$

Sine-wave inputs with amplitudes smaller than 1.6 mV will give rise to square wave outputs with slower edges (than the fastest possible of 10^7 V/s or 2 μs rise and fall times).

3.25 small-signal bandwidth $= f_t/(1+\frac{R_2}{R_1})$

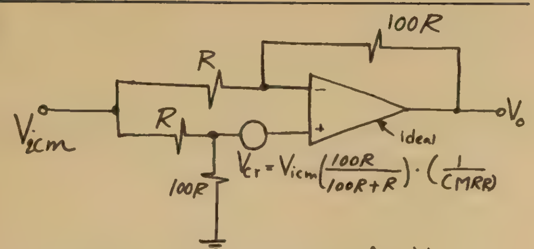
full-power bandwidth $= \dfrac{SR}{\hat{V}} = \dfrac{10^6}{10} = 10^5$ rad/s

The maximum gain $\left(\frac{R_2}{R_1}\right)$ for which the small-signal bandwidth is equal to the full-power bandwidth is obtained from

$$\frac{2\pi\times 10^6}{1+\frac{R_2}{R_1}} = 10^5$$

$$\frac{R_2}{R_1} = \underline{\underline{61.83}}$$

3.26



Using superposition we can write for V_o:

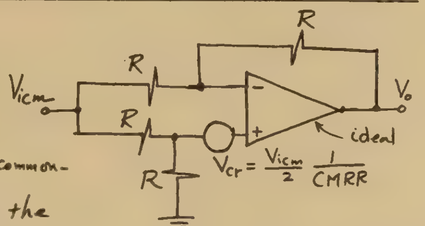
$$V_o = V_{icm} \times 0 + V_{icm}\cdot\frac{100}{101}\cdot\frac{1}{10^3}\left(1+\frac{100R}{R}\right)$$

$$= 0.1\ V_{icm}$$

Thus, the common-mode gain of the closed-loop amplifier is $\underline{\underline{0.1\ V/V}}$.

3.27

As in Problem 3.26 we can calculate the common-mode gain of the closed-loop amplifier using the scheme illustrated in the Figure. Thus



$$V_o = \frac{V_{icm}}{2}\times\frac{1}{CMRR}\times 2$$

From this we obtain

$$CMG = \frac{1}{CMRR} = 10^{-5}\ V/V$$

Amplitude of the 60-Hz common-mode signal at the output $= 10\times10^{-5} = 10^{-4}$ V.

Amplitude of the 1-kHz differential signal at the output $= 0.1\text{ mV}\times 1 = 10^{-4}$ V

Considering the 60 Hz common-mode signal as noise, the signal-to-noise ratio at the output is unity or $\underline{0\text{dB}}$.

3.28

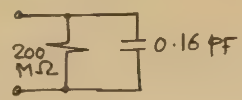
The input impedance can be obtained by substituting in Eq. (3.13) the following:

$R_{icm} = 10^8\Omega$, $R_{id} = 10^6\Omega$, $\frac{R_2}{R_1} = 0$ for the unity-gain case and 9 for the gain-of-10 case, and $A = \frac{\omega_t}{j\omega} = \frac{2\pi\times10^6}{j\omega}$.

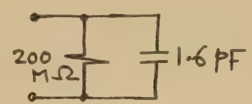
The result is

$$Z_{in} = (2\times10^8\Omega) // \left[\frac{2\pi\times10^6}{j\omega(1+\frac{R_2}{R_1})}\times10^6\right]$$

Thus, for the unity-gain case we have



and for the gain-of-10 case we have

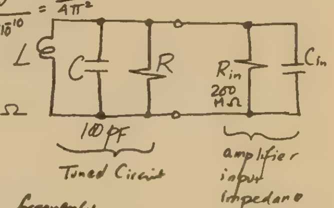


We see that the difference is that the input capacitance of the gain-of-10 amplifier is ten times that of the unity-gain amplifier.

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{\omega_o^2\times10^{-10}} = \frac{1}{(2\pi\times10^4)^2\times10^{-10}} = \frac{100}{4\pi^2}$$

$$\omega_o CR = Q = 1000$$

$$R = \frac{10^3}{2\pi\times10^4\times10^{-10}} = \frac{10^9}{2\pi}\ \Omega$$



With the unity-gain amplifier the measured frequency will be

$$\omega_o' = \frac{1}{\sqrt{L(C+C_{in})}} = \frac{1}{\sqrt{L(100+0.16)\times10^{-12}}}$$

$$= \frac{1}{\sqrt{\frac{100}{4\pi^2}\times100\times10^{-12}(1+0.0016)}} \simeq 2\pi\times10^4(1-0.0008)$$

Thus f_o' will be 0.08% lower than f_o.

With the gain-of-10 amplifier the measured resonance frequency will be 0.8% lower than f_o (10kHz).

In both cases the total resistance across the tuned circuit will be $(R//R_{in})$ and Q will be: $Q' = \omega_o' C(R//R_{in}) \simeq \underline{\underline{557}}$

3.29

From Eq. (3.14), $R_{out} \simeq \frac{R_o}{A\beta}$. Since A is a function of frequency then R_{out} should be relabeled Z_{out},

$$Z_{out} = \frac{R_o}{A\beta}$$

$$A = \frac{A_0}{1+j\frac{\omega}{\omega_b}} \Rightarrow \frac{1}{A} = \frac{1}{A_0} + j\frac{\omega}{\omega_t}$$

$$Z_{out} = \frac{R_o}{A_0\beta} + j\frac{\omega}{\omega_t}\frac{R_o}{\beta}$$

For $A_0 = 10^4$, $R_o = 10^3\ \Omega$, $f_t = 10^6$ Hz, and $\beta = \frac{1}{101} \simeq 0.01$ we have: $Z_{out} = 10 + j\omega\times0.016$.

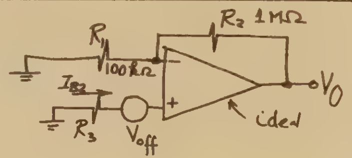
Thus the output impedance is equivalent to a series combination of a 10-Ω resistance and a 16 mH inductance. This impedance appears in parallel with (R_1+R_2).

3.30 * ac component at output has amplitude of $0.1\ mV \times 10 = 1\ mV$.

* dc component at output has magnitude of $1\,mV\left(1+\frac{R_2}{R_1}\right) = 1 \times 11 = 11\ mV$

* Capacitor coupling reduces the dc gain to unity and thus reduces the magnitude of the dc component at the output to 1 mV. Then the two components at the output become: 1 mV dc + ac signal of 1 mV amplitude.

3.31



$$V_O = V_{off}\left(1+\frac{R_2}{R_1}\right) - I_{B2}R_3 + I_{B1}R_2 - \frac{R_2R_3}{R_1}I_{B2}$$

$$I_{B1} = I_B + \frac{I_{off}}{2} \quad \& \quad I_{B2} = I_B - \frac{I_{off}}{2}$$

$$V_O = V_{off}\left(1+\frac{R_2}{R_1}\right) + I_B\left(R_2 - R_3 - \frac{R_2R_3}{R_1}\right) + \frac{I_{off}}{2}\left(R_2+R_3+\frac{R_2R_3}{R_1}\right)$$

Since the polarity of V_{off} and I_{off} are not known, all we can do to minimize V_O is to select R_3 so that the middle term is reduced to zero. This is obtained with

$$+R_2 - R_3 - \frac{R_2R_3}{R_1} = 0 \Rightarrow R_3 = \frac{R_1R_2}{R_1+R_2}$$

For our case, $R_3 = \frac{100\times1000}{100+1000} = \underline{90.9\ k\Omega}$

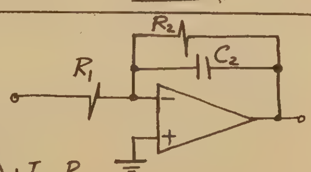
The remaining worst-case output offset voltage will be

$$V_O = V_{off}\left(1+\frac{R_2}{R_1}\right) + I_{off}R_2$$
$$= 2\,(1+10) + 3\times1 = \underline{25\ mV}$$

If R_3 is made zero the offset voltage becomes (in the worst-case)

$$V_O = V_{off}\left(1+\frac{R_2}{R_1}\right) + \left(I_B + \frac{I_{off}}{2}\right)R_2$$
$$= 2\times11 + 21.5\times1 = \underline{43.5\ mV}$$

3.32 $R_1 = \underline{100\ k\Omega}$



$C_2R_1 = 10^{-3}\,s$

$$C_2 = \frac{10^{-3}}{10^5} = \underline{10\,nF}$$

$$200\,mV = V_{off}\left(1+\frac{R_2}{R_1}\right) + I_B R_2$$
$$= 1\left(1+\frac{R_2}{R_1}\right) + 10R_2$$

Thus, $R_2 = \underline{10\,M\Omega}$

3.33 The response of the nonideal integrator will be that of a low-pass STC network with

$$\omega_{3dB} = \frac{1}{C_2R_2} = \frac{1}{10^{-8}\times10^7} = 10\ rad/s.$$

or, equivalently, a time-constant τ,

$$\tau = \frac{1}{\omega_{3dB}} = 0.1\ s.$$

The output waveform, rather than being perfectly triangular as that of an ideal integrator as shown in Fig.(a), will be exponential as indicated in Fig.(b).

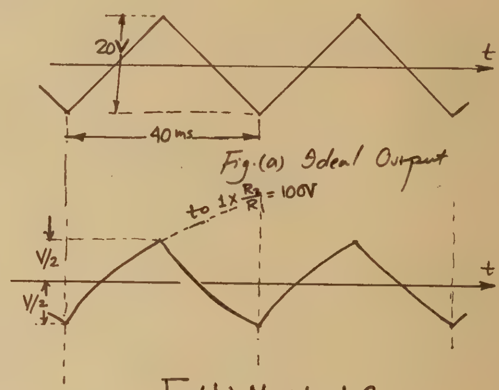


Fig.(a) Ideal Output

Fig.(b) Nonideal Output

The peak-to-peak amplitude of the ideal output waveform can be found from: $IT = I\frac{T}{2} = C\,V_{p\text{-}p}$

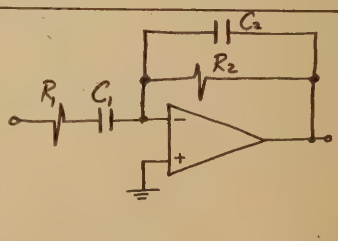
$$\frac{1}{100\,k\Omega}\times 20\,ms = 10^{-8}\times V_{p\text{-}p} \Rightarrow V_{p\text{-}p} = 20V$$

In the nonideal case the peak-to-peak amplitude (V) can be found from

$$v(t) = 100 - \left(100+\frac{V}{2}\right)e^{-t/\tau}$$
$$v\left(\frac{T}{2}\right) = 100 - \left(100+\frac{V}{2}\right)e^{-0.02/0.1} = \frac{V}{2}$$

$$V = \underline{19.93\ volts}$$

3.34 $R_{in}\big|_{\omega\to\infty} = R_1$



Thus, $R_1 = 10\ k\Omega$

Midband gain $= -\frac{R_2}{R_1} = -100$

Thus, $R_2 = 1\,M\Omega$

Lower $f_{3dB} = \frac{1}{2\pi C_1R_1} = 100 \Rightarrow C_1 = 0.159\ \mu F$

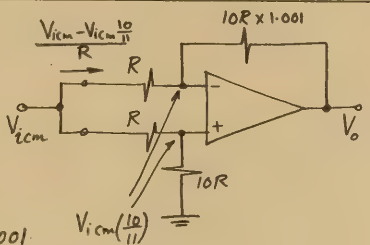
Upper $f_{3dB} = \frac{1}{2\pi C_2R_2} = 10^4 \Rightarrow C_2 = 15.9\ pF$

If C_2 were not present, the finite bandwidth of the op amp would cause the closed-loop amplifier to have a 3dB frequency at $f_t/(1+\frac{R_2}{R_1}) = \frac{f_t}{101}$. To minimize the effect of the op amp frequency response on the response of the bandbass amplifier we select an op amp with an f_t so that

$$\frac{f_t}{101} \geq 10 \times 10^4$$

Thus $f_t \geq \underline{10^7 \text{ Hz}}$

3.35



$V_o = \frac{10}{11}V_{icm} - \frac{V_{icm}}{11R}\times 10R \times 1.001$

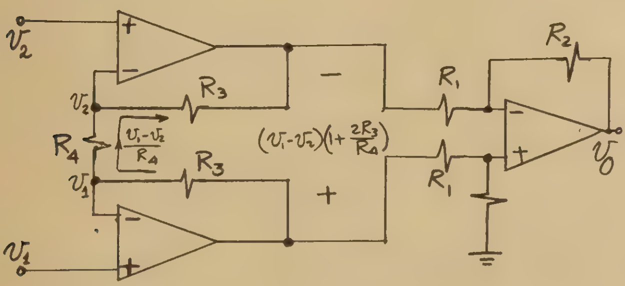
Common-Mode Gain

$$\equiv \frac{V_o}{V_{icm}} = -\frac{10}{11}\times 0.001$$

Differential Gain = −10

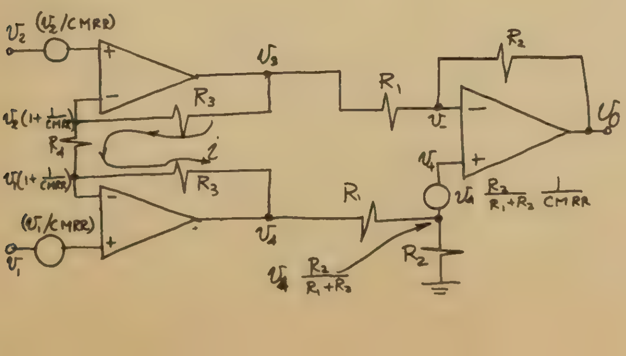
$$CMRR = 20\log\frac{10}{\frac{10}{11}\times 0.001} = \underline{\underline{80.8\ dB}}$$

3.36



$$v_0 = \frac{R_2}{R_1}\left(1+\frac{2R_3}{R_4}\right)(v_1-v_2)$$

3.37



$$i = \frac{(v_2-v_1)}{R_4}\left(1+\frac{1}{CMRR}\right)$$

$$v_3 = v_2\left(1+\frac{1}{CMRR}\right) + (v_2-v_1)\left(1+\frac{1}{CMRR}\right)\left(\frac{R_3}{R_4}\right)$$
$$= \left(1+\frac{1}{CMRR}\right)\left[v_2 + (v_2-v_1)\left(\frac{R_3}{R_4}\right)\right] \quad ....(1)$$

$$v_4 = v_1\left(1+\frac{1}{CMRR}\right) - \frac{v_2-v_1}{R_4}\left(1+\frac{1}{CMRR}\right)R_3$$
$$= \left(1+\frac{1}{CMRR}\right)\left[v_1 - (v_2-v_1)\left(\frac{R_3}{R_4}\right)\right] \quad ....(2)$$

$$v_- = v_+ = v_4\frac{R_2}{R_1+R_2}\left(1+\frac{1}{CMRR}\right)$$

$$v_0 = v_- - \frac{v_3-v_-}{R_1}R_2$$
$$= v_4\frac{R_2}{R_1+R_2}\left(1+\frac{1}{CMRR}\right)\left(1+\frac{R_2}{R_1}\right) - v_3\frac{R_2}{R_1}$$
$$= v_4\left(\frac{R_2}{R_1}\right)\left(1+\frac{1}{CMRR}\right) - v_3\frac{R_2}{R_1}$$

Substituting for v_3 from Eqn.(1) and for v_4 from Eqn. (2) results in

$$v_0 = \left(\frac{R_2}{R_1}\right)\left(1+\frac{1}{CMRR}\right)\left\{\frac{v_1}{CMRR} - (v_2-v_1)\left[1+\frac{2R_3}{R_4}\left(1+\frac{1}{2CMRR}\right)\right]\right\}$$

No letting $v_1 = v_{icm}$ and $v_2 = v_{icm} + v_{id}$ we obtain

$$v_0 = \left(\frac{R_2}{R_1}\right)\left(1+\frac{1}{CMRR}\right)\left\{\frac{v_{icm}}{CMRR} - v_{id}\left[1+\frac{2R_3}{R_4}\left(1+\frac{1}{2CMRR}\right)\right]\right\}$$

Thus,

$$\text{Common-mode Gain} = \left(\frac{R_2}{R_1}\right)\left(1+\frac{1}{CMRR}\right)\frac{1}{CMRR}$$
$$\simeq \left(\frac{R_2}{R_1}\right)\left(\frac{1}{CMRR}\right)$$

and,

$$\text{Differential Gain} = -\frac{R_2}{R_1}\left(1+\frac{1}{CMRR}\right)\left[1+\frac{2R_3}{R_4}\left(1+\frac{1}{2CMRR}\right)\right]$$
$$\simeq -\frac{R_2}{R_1}\left(1+\frac{2R_3}{R_4}\right)$$

The common-mode rejection ratio of the instrumentation amplifier is $\frac{|\text{Diff. Gain}|}{|\text{Common-mode Gain}|}$

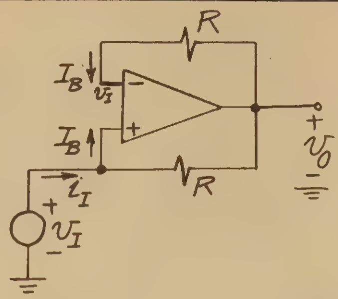
$$= \underline{\underline{CMRR \times \left(1+\frac{2R_3}{R_4}\right)}}$$

which is the required result.

3.38

$v_O = v_I + I_B R$

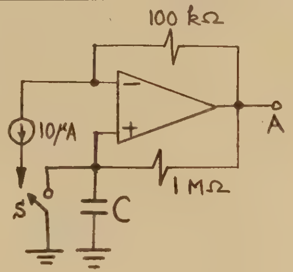
Thus, $i_I = 0$



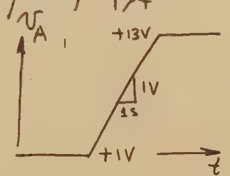
3.39 With S closed,

$v_A = +10\times10^{-6}\times100\times10^{3}$

$= +1\text{ V}.$

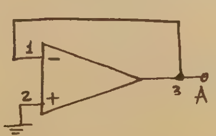


As S opens at $t=0$, capacitor C charges up through the 1 MΩ resistance towards the voltage at A. The positive input terminal of the op amp then rises in voltage and the negative input terminal tracks. The voltage drop across the 100-kΩ resistor remains constant at 1V. Thus the voltage difference between A and the positive input terminal of the op amp remains constant at +1V. In other words, there will be a constant voltage drop across the 1-MΩ resistor. It follows that C will charge up at a constant current of $\frac{1V}{1M\Omega} = 1\mu A$. Thus the voltage across C will be a linear ramp starting from 0V and having a slope of $\frac{1\mu A}{1\mu F} = 1V/s$. The voltage at A will be a linear ramp starting from +1V at $t=0$ and reaching the saturation voltage of +13 V in 12 s.

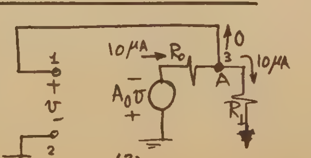


* 10s

3.40



(1)



(2)

With no load, $v_A = 0$. With a current of 10 mA extracted from the output the equivalent circuit of Fig. (2) applies. From this circuit we have,

$v_A = v = -A_0 v - 10\text{ mA}\times R_o$

$v(1+A_0) = -10\times10^{-3}\times R_o$

Substituting $A_0 = 10^3$ and $R_o = 10^3\,\Omega$ results in $v = -\frac{10\times10^{-3}\times10^{3}}{1001} \simeq -10\text{ mV}$

Thus $v_A = -10\text{ mV}$

The equivalent closed-loop output resistance $= \frac{10\text{ mV}}{10\text{ mA}}$

$= \underline{\underline{1\,\Omega}}$

3.41 Since the voltage at A falls, the op-amp bias current flows out of the negative input terminal. If the bias current is I_B then

$\frac{I_B}{C} = 10\text{ mV/s}$

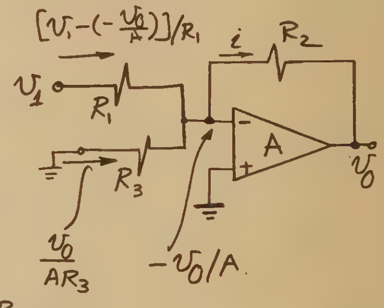
Thus $I_B = 10\times10^{-3}\times10^{-9} = \underline{\underline{10\text{ pA}}}$

3.42

$i = \frac{v_1 + \frac{v_O}{A}}{R_1} + \frac{v_O}{AR_3}$

$v_O = -\frac{v_O}{A} - iR_2$

Thus



$$\frac{v_O}{v_i} = \frac{-R_2/R_1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}+\frac{R_2}{R_3}\right)}$$

At dc $A = A_0$ and the gain becomes

$$\frac{v_O}{v_i} = \frac{\text{Nominal Gain}}{1+\frac{1}{A_0}\left(1+\frac{R_2}{R_1}+\frac{R_2}{R_3}\right)} \qquad (1)$$

Thus the existence of R_3 increases the gain error due to the finite A_0. By varying R_3 the gain error can be varied.

To find the effect of the finite bandwidth of the op amp substitute in (1) $A = \omega_t/j\omega$,

$$\frac{V_o}{V_i}(\omega) = \frac{-R_2/R_1}{1+j\frac{\omega}{\omega_t/(1+\frac{R_2}{R_1}+\frac{R_2}{R_3})}}$$

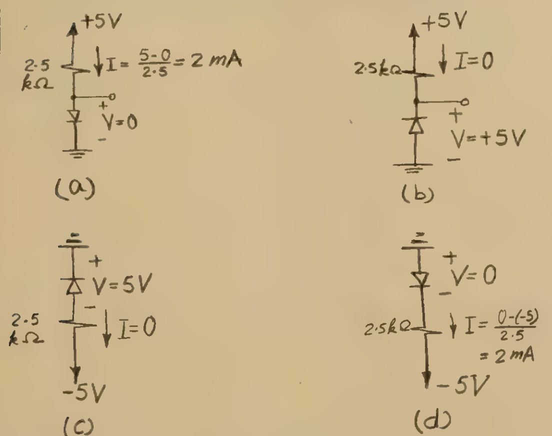
Thus the closed-loop amplifier has a 3-dB frequency ω_{3dB} given by

$$\omega_{3dB} = \frac{\omega_t}{1 + \frac{R_2}{R_1} + \frac{R_2}{R_3}}$$

As R_3 is reduced, ω_{3dB} is reduced. Thus R_3 can be used to change the bandwidth of the inverting amplifier without affecting its nominal low-frequency gain $(-\frac{R_2}{R_1})$, assuming that A_0 is large (as is usually the case).

CHAPTER 4 — EXERCISES

4.1



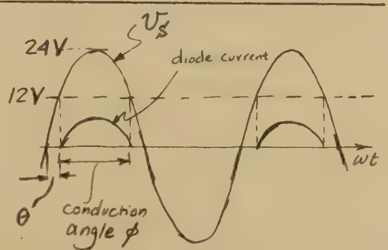
4.2

From the figure:

$\sin\theta = \frac{12}{24} = \frac{1}{2}$

$\theta = 30°$

Thus the diode conducts for $120°/360° = \underline{1/3}$ of every cycle. The peak of the diode current $= \frac{24-12}{100\,\Omega} = \underline{120\text{ mA}}$



4.3 The current through the meter consists of half sinusoids with amplitude of $10\text{ V}/(R+R_m)$ where R_m, the meter resistance, is $50\,\Omega$. The average of this current is $\left[\frac{1}{\pi}\frac{10}{R+R_m}\right]$. To obtain full-scale reading this average current must be equal to the specified 1 mA. Thus R is obtained from

$$\frac{1}{\pi}\,\frac{10}{R+50} = 10^{-3} \longrightarrow \underline{R = 3.133\text{ k}\Omega}$$

4.4

$$i = I_S\, e^{v/nV_T}$$

$$v_2 - v_1 = nV_T \ln\left(\frac{i_2}{i_1}\right)$$

$$= 1.5 \times 25 \ln\left(\frac{10}{0.1}\right) = \underline{172.7\text{ mV}}$$

4.5

$$v_2 - v_1 = nV_T \ln(i_2/i_1)$$

For $i_1 = 1\text{ mA}$, $v_1 = 0.7\text{ V}$

For $i_2 = 0.1\text{ mA}$, $v_2 = 0.7 + 2\times 0.025 \ln\left(\frac{0.1}{1}\right) = \underline{0.58\text{V}}$

For $i_2 = 10\text{ mA}$, $v_2 = 0.7 + 2\times 0.025 \ln\left(\frac{10}{1}\right) = \underline{0.82\text{V}}$

4.6 Since I_S doubles for every 10°C rise in temperature then

$$I_S = \alpha\, 2^{(TEMP/10)}$$

To find the value of α, substitute

$I_S = 10^{-14}\text{ A}$ for $TEMP = 22°C$

$$10^{-14} = \alpha\, 2^{2.2}$$

$$\alpha = 2.2 \times 10^{-15}$$

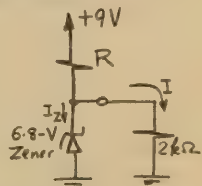
Thus: $\underline{I_S = 2.2 \times 10^{-15} \times 2^{(0.1 \times TEMP)}}$

4.7 Design #1 ($R = 110\,\Omega$)

$$I = \frac{6.8}{2} = 3.4\text{ mA}$$

$$I_Z \simeq \frac{9-6.8}{0.110} - 3.4$$

$$= 20 - 3.4 = 16.6\text{ mA}$$



Thus the zener will still have plenty of current. Its' current has simply decreased by 3.4 mA. Thus its voltage will decrease by

$3.4\text{ mA} \times r_Z = 3.4 \times 5 = \underline{17\text{ mV}}$

Design #2 ($R = 8.8\,k\Omega$)

$I_Z = \frac{9-6.8}{8.8} - 3.4$ = negative value.

Thus the zener will no longer be operating in the zener mode; it will operate as a reverse-biased diode conducting a negligible current. The output voltage will be determined by the voltage divider formed by R ($8.8\,k\Omega$) and the $2\text{-}k\Omega$ load resistance. Thus

$$V_O = 9\,\frac{2}{2+8.8} = 1.67\,V$$

and the change in output voltage is

$$\Delta V_O = 1.67 - 6.8 = \underline{\underline{-5.13\,V}}$$

4.8 (a) We iterate as follows:

$V = 0.7\,V \quad I = \frac{1-0.7}{1\,k\Omega} = 0.3\,mA$

$V = 0.7 + 2 \times 0.025 \times \ln\left(\frac{0.3}{1}\right) = 0.64\,V,\ I = \frac{1-0.64}{1} = 0.36\,mA$

$V = 0.7 + 2 \times 0.025 \times \ln\left(\frac{0.36}{1}\right) = 0.6489\,V,\ I = \frac{1-0.6489}{1} = 0.351\,mA$

$V = 0.7 + 2 \times 0.025 \times \ln\left(\frac{0.351}{1}\right) = \underline{0.6476\,V},\ I = \frac{1-0.6476}{1} = \underline{0.352\,mA}$

No further iterations are warranted.

$V_D = 0.7 + 2 \times 0.025 \times \ln\left(\frac{12.52}{1}\right) = 0.8264\,V$

$V_1 = 2.479\,V$

Thus the change in V_1 is $2.479 - 2.403 = \underline{\underline{76\,mV}}$

<u>With the dc supply voltage at +5V:</u>

$V_D = 0.7\,V \quad V_1 = 2.1\,V \quad I = \frac{5-2.1}{1} = 2.9\,mA$

$V_D = 0.7 + 2 \times 0.025\,\ln\left(\frac{2.9}{1}\right) = 0.7532\,V$

$V_1 = 2.260\,V \quad I = \frac{5-2.260}{1} = 2.74\,mA$

$V_D = 0.7 + 2 \times 0.025\,\ln\frac{2.74}{1} = 0.7503\,V$

$V_1 = 2.251\,V \quad I = \frac{5-2.251}{1} = 2.749\,mA$

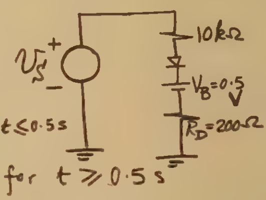
$V_D = 0.7 + 2 \times 0.025\,\ln 2.749 = 0.7506\,V$

$V_1 = 2.252\,V$

Thus the change in V_1 is $\underline{\underline{-151\,mV}}$

4.10 For a 1-mA diode, if we select $V_B = 0.5\,V$ then

$R_D = \frac{0.7-0.5}{1} = 200\,\Omega$



$i = 0$ for $v_S \leq V_B$, i.e. for $t \leq 0.5\,s$

$i = \frac{v_S - V_B}{10.2}\,mA = \frac{v_S - 0.5}{10.2}\,mA$, for $t \geq 0.5\,s$

(b) $V = 0.7\,V \quad I = \frac{10-0.7}{1\,k\Omega} = 9.3\,mA$

$V = 0.7 + 2 \times 0.025 \times \ln\left(\frac{9.3}{1}\right) = 0.811\,V,\ I = \frac{10-0.811}{1} = 9.188\,mA$

$V = 0.7 + 2 \times 0.025 \times \ln\left(\frac{9.188}{1}\right) = \underline{0.8109\,V},\ I = \frac{10-0.8109}{1} = \underline{9.189\,mA}$

(c) $V = 0.7\,V \quad I = \frac{10-0.7}{10} = 0.93\,mA$

$V = 0.7 + 2 \times 0.025\,\ln\left(\frac{0.93}{1}\right) = \underline{0.6964\,V},\ I = \frac{10-0.6964}{10}$

$= \underline{0.930\,mA}$

4.9 Iterate to find V_1:

$V_D = 0.7\,V \quad V_1 = 3 \times 0.7 = 2.1\,V \quad I = \frac{10-2.1}{1} = 7.9\,mA$

$V_D = 0.7 + 2 \times 0.025\,\ln\left(\frac{7.9}{1}\right) = 0.8033\,V \quad V_1 = 2.410\,V$

$I = \frac{10-2.410}{1} = 7.56\,mA$

$V_D = 0.7 + 2 \times 0.025\,\ln\left(\frac{7.56}{1}\right) = 0.8011\,V$

$\underline{V_1 = 2.403\,V}$

<u>With the dc supply voltage at +15V:</u>

$V_D = 0.7\,V \quad V_1 = 2.1\,V \quad I = \frac{15-2.1}{1} = 12.9\,mA$

$V_D = 0.7 + 2 \times 0.025 \times \ln\left(\frac{12.9}{1}\right) = 0.8279\,V$

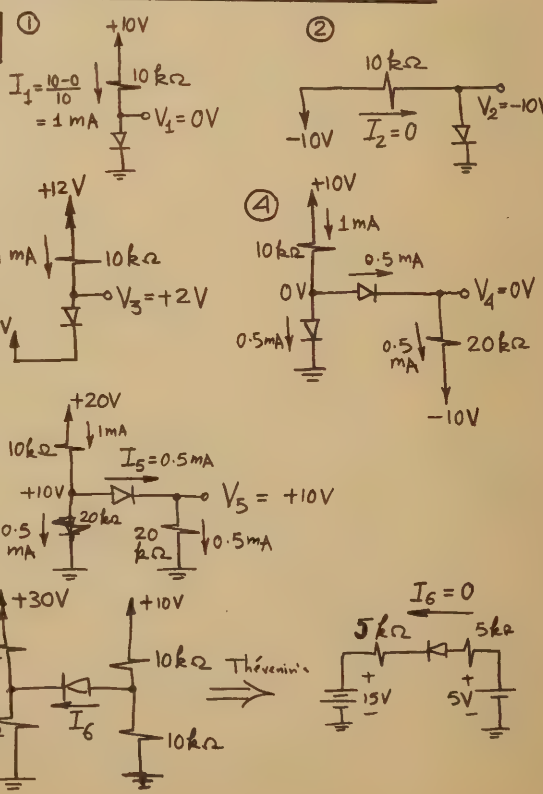
$V_1 = 2.483\,V \quad I = \frac{15-2.483}{1} = 12.52\,mA$

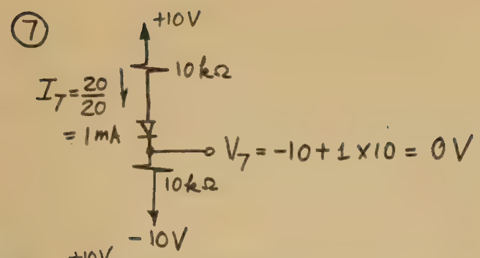
$V_D = 0.7 + 2 \times 0.025\,\ln\left(\frac{12.52}{1}\right) = 0.8264\,V$

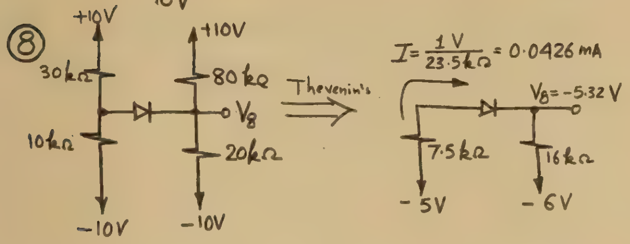
$V_1 = 2.479\,V \quad I = (15-2.479)/1 = 12.52\,mA$

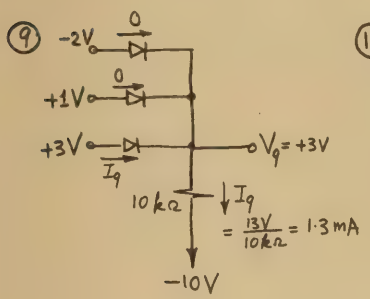
CHAPTER 4 — PROBLEMS

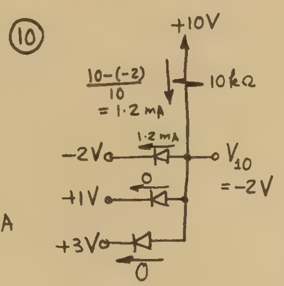
4.1

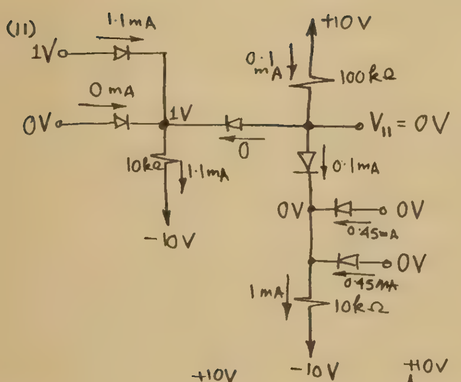


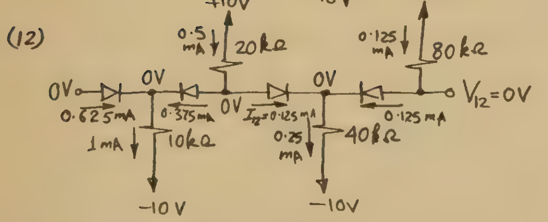




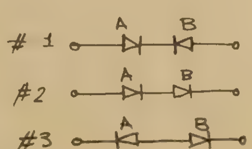




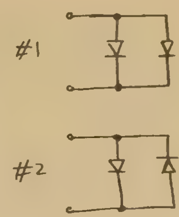




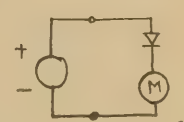
4.2 There are three possible connections. Only connection #2 retains the nonlinearity of a single ideal diode. The other two appear as open circuits.



4.3 Two connections. Only connection #1 retains the nonlinearity of a single ideal diode. Connection #2 provides a short-circuit.

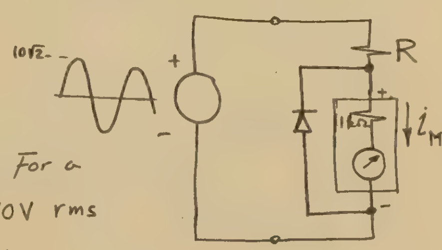


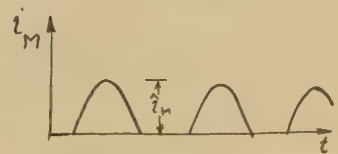
4.4 For a square wave of amplitude V_p, the meter reading will be $V_p/2$ (the average of the half waves). Thus if the meter reading is 5V then the amplitude of the square wave must be 10V peak or 20V peak-to-peak.



For a triangular wave of 20V peak-to-peak, the average of the half waves is $\frac{1}{2} \times \frac{T}{2} \times 10 \times \frac{1}{T} = 2.5\,V$. ($T$ is the period) Thus the meter reading will be 2.5V.

4.5 The current through the meter will consist of half sinusoids, as indicated. For a sinusoid of 10V rms the peak value is $10\sqrt{2}$ and the peak current will be $\hat{i}_M = \frac{10\sqrt{2}}{R + R_M}$



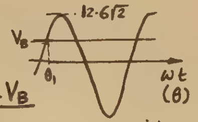


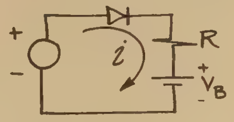
where $R_M = 1\,k\Omega$. The average meter current will be $i_{M_{av}} = \frac{\hat{i}_M}{\pi} = \frac{10\sqrt{2}}{\pi(R + R_M)}$. Now, for full-scale deflection, R should be slected so that $i_{M_{av}} = 1\,mA$. Thus

$$\frac{10\sqrt{2}}{\pi(R+1)} = 1 \implies R = \underline{\underline{3.5\,k\Omega}}$$

As indicated in the Figure the cathode of the diode should be connected to the positive terminal of the meter movement.

4.6





$$i = \frac{12.6\sqrt{2}\,\sin\theta - V_B}{R}$$

$$i_{av} = \frac{1}{2\pi}\int_{\theta_1}^{\pi-\theta_1} \frac{12.6\sqrt{2}\,\sin\theta - V_B}{R}\,d\theta$$

$$= \frac{1}{2\pi R}\left[-12.6\sqrt{2}\,\cos\theta\Big|_{\theta_1}^{\pi-\theta_1} - V_B\,(\pi - 2\theta_1)\right]$$

$$= \frac{1}{2\pi R}\left[12.6\sqrt{2}\times 2\,\cos\theta_1 - V_B\,(\pi-2\theta_1)\right]$$

For $V_B = 14$ V we desire that $i_{av} = 1$ A, thus

$$\theta_1 = \sin^{-1}\left(\frac{14}{12.6\sqrt{2}}\right) = 0.9 \text{ rad, and}$$

$$R = \frac{1}{2\pi}\left[12.6\sqrt{2}\times 2\times 0.622 - 14\times 1.34\right]$$

$$= \underline{0.54\,\Omega}$$

When $V_B = 12$ V, $\theta_1 = \sin^{-1}\frac{12}{12.6\sqrt{2}} = 0.739$ rad, and

$i_{av} = \underline{1.88\text{ A}}$

4.7 $i = I_S\, e^{v/nV_T} \Rightarrow I_S = i\, e^{-v/nV_T}$

Thus, $v_2 - v_1 = nV_T \ln\left(\frac{i_2}{i_1}\right)$

$$n = \frac{v_2 - v_1}{V_T \ln(i_2/i_1)}$$

For diode 1:

$$n = \frac{0.8-0.7}{0.025\,\ln(10/1)} = \underline{\underline{1.737}}$$

$$I_S = 10^{-3}\, e^{-0.7/(1.737\times 0.025)} = \underline{\underline{10^{-10}\text{ A}}}$$

For diode 2:

$$n = \frac{0.7-0.6}{0.025\,\ln(10/1)} = \underline{1.737}$$

$$I_S = 10\, e^{-0.7/(1.737\times 0.025)} = \underline{10^{-6}\text{ A}}$$

4.8 $i = I_S\, e^{v/nV_T}$

$$\frac{i_2}{i_1} = e^{(v_2-v_1)/nV_T}$$

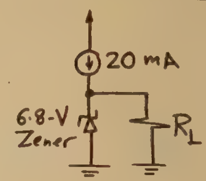
For the 0.1V/decade diode the current at $v = 0.5$ V will be two decades lower than 10 mA, that is $\underline{0.1\text{ mA}}$.

For a diode with $n = 2$, substitute $i_2 = 10$ mA, $v_2 = 0.7$ V, and $v_1 = 0.5$ V to obtain

$i_1 = \underline{0.18\text{ mA}}$

4.9 25°C to 125°C is an increase of 100 °C = 10 X 10. Thus the leakage current increases to $1\,\mu\text{A} \times 2^{10} = 1024\,\mu\text{A}$ or 1.024 mA.

4.10 With $R_L = 2$ kΩ, the zener current decreases by approximately $\frac{6.8\text{V}}{2\text{ k}\Omega} = 3.4$ mA. Thus the zener voltage drops by 3.4 mA X 5Ω = $\underline{17\text{ mV}}$.



With $R_L = 200\,\Omega$ the load current required is $\frac{6.8\text{V}}{0.2\text{ k}\Omega} = 34$ mA. Since the total current available is 20 mA, the zener will stop operating in the breakdown mode; it will simply operate as a reverse-biased diode and the output voltage will become

V_O = 20 mA X 0.2 kΩ = 4V.

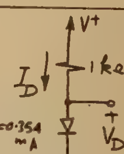
Thus the output voltage drops by $\underline{2.8\text{V}}$.

4.11 Reducing the zener current to a 1% level, that is by two decades, reduces the diode voltage to 0.5V; thus the zener voltage becomes 6.6V. The change in voltage (0.2V) can be expressed as $\frac{0.2}{6.8}\times 100 = 2.9\,\%$.

For a regulator formed by 10 forward-conducting diodes in series, reducing the current level by two decades results in 0.2V reduction in the voltage drop of each diode. Thus the overall reduction is 2V, or $\frac{2}{10\times 0.7}\times 100 = 28.6\,\%$. The zener is obviously superior.

4.12 When $V^+ = 1$ V:

$V_D = 0.7$ V $\quad I_D = \frac{1-0.7}{1} = 0.3$ mA



$V_D = 0.7 + nV_T \ln\frac{0.3}{1} = 0.646$ V. $\quad I_D = \frac{1-0.646}{1} = 0.354$ mA

$V_D = 0.7 + nV_T \ln\frac{0.354}{1} = 0.653$ V, $\quad I_D = \frac{1-0.653}{1} = 0.347$ mA

$V_D = 0.7 + nV_T \ln\frac{0.347}{1} = \underline{\underline{0.652\text{ V}}} \quad I_D = \frac{1-0.652}{1} = \underline{\underline{0.348\text{ mA}}}$

No further iterations are warranted.

When $V^+ = 10V$:

$V_D = 0.7V \qquad I_D = \frac{10-0.7}{1} = 9.3 \text{ mA}$

$V_D = 0.7 + nV_T \ln\frac{9.3}{1} = 0.8V \qquad I_D = \frac{10-0.8}{1} = 9.2 \text{ mA}$

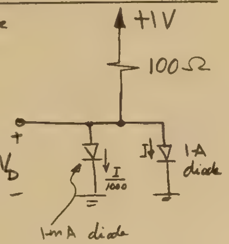
$V_D = 0.7 + nV_T \ln\frac{9.2}{1} = \underline{0.8V} \qquad I_D = \frac{10-0.8}{1} = \underline{9.2 \text{ mA}}$

No further iterations are warranted.

4.13 The current in each diode becomes $(I/2)$. Thus the voltage changes by $nV_T \ln\frac{1}{2} =$

$2 \times 0.025 \ln \frac{1}{2} = \underline{-34.7 \text{ mV}}$.

4.14 Since the two diodes are connected in parallel and thus have the same voltage drop, the current in the 1-A diode will be 1,000 times the current in the 1-mA diode. If we denote the current in the 1-A device by I then the current in the 1-mA diode will be $0.001 I$. The total current through the $100\text{-}\Omega$ resistor will be $1.001 I$. Let us now



perform few iterations to determine the value of I and V_D.

Let $V_D = 0.7V \qquad I = \frac{1-0.7}{1.001 \times 0.1} = 2.997 \text{ mA}$

$V_D = 0.7 + nV_T \ln\frac{2.997 \times 10^{-3}}{1}$ (where $n = 1.737$)

$= 0.448 \text{ V} \qquad I = \frac{1-0.448}{1.001 \times 0.1} = 5.5 \text{ mA}$

$V_D = 0.7 + 1.737 \times 0.025 \ln \frac{5.5 \times 10^{-3}}{1} = 0.474V$

$I = \frac{1-0.474}{1.001 \times 0.1} = 5.25 \text{ mA}$

$V_D = 0.7 + 1.737 \times 0.025 \ln \frac{5.25 \times 10^{-3}}{1} = 0.472V$

$I = \frac{1-0.472}{1.001 \times 0.1} = 5.27 \text{ mA}$

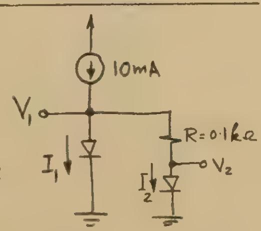
No further iterations are necessary and the voltage across the pair is $\underline{0.472 \text{ V}}$.

4.15 $\frac{I_1}{I_2} = e^{(V_1 - V_2)/nV_T}$

Thus $I_2 = \frac{I_1 + I_2}{1 + e^{(V_1-V_2)/nV_T}}$

But $I_1 + I_2 = 10 \text{ mA}$ and $V_1 - V_2 = I_2 R$

Thus $I_2 = \frac{10 \text{ mA}}{1 + e^{I_2 R / nV_T}}$



Solving this equation by iteration results in

$I_2 = 1.064 \text{ mA}$

Thus,

$I_1 = 10 - 1.064 = 8.936 \text{ mA}$

$V_1 = 0.7 + 2 \times 0.025 \times \ln\frac{8.936}{1} = 0.81V$

$V_2 = V_1 - I_1 R = \underline{0.703 \text{ V}}$

4.16 For the 1-mA diode at 0.7V:

$r = \frac{nV_T}{I} = \frac{2 \times 25 \text{ mV}}{1 \text{ mA}} = \underline{50\ \Omega}$

For the 1-A diode at 0.7V:

$r = \frac{nV_T}{I} = \frac{2 \times 25 \text{ mV}}{1 \text{ A}} = \underline{50 \text{ m}\Omega}$

For the 1-A diode at 1mA bias current:

$r = \frac{nV_T}{I} = \frac{2 \times 25 \text{ mV}}{1 \text{ mA}} = \underline{50\ \Omega}$

4.17 $v_d = v_s \frac{r_d}{r_d + R_s}$

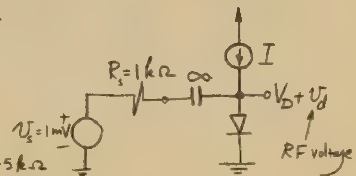
where $r_d = nV_T/I$.

Assuming $n = 2$ then

For $I = 10 \mu A \quad r_d = \frac{2 \times 25 \times 10^{-3}}{10 \times 10^{-6}} = 5 k\Omega$

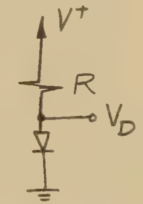
and $v_d = 1 mV \times \frac{5}{5+1} = \underline{\frac{5}{6} \text{ mV}}$

For $I = 10 mA \quad r_d = 5\ \Omega$ and $v_d = 1 mV \times \frac{5}{1{,}005} \simeq \underline{5 \mu V}$



4.18 (a) For $V^+ = 1V \pm 10\%$:

To establish a nominal V_D of 0.7V we need to supply the diode with 1 mA current. Thus



$R = \frac{1-0.7}{1 mA} = \underline{300\ \Omega}$

Since the variability of V^+ is small we shall use the small-signal model of the diode to determine the variability of the output voltage V_D. At the bias point, the incremental resistance of the diode is

$r_d = \frac{nV_T}{I} = \frac{2 \times 25 \text{ mV}}{1 \text{ mA}} = 50\ \Omega$

Thus for $\Delta V^+ = \pm 0.1V$, the variability of V_D is $\quad \Delta V_D = \pm 0.1 \frac{r_d}{r_d + R} = \pm 0.1 \times \frac{50}{350} = \pm 14.3 \text{ mV}$

This change is a bit too large for the small-signal model of the diode to be valid. Therefore we shall recalculate ΔV_D as follows:

With $V^+ = 1 + 0.1 = 1.1V$ we use iteration to obtain the high value of V_D, V_{DH}. Let

$V_{DH} = 0.75\,V \Rightarrow I = \frac{1.1 - 0.75}{0.3} = 1.17$ mA

$V_{DH} = 0.7 + 2 \times 0.025 \ln \frac{1.17}{1} = 0.708$ V

$I = \frac{1.1 - 0.708}{0.3} = 1.31$ mA

$V_{DH} = 0.7 + 2 \times 0.025 \ln \frac{1.31}{1} = 0.7135$

$I = \frac{1.1 - 0.7135}{0.3} = 1.288$ mA

$V_{DH} = 0.7 + 2 \times 0.025 \times \ln \frac{1.288}{1} = 0.7127$

$I = \frac{1.1 - 0.7127}{0.3} = 1.291$ mA.

No further iterations are warranted and

$\Delta V_{DH} = V_{DH} - V_D = \underline{12.7\text{ mV}}$

To find the low value of V_D, V_{DL}, which occurs when $V^+ = 0.9$ V we repeat the above procedure and obtain $V_{DL} = 0.6836$ V.

Thus $\Delta V_{DL} = \underline{\underline{16.4\text{ mV}}}$

Thus the variability of V_D is -16.4 mV to $+12.7$ mV or equivalently -2.34% to 1.81%.

(b) For the case $V^+ = 5\,V \pm 50\%$:

$R = \frac{5 - 0.7}{1\text{ mA}} = \underline{4.3\text{ k}\Omega}$

as large as possible. However IR is fixed from $IR = V^+_{nominal} - V_{O\,nominal} = 5.5 - 1.5 = 4$ V.

Consider next the changes in V_O due to loading with a 1kΩ resistor. The change in diode current will be approximately 1.5 mA and the change in V_O will be

$\Delta V_O \simeq 1.5 \times 2 r_d$. To minimize ΔV_O we ~~choose~~ design for as small r_d as possible. This is achieved by designing for I to equal the largest current available from the supply which is 15 mA. Thus

$15\text{ mA} \approx \frac{7 - 1.5}{R} \Rightarrow R = \frac{7 - 1.5}{15} = 366.7\,\Omega$

To allow for the additional current drawn from V^+ when $V^+ = +7$ V we shall select $R = \underline{400\,\Omega}$. This results in a nominal

$I = \frac{5.5 - 1.5}{400} = 10$ mA.

At 10 mA of nominal bias current we have $V_{O\,nominal} = 2 \times 0.757 = \underline{\underline{1.514\,V}}$

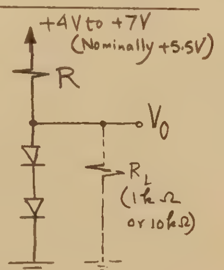
When $V^+ = 7.5$ V we use iteration to determine $V_{DH} = 722.7$ mV. Thus $\Delta V_{DH} = 22.7$ mV.

When $V^+ = 2.5$ V we use iteration to find $V_{DL} = 657.6$ mV. Thus $\Delta V_{DL} = 42.4$ mV.

Thus the variability in the output voltage is -42.4 mV to $+22.7$ mV or -6% to $+3.2\%$. Obviously the first design is better.

4.19 Assume $n = 1$.

Consider the no-load case. An approximate value for the change in V_O due to the ± 1.5 V variation in the supply voltage can be obtained using the small-signal model for the diodes. Thus



$$\Delta V_O = \pm 1.5 \frac{2\frac{nV_T}{I}}{2\frac{nV_T}{I} + R} = \pm 1.5 \frac{2V_T}{2V_T + IR}$$

To minimize the value of ΔV_O we make (IR)

We shall now evaluate our design by calculating the variabilities of output voltage.

(a) With No Load:

For $V^+ = +4$ V we iterate to determine the low value of $V_O \Rightarrow V_{OL} = 1.492$ V. For $V^+ = +7$ V we iterate to determine the high value of $V_O \Rightarrow V_{OH} = 1.531$ V. Thus the variation in V_O is -22 mV to $+17$ mV or -1.5% to 1.13%.

(b) With a 10-kΩ load:

$I_{Load} \simeq \frac{1.5}{10} = 0.15$ mA

$\Delta V_O = -I_L \times 2r_d = -0.15 \times 2 \times 5 = -1.5$ mV

or equivalently $\underline{-0.1\%}$

(c) With a 1-kΩ load:

$I_{Load} \simeq \frac{1.5}{1} = 1.5$ mA

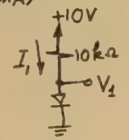
$\Delta V_O = -I_L \times 2r_d = -1.5 \times 2 \times 5 = -15$ mV

or equivalently -1%.

4.20 $n = 1.737$

$$v_D = 0.7 + 1.737 \times 0.025 \ln\left(\frac{i_D}{1\text{ mA}}\right)$$

(1) First Iteration: $V_1 = 0.7V$, $I_1 = \frac{10-0.7}{10} = 0.93\text{ mA}$



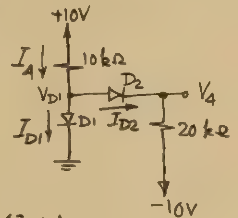
Second iteration:
$V_1 = 0.7 + 1.737 \times 0.025 \ln\left(\frac{0.93}{1}\right)$

$= \underline{0.697V}$ $\quad I_1 = \underline{0.93\text{ mA}}$

(2) As in Problem 4.1, $I_2 = \underline{0}$, $V_2 = \underline{-10V}$

(3) Same conditions as in (1) above; thus
$V_3 = 2 + 0.697 = \underline{2.697\text{ V}}$

(4) Both diodes are on.



First iteration:
$V_{D_1} = 0.7V$, $V_{D_2} = 0.7V$

$I_4 = \frac{10-0.7}{10} = 0.93\text{ mA}$

$V_4 = 0V$

$I_{D2} = \frac{0-(-10)}{20} = 0.5\text{ mA}$

$I_{D1} = I_4 - I_{D2} = 0.93 - 0.5 = 0.43\text{ mA}$

Second iteration

$V_{D_1} = 0.7 + 1.737 \times 0.025 \ln\left(\frac{0.43}{1}\right) = 0.663V$

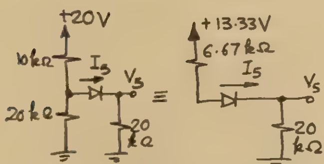
$V_{D2} = 0.7 + 1.737 \times 0.025 \ln\left(\frac{0.5}{1}\right) = 0.670\text{ V}$

$V_4 = 0.663 - 0.670 = \underline{\underline{-0.007\text{ V}}}$

$I_{D2} = \frac{-0.007-(-10)}{20} \simeq 0.5\text{ mA}$

$I_4 = \frac{10-0.663}{10} = 0.934\text{ mA} \quad I_{D1} = 0.934 - 0.5 = 0.434\text{ mA}$

(5) 1st iteration



Using Thévenin's theorem

$V_D = 0.7V$

$I_5 = \frac{13.33-0.7}{6.67+20} = 0.474\text{ mA}$

2nd iteration

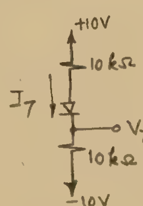
$V_D = 0.7 + 1.737 \times 0.025 \ln\left(\frac{0.474}{1}\right) = 0.668\text{ V}$

$I_5 = \frac{13.33-0.668}{26.67} = \underline{0.474\text{ mA}}$

$V_5 = 0.474 \times 20 = \underline{+9.48V}$

(6) As in Problem 4.1, $I_6 = 0$

(7) 1st iteration



$V_D = 0.7V \quad I_7 = \frac{20-0.7}{20} = 0.965\text{ mA}$

$V_7 = -10 + 0.965 \times 10 = -0.35V$

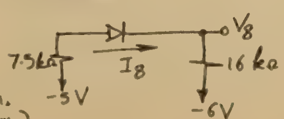
2nd iteration

$V_D = 0.7 + 1.737 \times 0.025 \ln\left(\frac{0.965}{1}\right) = 0.698\text{ V}$

$I_7 = \frac{20-0.698}{20} = \underline{0.965\text{ mA}}$

$V_7 = -10 + 0.965 \times 10 = \underline{-0.35V}$

(8) As in Problem 4.1 we obtain the equivalent circuit shown (using Thévenin's theorem).



First iteration: $V_D = 0.7V \quad I_8 = \frac{-5-0.7-(-6)}{23.5} = 0.013\text{ mA}$

$V_8 = -6 + 0.013 \times 16 = -5.8V$

Second iteration: $V_D = 0.7 + 1.737 \times 0.025 \ln\left(\frac{0.013}{1}\right)$

$= 0.511V$

$I_8 = \frac{1-0.511}{23.5} = 0.021\text{ mA}$

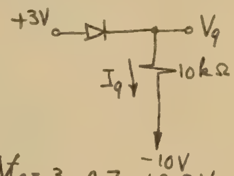
$V_8 = -6 + 0.021 \times 16 = -5.64V$

Third iteration: $V_D = 0.7 + 1.737 \times 0.025 \ln\left(\frac{0.021}{1}\right) = 0.532V$

$I_8 = \frac{1-0.532}{23.5} = 0.02\text{ mA}$

$V_8 = -6 + 0.02 \times 16 = \underline{\underline{-5.68V}}$

(9) As in Problem 4.1 only the diode to which +3V is applied will be conducting. Thus the circuit reduces to the one shown.



First iteration: $V_D = 0.7V \quad V_9 = 3 - 0.7 = +2.3V$

$I_9 = \frac{2.3-(-10)}{10} = 1.23\text{ mA}$

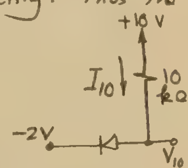
Second iteration: $V_D = 0.7 + 1.737 \times 0.025 \ln\left(\frac{1.23}{1}\right)$

$= 0.709\text{ V}$

$V_9 = 3 - 0.709 = \underline{+2.29V}$

$I_9 = \frac{2.29-(-10)}{10} = \underline{\underline{1.229\text{ mA}}}$

(10) As in Problem 4.1, only the diode to which −2V is applied will be conducting. Thus the circuit reduces to the one shown.



1st iteration: $V_D = 0.7\text{ V}$

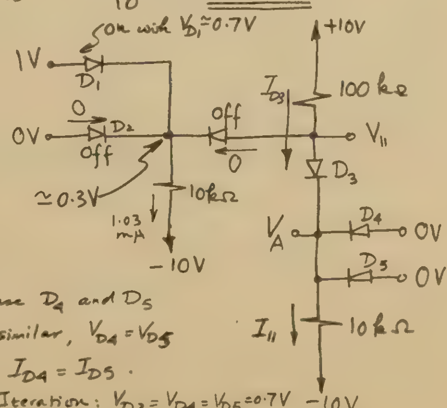
$V_{10} = -2 + 0.7 = -1.3V \quad I_{10} = \frac{10-(-1.3)}{10}$
$= 1.13\text{ mA}$

2nd iteration: $V_D = 0.7 + 1.737 \times 0.025 \ln\left(\frac{1.13}{1}\right) = 0.705\text{ V}$

$V_{10} = -2 + 0.705 = \underline{-1.295\text{ V}}$

$I_{10} = \frac{10-(1.295)}{10} \simeq \underline{\underline{1.13\text{ mA}}}$

(11)



Because D_4 and D_5 are similar, $V_{D4} = V_{D5}$ and $I_{D4} = I_{D5}$.

First Iteration: $V_{D3} = V_{D4} = V_{D5} = 0.7V$

$V_A = -0.7\text{ V} \quad I_{11} = \frac{-0.7-(-10)}{10} = 0.93\text{ mA}$

$V_{11} = 0\,V \qquad I_{D3} = \frac{10-0}{100} = 0.1\,mA$

2nd iteration: $I_{D4} = I_{D5} = \frac{1}{2}(I_{11} - I_{D3}) = 0.415\,mA$

$$V_{D3} = 0.7 + 1.737 \times 0.025\,\ln\left(\frac{0.1}{1}\right) = 0.6\,V$$

$$V_{D4} = V_{D5} = 0.7 + 1.737 \times 0.025\,\ln\left(\frac{0.415}{1}\right) = 0.662\,V$$

$$V_A = 0 - 0.662 = -0.662\,V$$

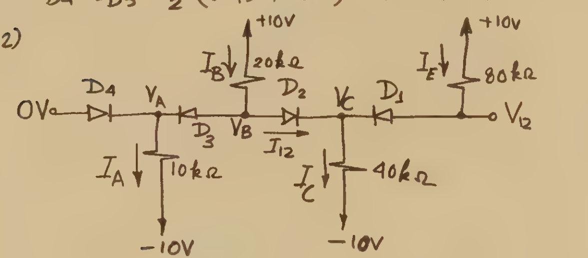
$$I_{11} = \frac{-0.662+10}{10} = 0.934\,mA$$

$$V_{11} = -0.662 + 0.6 = \underline{\underline{-0.062\,V}}$$

$$I_{D3} = \frac{10-(-0.062)}{100} \simeq 0.1\,mA$$

$$I_{D4} = I_{D5} = \frac{1}{2}(0.934 - 0.1) = 0.417\,mA$$

(12)



First iteration: $V_{D1} = V_{D2} = V_{D3} = V_{D4} = 0.7\,V$

$$V_A = 0 - V_{D4} = -0.7\,V \qquad I_A = \frac{V_A - (-10)}{10} = 0.93\,mA$$

$$V_B = V_A + V_{D3} = 0\,V \qquad I_B = \frac{10 - V_B}{20} = 0.5\,mA$$

$$V_C = V_B - V_{D2} = -0.7\,V \qquad I_C = \frac{V_C - (-10)}{40} = 0.2325\,mA$$

$$V_{12} = V_C + V_{D1} = 0\,V \qquad I_E = \frac{10 - V_{12}}{80} = 0.125\,mA$$

$$I_{12} = I_{D2} = I_C - I_E = 0.1075\,mA$$

$$I_{D3} = I_B - I_{12} = 0.3925\,mA$$

$$I_{D4} = I_A - I_{D3} = 0.5375\,mA$$

Second Iteration:

$$V_{D4} = 0.7 + 1.737 \times 0.025 \times \ln\left(\frac{0.5375}{1}\right) = 0.673\,V$$

$$V_A = -0.673\,V$$

$$I_A = \frac{-0.673-(-10)}{10} = 0.933\,mA$$

$$V_{D3} = 0.7 + 1.737 \times 0.025 \times \ln\left(\frac{0.3925}{1}\right) = 0.659\,V$$

$$V_B = 0.659 - 0.673 = -0.014\,V$$

$$I_B = \frac{10-(-0.014)}{20} = 0.5\,mA$$

$$V_{D2} = 0.7 + 1.737 \times 0.025 \times \ln\left(\frac{0.1075}{1}\right) = 0.603\,V$$

$$V_C = -0.603 - 0.014 = -0.617\,V$$

$$I_C = \frac{-0.617-(-10)}{40} = 0.2346\,mA$$

$$V_{D1} = 0.7 + 1.737 \times 0.025 \times \ln\left(\frac{0.125}{1}\right) = 0.610\,V$$

$$V_{12} = 0.610 - 0.617 = \underline{-0.007\,V}$$

$$I_E = \frac{10-(-0.007)}{80} = 0.125\,mA$$

$I_{12} = I_{D2} \simeq \underline{0.11\,mA} \quad I_{D3} = 0.39\,mA \quad I_{D4} = 0.543\,mA.$

$\boxed{4.21}$ With $v_I = 0V$, all diodes will be conducting, $v_A = +0.7V$, $v_B = -0.7V$, and $v_O = 0V$. $i_A = 0.93\,mA$, $i_L = 0$, $i_B = 0.93\,mA$. Thus i_A divides between D_1 and D_2 and i_I will be equal to $0V$. $(i_A + i_I = i_B + i_L)$

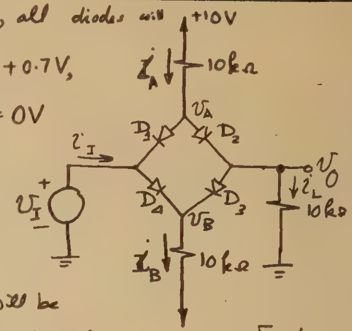
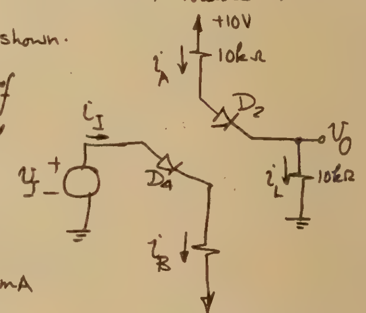


Fig. 1

For $v_I = +2V$ all diodes will be conducting and: $v_A = +2.7V$, $i_A = 0.73\,mA$, $v_O = +2V$, $i_L = 0.2\,mA$, $i_B = 1.13\,mA$. Since $i_A + i_I$ must equal $i_B + i_L$ we see that $i_I = 0.6\,mA$. More importantly note that the distribution of i_A (0.73 mA) between D_1 and D_2 must be such that the current through D_2 is greater than i_L, otherwise D_3 will turn off. We see that as v_I is increased i_A decreases and i_L increases. Thus a value of v_I will be reached at which i_A is just sufficient to supply i_L; in other words there will be no current left for D_1 and for D_3. These two diodes will then turn off and for v_I greater than this critical value the circuit becomes as shown.

The critical value of v_I can be determined from this circuit as: $i_A = i_L = \frac{10-0.7}{20} = 0.465\,mA$



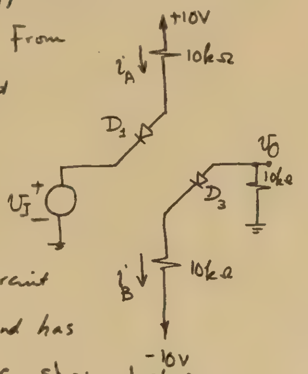
$$v_O = 4.65\,V$$

Fig. 2

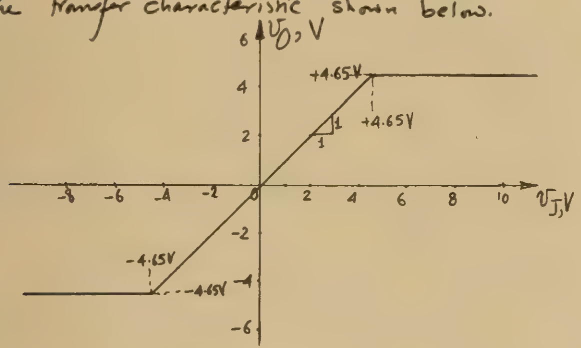
Since at the critical point $v_I = v_O$ we see that the threshold is $v_I = +4.65V$. For $v_I \geqslant 4.65V$, the circuit in Fig. 2 applies and v_O remains constant at $v_O = +4.65V$.

For negative values of v_I one can show that the complement of the above occurs. Thus

for v_I in the range 0 to −4.65 V, all diodes will be conducting and $v_O = v_I$. For $v_I \le -4.65$V diodes D_2 and D_4 turn off and the circuit reduces to that in Fig. 3. From this circuit we see that v_O remains constant at −4.65 V.

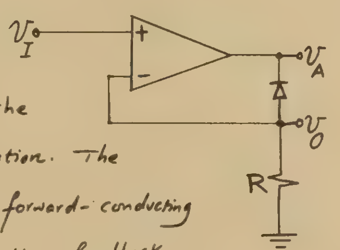


From the above we conclude that the circuit behaves as a limiter and has the transfer characteristic shown below.



CHAPTER 5—EXERCISES

5.1



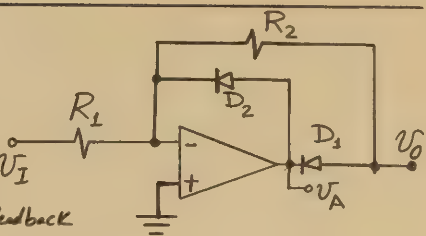
As v_I goes negative, v_A goes negative and current flows through the diode - resistor combination. The low resistance of the forward-conducting diode closes the negative-feedback loop of the op amp, thus causing v_O to equal v_I:

For $v_I \le 0$, $v_O = v_I$

When v_I goes positive the diode cuts off and the op amp feedback loop is opened. Thus the op amp saturates at the positive limit and $v_O = 0$:

For $v_I \ge 0$, $v_O = 0$.

5.2



When v_I goes positive, D_1 turns on and the negative-feedback loop of the op amp is closed through D_1 and R_2. Thus, $v_O = -\left(\frac{R_2}{R_1}\right) v_I$ and v_O will be negative while v_A will be more negative (by the diode drop V_{D1}). Hence D_2 will be off.

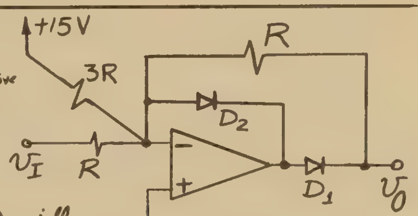
When v_I goes negative, D_2 turns on and closes the negative-feedback loop of the op amp. v_A will then be approximately +0.7 V. Thus D_1 will be off and no current will flow through R_2. Thus, $v_O = 0$ V.

In summary:

For $v_I \le 0$, $v_O = 0$, and

for $v_I \ge 0$, $v_O = -\frac{R_2}{R_1} v_I$

5.3



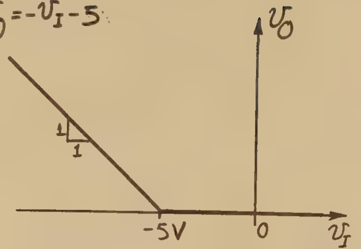
When v_I is positive D_2 will be on and D_1 will be off. D_2 will close the feedback loop and will conduct a current of $\left(\frac{15}{3R} + \frac{v_I}{R}\right)$. In this case $v_O = 0$. This situation remains even if v_I goes negative. In fact this situation obtains as long as a net positive current is forced through D_2. Thus the situation changes when the current $\left(\frac{15}{3R} + \frac{v_I}{R}\right)$ is reduced to zero, which occurs when $v_I = -5$V.

For $v_I \le -5$V, D_2 will be off and D_1 will turn on and close the loop through the resistor R. In this case: $v_O = -\left(\frac{15}{3R} + \frac{v_I}{R}\right) R$

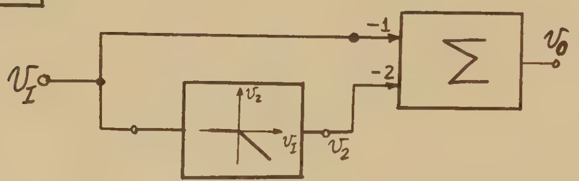
In summary:

For $v_I \geq -5V$, $v_O = 0$,

and for $v_I \leq -5V$, $v_O = -v_I - 5$.



5.4



For $v_I \geq 0$, i.e. $v_I = +|v_I|$, $v_2 = -|v_I|$, and

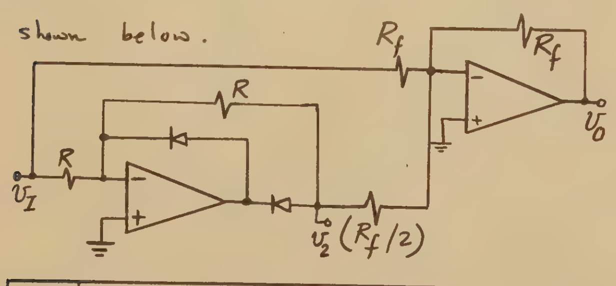
$$v_O = -1 \times |v_I| - 2 \times -|v_I| = +|v_I|$$

For $v_I \leq 0$, i.e. $v_I = -|v_I|$, $v_2 = 0$, and

$$v_O = -1 \times -|v_I| - 2 \times 0 = +|v_I|$$

Thus, the block diagram implements the absolute value operation.

The circuit in Exercise 5.2 implements the half-wave rectifier needed. Using this circuit together with a weighted summer results in the absolute value circuit shown below.



5.5

Assuming ideal diodes:

(a) The peak current in each diode $= \frac{100V}{1k\Omega} = 100\,mA$

(b) Consider the half cycle during which v_B and v_C are positive: D_1 is on and acts as a short circuit while D_2 is off. The reverse voltage across D_2 will be $(v_L + v_C)$ which attains a peak value of 200V.

(c) The average voltage across the load is

$$\frac{2}{\pi} V_p = \frac{2}{\pi} \times 100 = 63.7\,V$$

(d) During each half cycle only half of the transformer secondary is active and supplying a peak current of 100 mA. Thus the peak current in the primary will be 100 mA.

(e) The sinusoidal source has a peak value of 100 V (i.e. $\frac{100}{\sqrt{2}}$ RMS) and supplies a peak current of 100 mA ($\frac{0.1}{\sqrt{2}}$ A RMS). Thus the power supplied by the source

$$= \frac{100}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} = 5\,W.$$

5.6

v_A is a sinusoid of 5V RMS ($5\sqrt{2}$ V peak). The average current through the meter will be $\frac{2}{\pi} \times \frac{5\sqrt{2}}{R}$. To obtain full-scale reading this current must be equal to 1 mA. Thus: $\frac{2}{\pi} \times \frac{5\sqrt{2}}{R} = 1\,mA$, which leads to $R = 4.5\,k\Omega$.

v_C will be maximum when v_A is at its positive peak, i.e. $v_A = 5\sqrt{2}$. At this value of v_A we obtain

$$v_C = V_{D1} + V_M + V_{D3} + V_R$$

Assuming $V_{D1} = V_{D3} \simeq 0.7V$ and calculating V_M from

$$V_M = \frac{5\sqrt{2}}{4.5} \times 0.05 = 0.08\,V,$$

and $V_R = 5\sqrt{2}$ we obtain

$$v_C = 8.55\,V.$$

Similarly we can calculate the minimum of v_C to be $-8.55V$.

5.7

Consider the half cycle during which v_A is positive. D_1 and D_3 will be on and, assuming ideal diodes, will act as short circuit. Thus v_L will equal v_A and the reverse bias across each of D_2 and D_4 will be equal to v_L and thus equal to v_A. Thus the peak reverse voltage across each of the diodes will be equal to the peak of the input (100V).

5.8 For a full-wave rectifier:

$$V_r = \frac{V_p}{2fCR}$$

Thus

$$2 = \frac{100}{2 \times 60 \times C \times 10 \times 10^3}$$

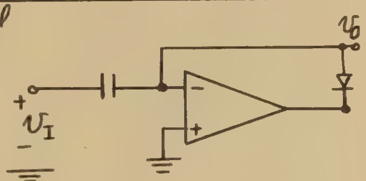
$$C = 41.7\ \mu F$$

5.9 The maximum rate of change of the output voltage is the lesser of the op-amp slew rate; $SR = 0.1 V/\mu s$, and the rate determined by the charging of C with the maximum possible output current, I_{max}, namely;

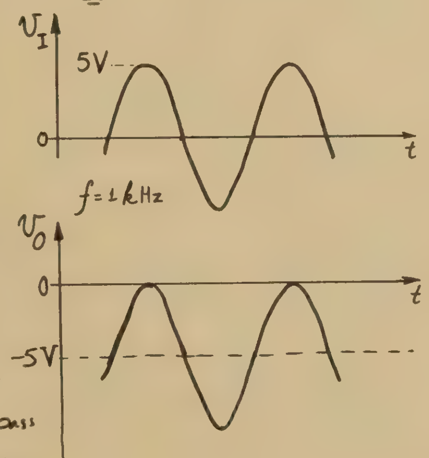
$$\frac{I_{max}}{C} = \frac{10 \times 10^{-3}}{10^{-6}}\ V/s = 10\ mV/\mu s.$$

It follows that the maximum rate of change of the output voltage is $10\ mV/\mu s$ which can be expressed more conveniently as $10 V/ms$.

5.10 Assuming ideal operation, V_O will be as shown:



It consists of a dc component of $-5V$ on which a 1 kHz sinusoid with 5V peak amplitude is superimposed.

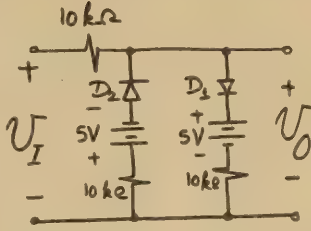


The RC low-pass filter attenuate this latter component to an amplitude of

$$\frac{5}{\sqrt{1+\left(\frac{1000}{10}\right)^2}} \simeq 0.05V.$$

Thus, at the output of the filter we have $-5V$ dc and a 0.05-V peak, 1 kHz sinusoid.

5.11 Assuming ideal diodes:



For D_1 to conduct, V_I has to exceed $+5V$. For D_2 to conduct, V_I has to be lower than $-5V$.

It follows that in the range $-5V \le V_I \le +5V$ both D_1 and D_2 will be off, no current will flow and $V_O = V_I$.

For $V_I \geqslant +5V$, D_1 will be on and acting as a perfect short circuit, then the current flowing becomes

$$I = \frac{V_I - 5}{20 k\Omega}$$

Thus,

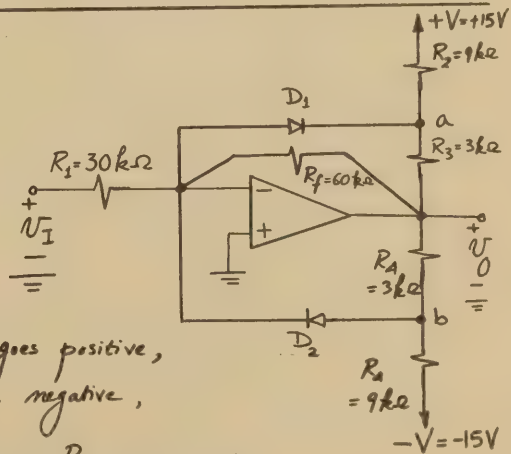
$$V_O = I \times 10\ k\Omega + 5$$

i.e. $V_O = \frac{1}{2} V_I + 2.5$

From symmetry we see that for $V_I \le -5V$,

$$V_O = \tfrac{1}{2} V_I - 2.5\ V$$

5.12



As V_I goes positive, V_O goes negative,

$$V_O = -\frac{R_f}{R_1} V_I = -2V_I$$

Node "b" will be at a negative voltage and D_2 will obviously be off. Assume for the moment that D_1 also is off: We see that the voltage at node "a" will be

$$V_A = V_O + \frac{15 - V_O}{R_2 + R_3} R_3$$

Thus, $v_A = v_O + \frac{1}{4}(15 - v_O)$

As long as v_A is positive, D_1 will be off and $v_O = -2v_I$. As v_O increases in the negative direction, a value will be reached at which v_A is reduced to zero and begins to go negative. This value is obtained from

$$v_A = 0 = v_O + \frac{1}{4}(15 - v_O) \Rightarrow v_O = -5\text{V}$$

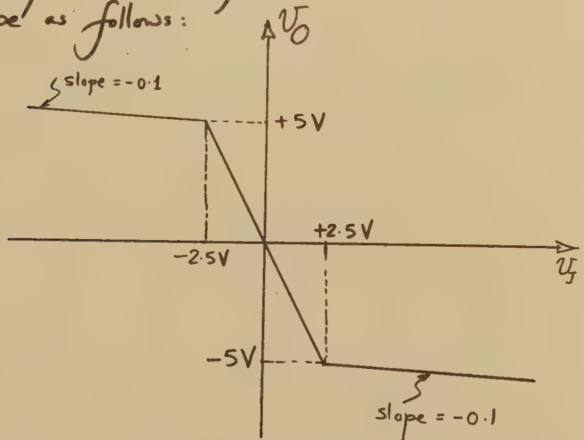
which corresponds to $v_I = +2.5\text{ V}$.

For $v_I \geq +2.5\text{ V}$, diode D_1 conducts and thus clamps node "a" to 0 V. From that point on, v_O decreases only slightly below the limiting level of -5V. Specifically,

$$v_O = -5 - \frac{(R_f // R_3)}{R_1} v_I$$

$$\simeq -5 - 0.1\, v_I$$

For negative v_I, similar arguments apply to derive the answer given in the book. Assuming ideal diodes the complete transfer characteristic will be as follows:



5.13

$$V_{T1} = -L_+ \frac{R_1}{R_2} = -10 \times \frac{10}{20} = -5\text{V}$$

$$V_{T2} = -L_- \frac{R_1}{R_2} = 10 \times \frac{10}{20} = +5\text{V}$$

5.14

For a 100-mV hystresis, $V_{T2} = -V_{T1} = 50$ mV

$$V_{T2} = -L_- \frac{R_1}{R_2} \Rightarrow 0.05 = 10\, \frac{1}{R_2}$$

$$R_2 = \frac{10}{0.05} = 200\text{ k}\Omega$$

5.15

From Eqn. (5.8) the period is given by

$$T = 2\tau \ln \frac{1+\beta}{1-\beta}$$

where $\tau = C_1 R_3 = 0.01 \times 10^{-6} \times 10^6 = 0.01$ s

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{100}{100 + 1000} = \frac{1}{11}$$

Thus, $T = 2 \times 0.01 \ln \dfrac{1 + \frac{1}{11}}{1 - \frac{1}{11}}$

$= 0.02 \ln 1.2 = 0.00365$ s

Thus, $f = \frac{1}{T} = 274.2$ Hz

5.16

To obtain triangular wave with 10 V peak-to-peak amplitude we should have

$$V_{TH} = -V_{TL} = 5\text{V}$$

But $V_{TL} = -L_+ \dfrac{R_1}{R_2}$

Thus $-5 = -10\, \frac{10}{R_2}$

$R_2 = \underline{20\text{ k}\Omega}$

For 1 kHz frequency, $T = 1$ ms.

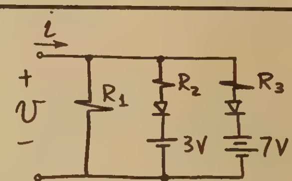
Thus, $\frac{T}{2} = 0.5 \times 10^{-3} = CR\, \dfrac{V_{TH} - V_{TL}}{L_+}$

$= 0.01 \times 10^{-6} \times R \times \frac{10}{10}$

$R = \underline{\underline{50\text{ k}\Omega}}$

5.17

$i = 0.1 v^2$



At $v = 2$ V, $i = 0.4$ mA

Thus $R_1 = \frac{2}{0.4} = \underline{\underline{5\text{ k}\Omega}}$

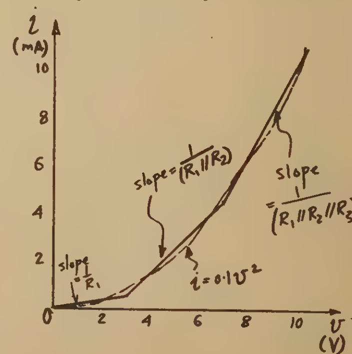
For $3\text{V} \leq v \leq 7\text{V}$,

$$i = \frac{v}{R_1} + \frac{v - 3}{R_2}$$

To obtain a perfect match at $v = 4$ V (i.e. to obtain $i = 1.6$ mA),

$$1.6 = \frac{4}{5} + \frac{4-3}{R_2}$$

$R_2 = 1.25$ kΩ



For $v \geq 7V$,

$$i = \frac{v}{R_1} + \frac{v-3}{R_2} + \frac{v-7}{R_3}$$

To obtain perfect match at $v = 8V$ we have to select R_3 so that $i = 6.4$ mA,

$$6.4 = \frac{8}{5} + \frac{8-3}{1.25} + \frac{8-7}{R_3}$$

$$\underline{\underline{R_3 = 1.25\ k\Omega}}$$

* At $v = 3V$, circuit provides $i = \frac{3}{5} = 0.6$ mA while ideally i should be $0.1 \times 9 = 0.9$ mA. Thus the error is -0.3 mA.

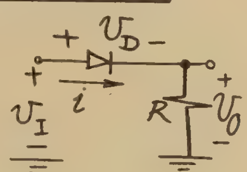
* At $v = 5V$, circuit provides $i = \frac{5}{5} + \frac{5-3}{1.25} = 2.6$ mA while ideally i should be $0.1 \times 25 = 2.5$ mA. Thus the error is $+0.1$ mA.

* At $v = 7V$, circuit provides $i = \frac{7}{5} + \frac{7-3}{1.25} = 4.6$ mA while ideally $i = 0.1 \times 49 = 4.9$ mA. Thus the error is -0.3 mA.

* At $v = 10V$, circuit provides $i = \frac{10}{5} + \frac{10-3}{1.25} + \frac{10-7}{1.25} = 10$ mA while ideally $i = 0.1 \times 100 = 10$ mA. Thus the error is 0.

CHAPTER 5 — PROBLEMS

5.1 $i = I_S e^{v_D / nV_T}$



$$v_O = iR = I_S R e^{v_D/nV_T}$$

$$v_D = v_I - v_O$$

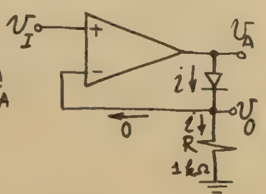
$$v_O = I_S R e^{(v_I - v_O)/nV_T}$$

$$v_I - v_O = nV_T \ln\left(\frac{v_O}{I_S R}\right)$$

Thus:

$$v_I = v_O + nV_T \ln(v_O / I_S R)$$

5.2 $v_I = 1$ mV, $v_O = 1$ mV



$$i = \frac{1\text{mV}}{1k\Omega} = 1\mu A$$

$$v_D = 0.7 + 2 \times 0.025 \ln \frac{1\mu A}{1\text{mA}}$$

$$= 0.355\ V$$

$$v_A = 0.356\ V$$

For $v_I = 1V$, $v_O = 1V$

$$i = \frac{1V}{1k\Omega} = 1\text{mA} \qquad v_D = 0.7V$$

$$v_A = 1.7V$$

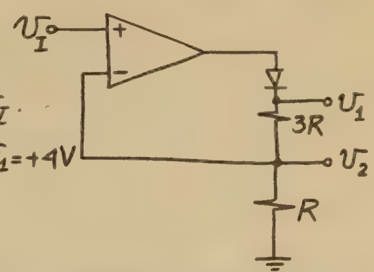
5.3

For $v_I < 0$, $v_1 = 0$.

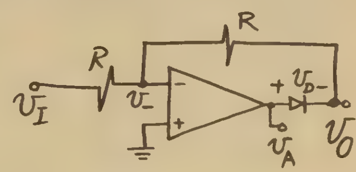
For $v_I \geq 0$, $v_2 = v_I$,

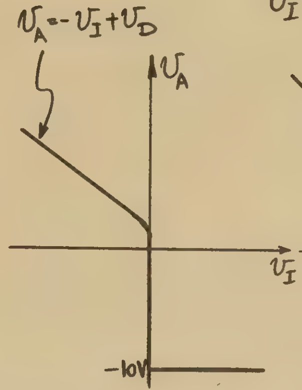
and $v_1 = 4v_2 = 4v_I$.

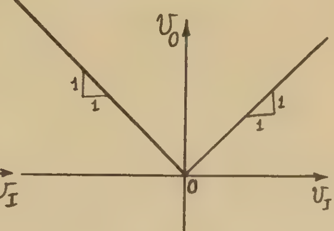
For $v_I = +1V$, $v_1 = +4V$



5.4



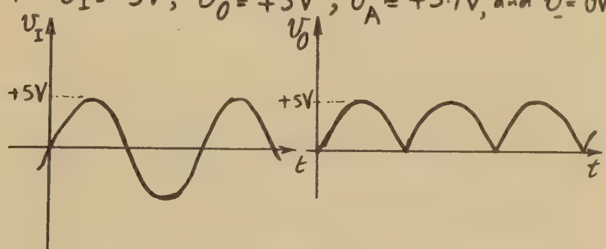




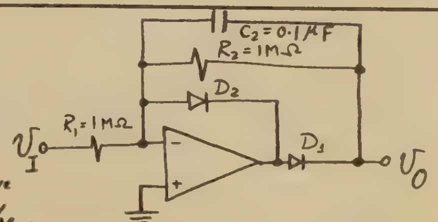
$v_O = |v_I|$

For $v_I = +5V$, $v_O = +5V$, $v_A = -10V$, and $v_- = +5V$

For $v_I = -5V$, $v_O = +5V$, $v_A \simeq +5.7V$, and $v_- = 0V$

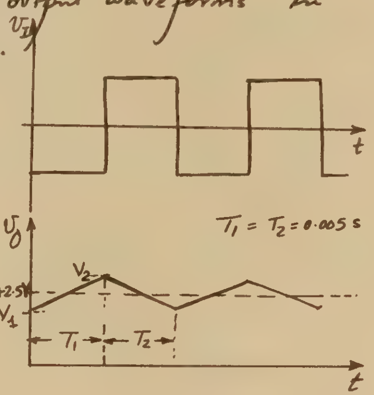


5.5



Without the capacitor, the output consists of the positive halves of the square wave. That is, the output consists of periodic positive pulses going from 0 to +5V with a 100-Hz frequency. This waveform has an average value of +2.5V. Adding the capacitor provides a low-pass filter effect. Specifically, C_2

together with R_2 act as a low-pass filter with a time constant $\tau = 0.1\,s$. This time constant is much longer than half the period of the square wave ($\frac{T}{2} = 0.005s$). We should therefore expect that the output will contain a dc component equal to $+2.5V$ superimposed on which will be a ripple waveform that will look almost triangular. The Figure shows the input and output waveforms in the steady state.



The values of V_1 and V_2 can be found as follows.

For the interval T_1 we can write:

$$V_2 = 5 - (5 - V_1)\, e^{-0.005/0.1}$$

$$\simeq 5 - (5 - V_1)(1 - 0.05)$$

$$V_2 = 0.95\,V_1 + 0.25 \qquad (1)$$

For the interval T_2 we can write:

$$V_1 = V_2\, e^{-0.005/0.1} \simeq 0.95\,V_2 \qquad (2)$$

Substituting in (1) provides

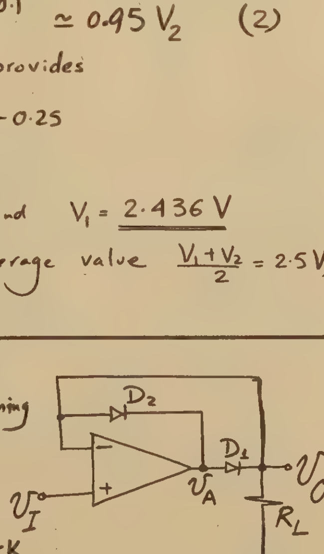
$$V_2 = 0.95 \times 0.95\,V_2 + 0.25$$

which results in

$\underline{V_2 = 2.564V}$ and $\underline{V_1 = 2.436\,V}$

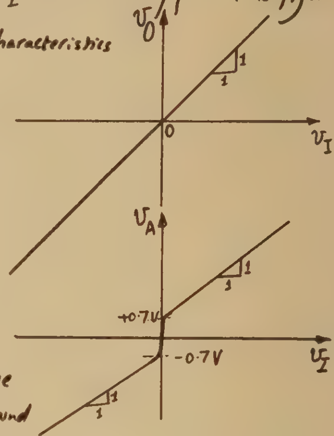
Note that the average value $\frac{V_1 + V_2}{2} = 2.5V$, as expected.

$\boxed{5.6}$ When $v_I > 0$, v_A goes positive turning D_1 on. D_1 then conducts and closes the negative-feedback loop, thus causing $v_O = v_I$. In this



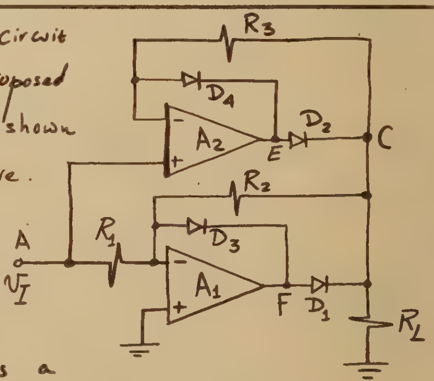
case v_A is one diode drop greater than v_I and D_2 will be off.

When $v_I < 0$, v_A goes negative, turning D_2 on. Current now flows from ground through R_L and D_2 and into the output terminal of the op amp. D_2 thus closes the negative-feedback loop of the op amp, causing $v_O = v_I$. v_A will be one diode drop below v_I and D_1 will be off. The Figure shows the transfer characteristics v_O vs. v_I and v_A vs. v_I. It is obvious that the circuit no longer operates as a half-wave rectifier. Note that the change of polarity of v_A around



$v_I = 0$ may make the circuit useful as a comparator for detecting zero crossings.

$\boxed{5.7}$ The circuit with the proposed additions is shown in the Figure. We shall now show that it indeed operates as a full-wave rectifier and that op-amp saturation is avoided.



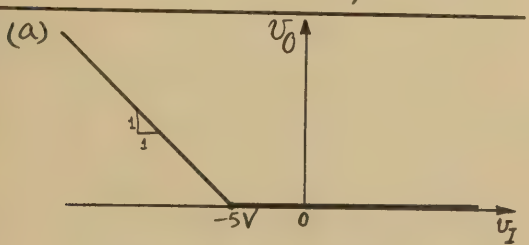
For $v_I > 0$, D_3 turns on and closes the loop around A_1. v_F will be at about $-0.7V$ and diode D_1 will be off. v_E will go positive, thus turning D_2 on. Diode D_2 conducts

through R_L and no current flows through R_3. The negative-feedback loop will thus be closed and v_C will be equal to v_I (and thus positive). v_E will be one diode drop higher than v_I and thus D_4 will be off. Note that current will flow through R_2. This current adds to the current through D_3, keeping it on as already assumed.

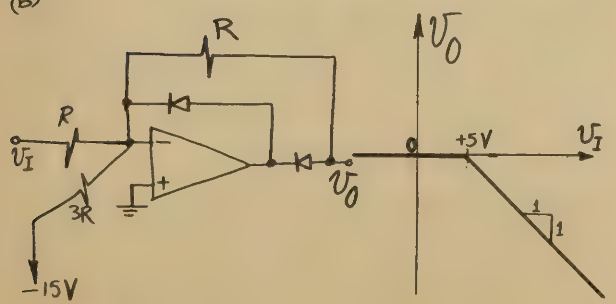
For $v_I < 0$, v_F will go positive turning D_1 on. D_1 and R_2 close the negative-feedback loop of A_1, and assuming $R_1 = R_2$ then $v_C = -v_I$. Thus v_C will be positive and v_F will be greater than v_C by a diode drop. Consider now the operation of A_2. The combination of positive v_C and negative v_I causes current to flow through R_3 and D_4. The loop around A_2 will thus be closed and the voltage drop across R_3 will be $2|v_I|$. v_E will be one diode drop below v_I and thus D_2 will obviously be off.

Thus the circuit operates as a full-wave rectifier and avoids op-amp saturation.

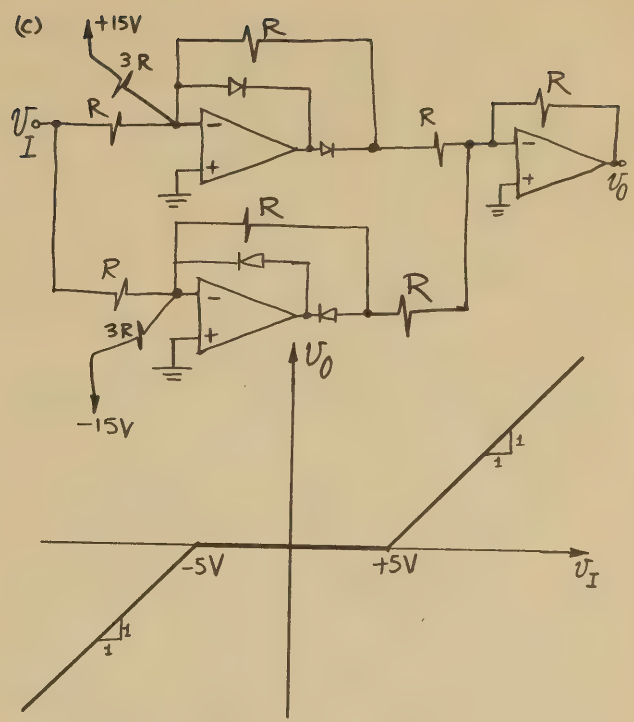
5.8 (a)



(b)



(c)



5.9 Peak current in each diode = Peak current in load = $\frac{\text{average current in load}}{(2/\pi)} = 1.57\ \text{A}$

5.10 The voltage across the load is dc of value $5 - 2 \times 0.7 = \underline{3.6\ V}$

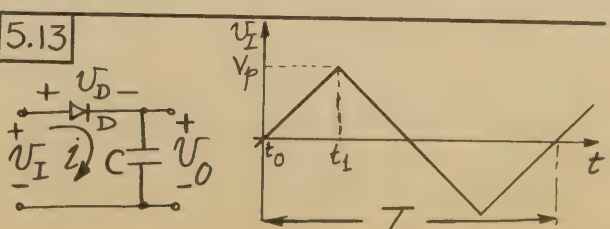
5.11 Average current in meter $= \frac{2}{\pi} \times \frac{0.1\sqrt{2}}{0.1\,k\Omega} = 0.9\,mA$

* Nothing happens because operation is independent of meter resistance.

5.12 Assume that the op amp saturates at ± 10V.

A	B	C
−1V	−1V	−10V
−1V	+1V	+1V
+1V	−1V	+1V
+1V	+1V	+1V

5.13



$$i = C\frac{dv_O}{dt} = I_S\, e^{v_D/nV_T}$$

Since $v_D = v_I - v_O$, then

$$C\frac{dv_O}{dt} = I_S\, e^{(v_I - v_O)/nV_T} \qquad ----(1)$$

For $t_0 \le t \le t_1$ we have

$$v_I = \left(\frac{V_p}{T/4}\right) t$$

Substituting in (1) yields

$$C\frac{dv_O}{dt} = I_S\, e^{\left(\frac{4V_p}{nV_T}\right)\left(\frac{t}{T}\right)}\, e^{-(v_O/nV_T)}$$

Thus

$$\frac{C}{I_S}\frac{dv_O}{dt}\, e^{v_O/nV_T} = e^{\left(\frac{4V_p}{nV_T}\right)\left(\frac{t}{T}\right)}$$

$$\frac{C}{I_S}\int_0^{v_O(t)} e^{v_O/nV_T}\, dv_O = \int_0^t e^{\left(\frac{4V_p}{nV_T}\right)\left(\frac{t}{T}\right)} dt$$

$$\left(\frac{C}{I_S}\right)(nV_T)\left[e^{v_O/nV_T}\right]_0^{v_O(t)} = \left(\frac{nV_T}{4V_p}\right)T\left[e^{\left(\frac{4V_p}{nV_T}\right)\left(\frac{t}{T}\right)}\right]_0^t$$

$$\left[e^{v_O/nV_T} - 1\right] = \left(\frac{I_S T}{4CV_p}\right)\left[e^{\frac{4V_p}{nV_T}\frac{t}{T}} - 1\right]$$

For t slightly greater than t_0 this can be approximated as follows

$$e^{v_O/nV_T} \simeq \left(\frac{I_S T}{4CV_p}\right) e^{\left(\frac{4V_p}{nV_T}\right)\left(\frac{t}{T}\right)}$$

Thus

$$\boxed{v_O = nV_T \ln\left(\frac{I_S T}{4CV_p}\right) + \left(\frac{V_p}{T/4}\right) t}$$

For the case $T = 40$ ms, $C = 10\,\mu$F, $V_p = 10$ V and for a 1-mA diode that follows the 0.1 V/decade model (i.e. has $v_D = 0.7$ V at $i_D = 1$ mA and $n = 1.737$) we have

$$I_S = 10^{-3}\, e^{-0.7/(1.737 \times 0.025)} \qquad (2)$$

$$v_O(t_1) = +9.2\text{V}$$

Analysis beyond $t = t_1$

Eqn. (1) still applies but now v_I is given by

$$v_I = 10 - \left(\frac{V_p}{T/4}\right) t$$

where for simplicity we have taken $t=0$ at t_1. Substituting in Eqn. (1),

$$C\frac{dv_O}{dt}\, e^{v_O/nV_T} = I_S\, e^{\left\{\left[10 - \frac{V_p}{T/4}t\right]/nV_T\right\}}$$

$$\int_{9.2}^{v_O(t)} \frac{C}{I_S}\, e^{v_O/nV_T}\, dv_O = \int_0^t e^{\left[10 - \frac{V_p}{T/4}t\right]/nV_T} dt$$

which results in

$$e^{(v_O - 9.2)/nV_T} = 1 + \left(\frac{I_S T}{4CV_p}\right) e^{0.8/nV_T}\left[1 - e^{\frac{-4V_p}{nV_T}\frac{t}{T}}\right]$$

Substituting for I_S from (2)

$$e^{(v_O - 9.2)/nV_T} = 1 + \frac{10^{-2} \times T}{4CV_p}\left[1 - e^{-\frac{4V_p}{nV_T}\frac{t}{T}}\right]$$

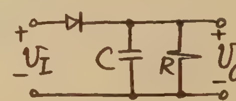
As t increases, $e^{\frac{-4V_p}{nV_T}\frac{t}{T}}$ becomes negligibly small and we can make the approximation

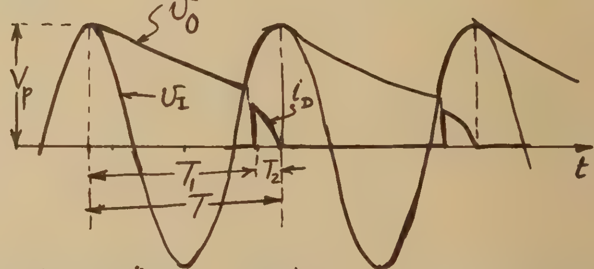
$$e^{(v_O - 9.2)/nV_T} \simeq 1 + \frac{10^{-2} \times T}{4CV_p}$$

which we can use to find the maximum value reached by v_O as,

$$v_O = 9.2 + nV_T \ln\left[1 + \frac{10^{-2} \times T}{4CV_p}\right] = \underline{\underline{9.23\text{V}}}$$

5.14





The peak output voltage will be approximately equal to the peak of the input, $V_p = 10\sqrt{2} = 14.14$ V. For the exponential discharge of the capacitor we can write

$$V_p\, e^{-T_1/\tau} = V_p \cos(\omega T_2)$$

where $T_1 = T - T_2$, and $T = \frac{1}{f} = \frac{1}{60}$ s. τ is given by $\tau = CR = 0.1$ s.

Thus, $e^{-T/\tau}\, e^{T_2/\tau} = \cos(\omega T_2)$

For $T_2 \ll \tau$, ωT_2 will be a small angle and we may make the approximation

$$e^{-\frac{1}{60\times 0.1}}\left(1+\frac{T_2}{\tau}\right) \simeq 1-\frac{(\omega T_2)^2}{2}$$

which yields $\omega T_2 \simeq 0.526$ rad.

The peak-to-peak ripple voltage V_r is given by

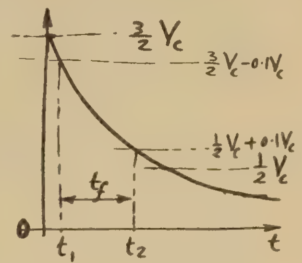
$$V_r = V_p - V_p \cos\omega T_2 = 14.14(1-\cos 0.526)$$
$$= \underline{\underline{1.91\,V}}$$

The average value of the output voltage $= 14.14 - \frac{1}{2}\times 1.91 = \underline{\underline{13.2\,V}}$

Since the diode is assumed ideal, when it conducts the capacitor voltage is equal to v_I. Thus the capacitor current will be $C\frac{dv_I}{dt}$ which is maximum at the start of diode conduction. Neglecting the current through R, the peak diode current is

$$\hat{I}_d = \omega C_f V_p \sin(\omega T_2) = \underline{\underline{0.27A}}$$

To find the fall time, we note that the capacitor discharges through the 10-kΩ load resistor. Because $m = 0.5$, the capacitor voltage starts at $\frac{3}{2}V_c$ volts and heads toward zero volts but stops at $\frac{1}{2}V_c$.



$$v(t) = \frac{3}{2}V_c\, e^{-t/\tau}$$

$$\frac{3}{2}V_c - 0.1V_c = \frac{3}{2}V_c\, e^{-t_1/\tau}$$

$$t_1 = 0.07\tau$$

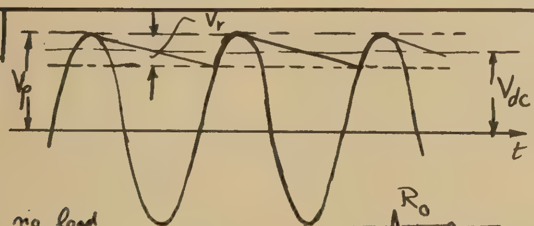
$$\frac{1}{2}V_c + 0.1V_c = \frac{3}{2}V_c\, e^{-t_2/\tau}$$

$$t_2 = 0.92\tau$$

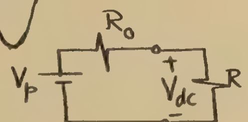
$$t_f = t_2 - t_1 = 0.85\tau = \underline{\underline{27.2\,\mu s}}$$

$\boxed{5.17}$ The output signal will vary between 0 V and −15 V and will have an average of $\underline{\underline{-10\,V}}$.

$\boxed{5.15}$



With no load, the output voltage is equal approximately to V_p.



With a finite R, the output dc voltage is $V_{dc} = V_p - \frac{V_r}{2} = V_p - \frac{V_p}{2fCR} = V_p\left(1-\frac{1}{2fCR}\right)$

From the equivalent circuit:

$$V_{dc} = V_p\frac{R}{R+R_0} = V_p\frac{1}{1+\frac{R_0}{R}} \simeq V_p\left(1-\frac{R_0}{R}\right)$$

Thus $\frac{R_0}{R} = \frac{1}{2fCR}$

$$R_0 = 1/2fC$$

$\boxed{5.16}$ As in Example 5.2, choose $C = 3{,}200$ pF. Thus,

Rise Time $\simeq 2.2\,C R_s = 2.2\times 3200\times 10^{-12}\times 1\times 10^{3} = 7\,\mu s.$

$\boxed{5.18}$

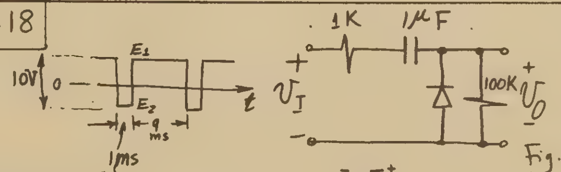


Fig. 1

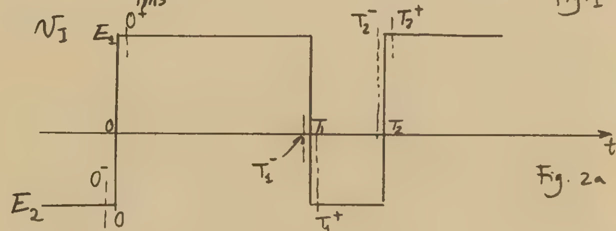


Fig. 2a

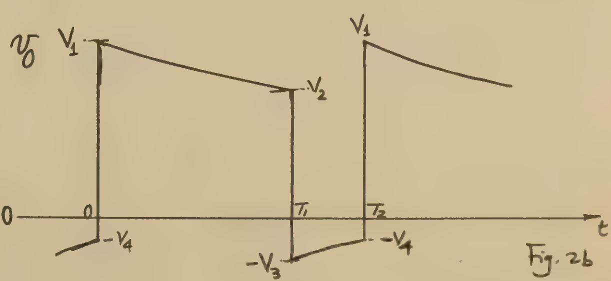


Fig. 2b

We shall provide a detailed step-by-step solution of the steady-state response of this nonlinear circuit.

At time $t=0^-$:

$v_I = E_2$, $v_O = -V_4$ and the diode is still conducting. If we neglect the current through the 100 kΩ resistor, the circuit reduces to that shown in Fig. 3, for which we write

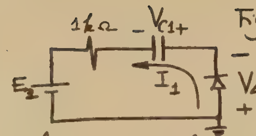


Fig. 3

$$V_4 + V_{C1} + I_1 \times 10^3 + E_2 = 0$$

i.e. $E_2 = -V_4 - V_{C1} - I_1 \times 10^3$ - - - (1)

Also, $I_1 = I_S\, e^{V_4/nV_T}$ — - - (2)

Thus, $E_2 = -V_{C1} - V_4 - I_S \times 10^3 \times e^{V_4/nV_T}$ - - (3)

At time $t=0^+$

$v_I = E_1$, $v_O = V_1$, the capacitor voltage remains unchanged (i.e. $=V_{C1}$), and the diode cuts-off. The circuit reduces to that in Fig. 4 for which we can write

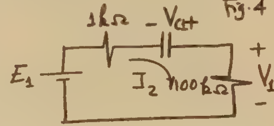


Fig. 4

$$E_1 = I_2 \times 1\,k\Omega - V_{C1} + I_2 \times 100\,k\Omega \qquad (4)$$

where $I_2 = V_1 / 100\ k\Omega$ (5)

Thus,

$$E_1 = 1.01\, V_1 - V_{C1} \qquad (6)$$

Subtracting (3) from (6) yields

$$10 = 1.01\, V_1 + V_4 + I_S \times 10^3\, e^{V_4/nV_T} \quad \text{------(7)}$$

For time $0 \leq t \leq T_1^-$:

The capacitor discharges through the series combination of the 1kΩ and 100-kΩ resistors, as shown in Fig. 5. We can write: $v_O = V_1\, e^{-t/\tau}$ (8)

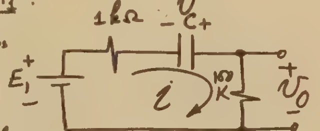


Fig. 5.

where $\tau = 1 \times 10^{-6} \times 101 \times 10^3 = 0.101$ s

At time $t = T_1^-$

$v_I = E_1$, $v_O = V_2$, and the circuit is shown in Fig. 6. V_2 can be obtained by substituting $t = T_1$ in (8),

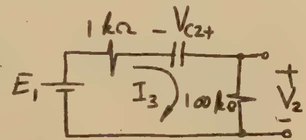


Fig. 6

$$V_2 = V_1\, e^{-T_1/\tau}$$

Since $\tau = 0.101$ s and $T_1 = 9 \times 10^{-3}$ s we have

$$V_2 = 0.91\, V_1 \quad \text{- - - - - (9)}$$

We also have

$$I_3 = V_2 / 100\ k\Omega \qquad (10)$$

and for the circuit in Fig. (6) we can write

$$E_1 = 1.01 V_2 - V_{C2} \quad \text{- - - - - (11)}$$

At time $t = T_1^+$:

$v_I = E_2$, the diode conducts, and $v_O = -V_3$. The capacitor voltage remains unchanged at V_{C2}. We can neglect the current through the 100-kΩ resistor and reduce the circuit to that in Fig. 7. Note that the capacitor begins to charge up to it's other voltage of V_{C1}. We can write:

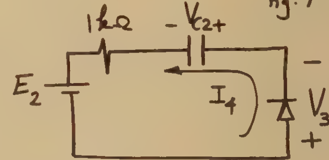


Fig. 7

$$E_2 = -V_3 - V_{C2} - I_4 \times 10^3 \qquad (12)$$

where $I_4 = I_S\, e^{V_3/nV_T}$ (13)

Thus, $E_2 = -V_{C2} - V_3 - I_S \times 10^3 \times e^{V_3/nV_T}$ (14)

Subtracting (14) from (11) results in

$$10 = 1.01\, V_2 + V_3 + I_S \times 10^3 \times e^{V_3/nV_T} \qquad (15)$$

For time $T_1^+ \leq t \leq T_2^-$

This is the interval during which the diode conducts and replenishes the charge lost by the capacitor during the $t=0$ to T_1 interval. The circuit reduces to that in Fig. 8 and we can write

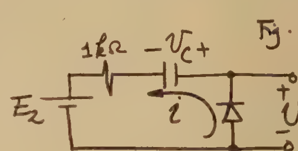


Fig. 8

$$-v_O + v_C + i \times 10^3 + E_2 = 0$$

Thus, $-\dfrac{dv_O}{dt} + \dfrac{dv_C}{dt} + 10^3 \times \dfrac{di}{dt} = 0$

But $i = C\, dv_C/dt$.

Thus $-\dfrac{dv_O}{dt} + \dfrac{i}{C} + 10^3 \times \dfrac{di}{dt} = 0$ (16)

Since $i = I_S\, e^{-v_O/nV_T}$

then $\dfrac{di}{dt} = -\dfrac{I_S}{nV_T}\, e^{-v_O/nV_T}\, \dfrac{dv_O}{dt}$

Thus we can obtain by substituting in (16)

$$\frac{dv_O}{dt}\left[e^{v_O/nV_T} + \frac{10^3\, I_S}{nV_T}\right] = \frac{I_S}{C} \qquad (17)$$

Since the diode is a 1-mA device we have

$$I_S = 10^{-3}\, e^{-0.7/nV_T} \qquad (18)$$

Substituting in (17) provides

$$e^{v_O/nV_T}\, dv_O + \frac{10^3 \times 10^{-3}\, e^{-0.7/nV_T}}{nV_T}\, dv_O = \frac{10^{-3}\, e^{-0.7/nV_T}}{1\times 10^{-6}}\, dt$$

Integrating this equation over the interval $T_1^+ \leq t \leq T_2^-$ provides

$$\int_{-V_3}^{-V_4} e^{v_O/nV_T}\, dv_O + \int_{-V_3}^{-V_4} \frac{e^{-0.7/nV_T}}{nV_T}\, dv_O = 10^3 \times e^{-0.7/nV_T} \times \underbrace{10^{-3}}_{1\,ms}$$

This results in

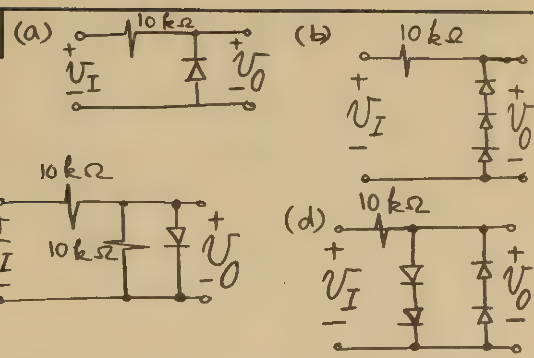
$$nV_T \left[e^{(0.7-V_4)/nV_T} - e^{(0.7-V_3)/nV_T} \right] + \frac{V_3 - V_4}{nV_T} = 1 \qquad \text{------} (19)$$

This completes the analysis! We now have four equations (Equations (7), (9), (15) and (19)) in the four unknowns V_1, V_2, V_3 and V_4. Solution can be obtained as follows: Combine (7) and (15) to eliminate V_1, thus obtaining an equation in V_3 and V_4 that be solved together with (19) to obtain V_3 and V_4. This leads to:

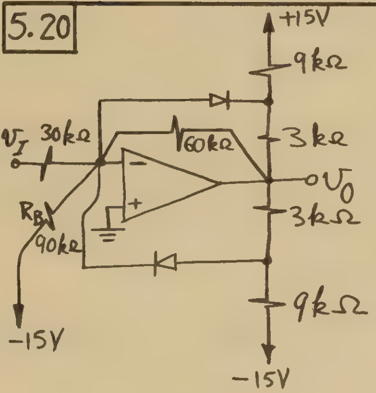
$V_1 = +8.74V$, $V_2 = 7.95V$, $V_3 = 0.71V$, and $V_4 = 0.67V$.

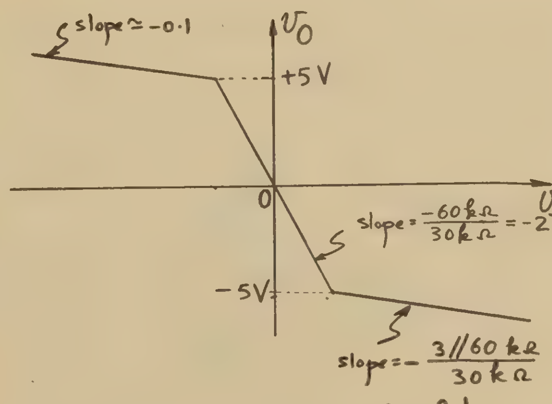
Finally note that this detailed solution should be contrasted with the much more approximate but quicker method used in Example 5.3. This problem, however, is more difficult than that of the Example because of the inclusion of source resistance.

5.19

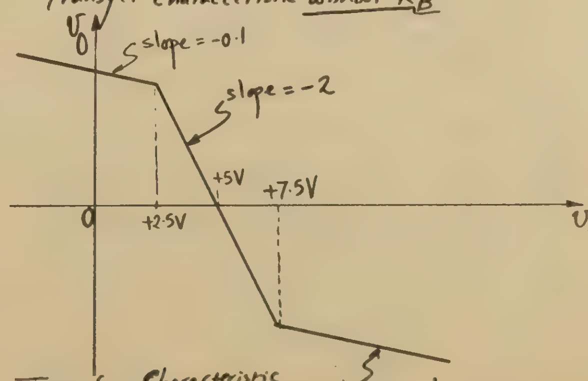


5.20





Transfer characteristic without R_B

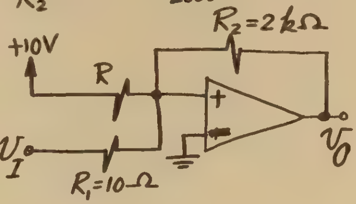


Transfer Characteristic With R_B

5.21 $V_{T1} = -L_+ \frac{R_1}{R_2} = -10 \times \frac{10}{2000} = -50\text{mV}$

$V_{T2} = -L_- \frac{R_1}{R_2} = -1 \times -10 \times \frac{10}{2000} = +50\text{ mV}$

With R included:

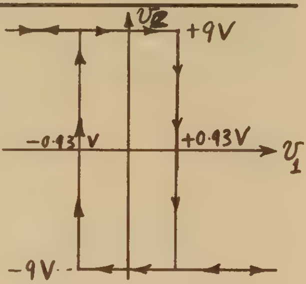


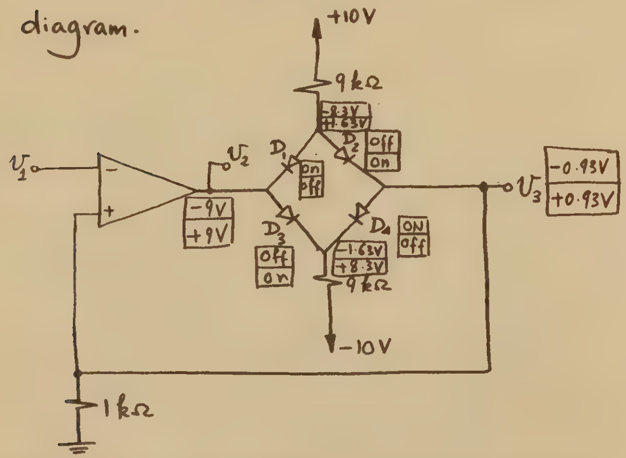
$$v_+ = v_O \frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R}} + 10 \frac{\frac{1}{R}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R}} + v_I \frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R}}$$

Thus: $V_{T1} = R_1 \left(\frac{-10}{R} - \frac{L_+}{R_2} \right) = -60\text{ mV}$

$V_{T2} = R_1 \left(\frac{-10}{R} - \frac{L_-}{R_2} \right) = +40\text{mV}$

5.22 The circuit behaves as a bistable and has the transfer characteristics shown. The voltage levels and the diodes condition (i.e. on or off) in the two states are indicated on the circuit diagram.





5.23

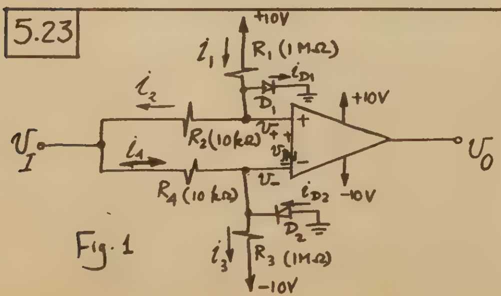


Fig. 2 shows the transfer characteristic of the circuit. As indicated the output will be high, at the positive saturation voltage of the op amp, for input voltages in a narrow range centered around $v_I = 0$. Specifically $v_O = +10\text{V}$ for $-V \le v_I \le +V$, otherwise $v_O = -10\text{V}$ (the negative saturation voltage of the op amp).

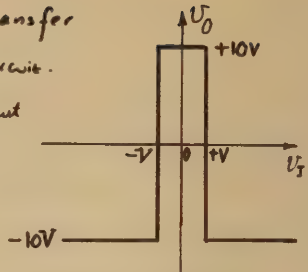


Fig. 2

If the input is a triangular waveform, as indicated in Fig. 3, the output will consist of narrow pulses at <u>twice</u> the frequency of the input. This comes about because of the unusual transfer characteristic depicted in Fig. 2.

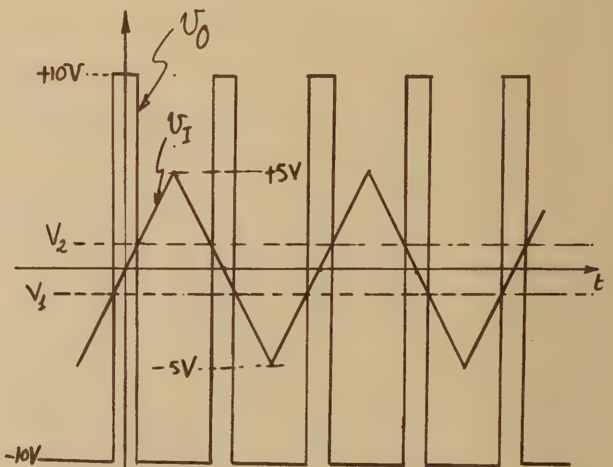


Fig. 3

We shall now explain how the transfer characteristic of Fig. 2 comes about and in the process we shall find the value of the threshold voltages, i.e. the value of V. We shall assume that the diodes are 1-mA units with 0.1V/decade-of-current model.

For $v_I = 0\text{V}$ both diodes will be conducting

negligible current ; $i_{D_1} = i_{D_2} \simeq 0$, and $i_1 = i_2 = i_3 = i_4 = 10/1.01 \simeq 9.9\ \mu A$. Thus $V_+ = +99$ mV and $V_- = -99$mV which means that the op amp input voltage $V_{IN} \equiv V_+ - V_- = +198$ mV. Thus the output will be saturated at +10V. Since each diode is forward biased with approx. 0.1V it will be conducting a current of $10^{-3} \times 10^{-6} = 10^{-9} A = 10^{-3} \mu A$ which is indeed negligible as assumed.

As V_I is increased in the positive direction both V_+ and V_- increase. Thus diode D_1 conducts more and more current while D_2 turns off completely. For instance for $V_I = +0.2V$, $V_+ = +297$ mV, $V_- = +99$ mV, $V_{IN} = 198$ mV, and $V_O = +10V$. At this point $i_{D_1} \simeq 0.1 \mu A$ (still negligible) and $i_{D_2} = 0$. This continues until V_I increases to the point that $i_{D_1} = i_1$, at which point $i_2 = 0$ and $V_+ = V_I$. This value of V_I is approx. 0.5V, for at 0.5 V $i_{D_1} = 10\mu A$ and $i_1 = 9.5\ \mu A$. Beyond this point, increases in V_I cause i_2 to reverse direction and thus add to i_1 and increase i_{D_1}. This in turn causes V_{D_1} and hence V_+ to increase. Such increase, however, is less than that in V_- and a point is reached at which V_- exceeds V_+ and the op amp saturates at the negative level of $-10V$. For instance, for $V_I = +1V$, $V_+ \simeq 0.6$ V and $V_- = 0.9V$; thus $V_{IN} = -0.3V$ and $V_O = -10V$. Switching occurs at $V_I \simeq 0.6V$. Thus, $V_1 \simeq +0.6$ V.

For V_I going negative the exact complement of the above occurs. In fact because of the complementary symmetry of the circuit, $V_2 = -V_1 \simeq -0.6V$.

5.24

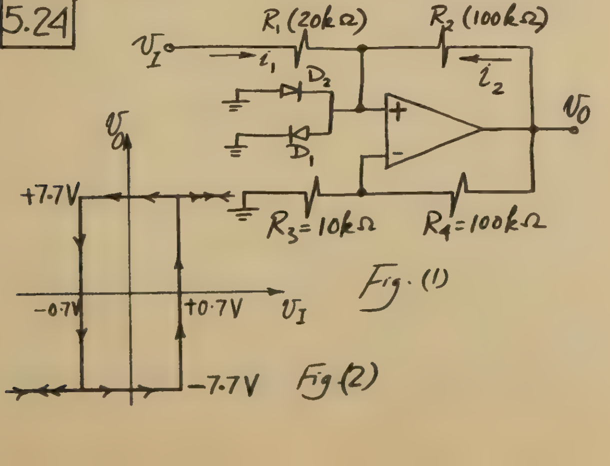


Fig. (1)

Fig (2)

Please note that in the first printing of the Text the circuit diagram had an error ; the op-amp input terminals were interchanged.

The circuit is a bistable having the transfer characteristic shown in Fig (2). This can be verified as follows: Assume D_1 to be conducting and thus $V_+ = +0.7V$. This voltage will be amplified by the op amp which has a closed-loop gain of $\left(1 + \frac{R_4}{R_3}\right) = 11$, and thus V_O will be +7.7V. Thus the current i_2 will be $\frac{7.7 - 0.7}{100} = 0.07$ mA which adds to whatever the value of i_1 happens to be and causes a net positive current to flow through D_1, thus maintaining it on and keeping $V_+ = +0.7V$ as initially assumed.

This stable state persists for all positive values of V_I and even for $V_I = 0$. In fact for this stable state to change we have to make V_I sufficiently negative so that $i_1 = -0.07$ mA at which point D_1 turns off. This occurs when $V_I = -0.7V$ which is the value of the lower threshold. For $V_I < -0.7V$, D_2 will turn on, $V_+ = -0.7V$ and $V_O = -7.7V$, which is the other stable state. One can easily show that the circuit will remain in this stable state unless V_I is made positive and greater than +0.7V. Note that the threshold voltages and the output voltages are independent of the characteristics of the op amp.

The op amp together with R_3, R_4 and R_2 forms a negative resistance between the positive input terminal and ground (see Example 3.5). This resistance is $-10k\Omega$. The net resistance of the source that feeds the two diodes will be $\frac{-10 \times +20}{-10 + 20} = -20\ k\Omega$. It is this net negative resistance that gives the circuit its bistable behaviour.

As R_1 is reduced toward 10 $k\Omega$, the width of the hysteresis is reduced. This can

be readily verified numerically. As R_1 reaches 10 kΩ the hysteresis becomes of zero width, as shown in Fig. 3. At this value of R_1 the net resistance is ∞ and the diodes are in effect fed by constant current; positive for $v_I > 0$ and negative for $v_I < 0$.

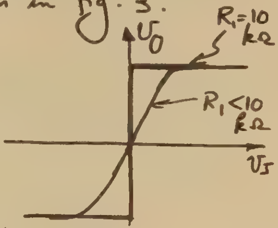
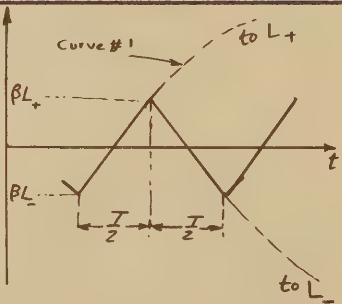


Fig. 3

For $R_1 < 10$ kΩ the transfer characteristic takes the shape shown in Fig. 3. Here the net resistance becomes positive and no bistability exists. The diodes are fed in effect by a voltage source with a finite positive source resistance.

5.25



Initial slope of the exponential curve #1

$$= \frac{L_+ - \beta L_-}{\tau}$$

$$= \frac{L_+ + \beta L_+}{\tau}$$

$$= \frac{(1+\beta)L_+}{\tau}$$

Assuming that the waveform is almost linear then this slope is equal to $\frac{\beta L_+ - \beta L_-}{T/2} =$

$$\frac{2\beta L_+}{T/2} = \frac{4\beta L_+}{T}$$

Thus $\frac{(1+\beta)L_+}{\tau} = \frac{4\beta L_+}{T}$

$$T = \tau\,\frac{4\beta}{1+\beta} = C_1 R_3\,\frac{4\beta}{1+\beta}$$

which is the required result.

For $f = 1$ kHz, $T = 1$ ms. If $R_2 = 10 R_1$ then $\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{11}$. Thus $C_1 R_3 = \frac{(1+\beta)T}{4\beta} = 3$ ms

Selecting $R_3 = 100$ kΩ then $C_1 = 0.03\ \mu F$.

Selecting $R_1 = 10$ kΩ then $R_2 = 100$ kΩ.

5.26 Refer to Fig. 5.35. For $f = 1$ kHz, $T = 1$ ms; thus $T_1 = T_2 = 0.5$ ms.

$$T_1 = CR\,\frac{V_{TH} - V_{TL}}{L_+}$$

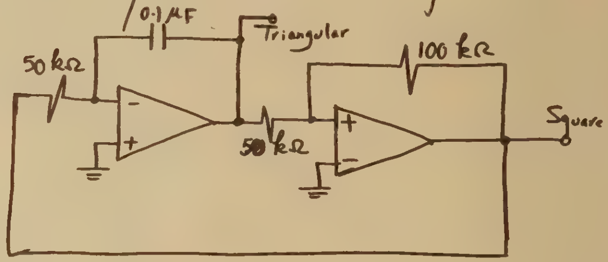
For 5-V amplitude for the triangular waves, $V_{TH} = -V_{TL} = 5$ V. Thus,

$$T_1 = CR\,\frac{5-(-5)}{10} = CR$$

$$CR = 0.5 \text{ ms}$$

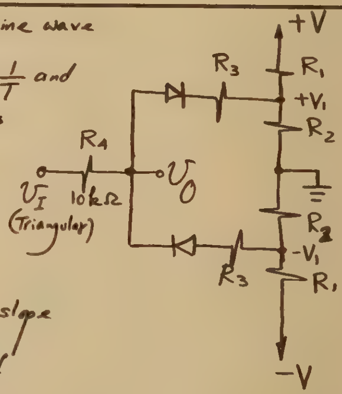
$C = 0.1\ \mu F$ $\quad R = 50$ kΩ

The complete circuit is as follows:



5.27 Consider a sine wave with frequency $f = \frac{1}{T}$ and amplitude $\hat{V}$. Its zero-crossing slope is

$$\omega \hat{V} = \frac{2\pi}{T}\hat{V}$$

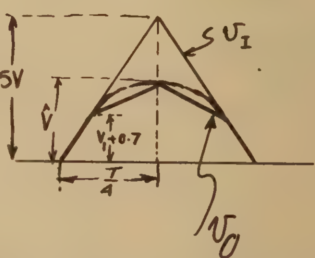


To make this slope equal to that of the triangle then

$$\frac{2\pi}{T}\hat{V} = \frac{5}{T/4}$$

$$\hat{V} = \frac{10}{\pi} = 3.18 \text{ V}$$

Selecting $V_1 + 0.7 = 2$ V yields $V_1 = 1.3$ V.



From the circuit we see that for $v_I \geq V_1 + 0.7$ we have $v_O = V_1 + 0.7 + \frac{v_I - V_1 - 0.7}{R_3 + R_4} R_3$

We require that $v_O = 3.18$ V at $v_I = +5$ V. Substituting these values and $V_1 = 1.3$ V

results in

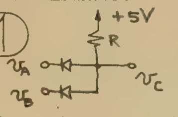
$$3.18 = 2 + \frac{3R_3}{R_3 + R_4}$$

$$\frac{R_4}{R_3} = 1.54$$

$$R_3 = \frac{10}{1.54} = \underline{\underline{6.48\ k\Omega}}$$

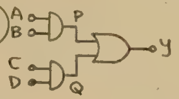
CHAPTER 6 - EXERCISES

(6.1)



For real diodes, the forward drop is 0.7 volts. The lowest input extracts all of the current from R with C held 0.7 volts higher. Since the lower input is +1.0, the output falls to +1.7 volts.

(6.2)



$P = A.B \ ; Q = C.D$

$y = P + Q = A.B + C.D$
$= AB + CD$
$= AB \vee CD$

where v is a notation which is also used for "OR" by some designers

(6.3)

A	B	f	minterms
0	0	0	$\bar{A}\bar{B}$
0	1	1	$\bar{A}B$
1	0	1	$A\bar{B}$
1	1	0	AB

See from the 1's of f that
$f = \bar{A}B + A\bar{B}$
See from the 0's of f that
$\bar{f} = \bar{A}\bar{B} + AB$

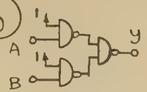
(6.4) N binary variables may be combined in 2^N different ways. Thus there are 2^N minterms of N variables and thus 2^3 or 8 for 3 variables

(6.5) $\bar{f}_2 = \bar{A}BC + ABC + AB\bar{C}$ using idempotence and distribution:

$= BC(\bar{A} + A) + AB(\bar{C} + C)$ using complementation:
$= BC + AB$
$= B(C + A)$

Thus $f_2 = \overline{B(C+A)} = \bar{B} + \overline{A+C} = \bar{B} + \bar{A}\bar{C}$ ie Eq 6.3

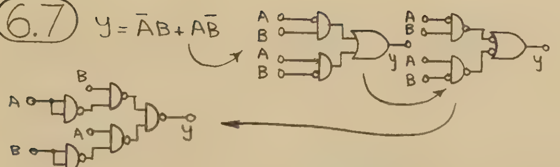
(6.6)



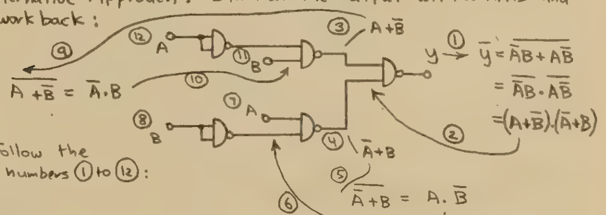
a) $y = \overline{(\overline{A.1}).(\overline{B.1})} = \bar{\bar{A}}.\bar{\bar{B}}$ ... $= \overline{\bar{A}.\bar{B}} = A + B$

b) $y = \overline{(\overline{A.B}).1} = \overline{\overline{A.B}} = A.B$

(6.7) $y = \bar{A}B + A\bar{B}$



Alternative Approach: Start at the output with a NAND and work back:



Follow the numbers ① to ⑫:

(6.8) a) Noise Margins: $\Delta 0 = V_{IL} - V_{OL} = 0.8 - 0.4 = 0.4V$ where the highest low input and highest low output are chosen. $\Delta 1 = V_{OH} - V_{IH} = 2.4 - 2.0 = 0.4V$ where the lowest high output and lowest high input are chosen.

b) Current with output low is typically 12mA and with output high is about 4mA. Thus for 50% duty cycle, the average current is $1/2(12) + 1/2(4) = 8mA$, while the voltage is 5V. Thus the average power per gate is $\frac{8 \times 5}{4} = 10\,mW$.

c) Output switches from 0.22V to 3.3V or by 3.08V each 1µs. $CV = IT \rightarrow$ Average current from the supply to charge 45pF by 3.08V is $\frac{45 \times 10^{-12} \times 3.08}{10^{-6}} = 138.6\,\mu A$. Thus the average supply power is $5 \times 139 = 0.69\,mW$.

d) The average propagation delay is $\frac{11+7}{2}$ or 9 ns, which with an average power dissipation of 10 mw gives a delay-power product of 9×10 or 90 pJ

(6.9) a) When $\bar{S}_d = 0$, the output of the lower right AND (ie $\bar{Q}$) is forced to 0 causing the output of the connected NAND to rise to 1 independant of its other input. Since $\bar{R}_d = 1$ also, the output of the right top AND (ie Q) is forced to 1. The same argument applies to $\bar{R}_d = 0$ and $\bar{S}_d = 1$ such that Q is forced to 0 (and $\bar{Q}$ to 1) all independantly of C, S or R

b) Only when $\bar{S}_d = \bar{R}_d = 1$ is the feedback loop closed and R, S, C able to control.

CHAPTER 6 - AIDS

VENN DIAGRAMS - AN AID TO LOGIC ANALYSIS

A Venn diagram provides a pictorial representation of logical relationships. On this diagram areas are used to represent logical variables, a variable being true within a labelled closed boundary and false outside it. Conventionally the boundaries of the area representing a single variable are circular (or nominally so). Where regions overlap a combination of logical truths will be seen to apply. Thus in Fig. A6.1 A=1 within the region bounded by the curve A while A=0 outside. Likewise in Fig. A6.2, A=1 in the horizontally hatched region while B=1 inside the vertically hatched region. Both A=1 and B=1 in the doubly hatched region. In fact as indicated in Fig. A6.3, 4 distinct regions are defined by a general Venn diagram for 2 variables. Fig. A6.4 shows the corresponding representation for 3 variables A, B, C with all regions labelled. Note that just as there are 4 regions for 2 variables and 2^N for N variables, there are 2^3 or 8 for 3 variables.

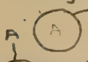


Fig. A6.1

Fig. A6.2

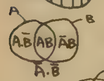


Fig A6.3

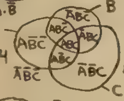


Fig A6.4

As well, special relationships which occasionally apply to physical systems can be represented: For example Fig. A6.5 shows that B is a (proper) subset of A. It also shows that B and C are mutually exclusive, that is B is true only when C is not. Be aware that diagrams such as Fig A6.5 are incomplete; since they implicitly include special relationships they must be used very carefully. Normally in logic analysis/design one uses complete diagrams (that is with 2^N distinct regions for N variables) while acknowledging special relationships algebraically.

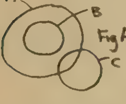


Fig A6.5

Let us now as an example use a Venn diagram to interpret and minimize the Boolean expression $X = AB + \bar{B}$. To proceed we draw a two variable diagram and shade various areas. AB is the horizontally hatched area while $\bar{B}$, all of the region outside B is hatched vertically. From the diagram one can see relationships that may be simpler in a particular context: Since the unhatched region is $\bar{A}B$, the $X = \overline{\bar{A}B}$ and as AB is included in A, then $X = A \vee \bar{B}$ as well.

Fig A6.6

(6.10) a) When $C=0$, the output of G_2 and G_3 are forced to 1 independent of all other conditions, causing $\bar{S}$ and $\bar{R}$ to be both 1.

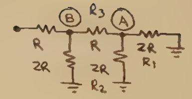
b) With $D=0$ and $C=0$, the outputs of G_4, G_3 and G_2 are 1 while that of G_1 is permitted to be 0, holding G_2 out at 1 independent of C. Since the outputs of G_2 and G_3 are each 1, $\bar{S}=\bar{R}=1$ and the flipflop G_5G_6 is unaffected. Note that two of the inputs to G_3 are 1 and G_3 is controlled by C. Now as C rises, the output of G_3, ie $\bar{R}$, drops, forcing $\bar{Q}$ high. Note that G_2 out and $\bar{S}$ remain high since as C rises the connection from G_1 to G_2 holds the previous state coupled through G_1. Since $\bar{R}$ goes low while $\bar{S}$ remains high, Q falls to 0. With the output of G_3 low, the input through G_4 from D is disabled.

c) Now with $D=1$ and $C=0$, the output of G_3 is 1, and the inputs of G_4 are both 1 so that its output is low maintaining the output of G_3 high independent of C. At the same time the output of G_4, being low, holds the output of G_1 high while the output of G_2 is held high by C at 0. When C rises, the action proceeds as in b) with the roles of G_1, G_2 and G_3, G_4 reversed.

(6.11) With $J=K=0$, the outputs of G_1 and G_2 are high independent of C, leaving the flipflop in its previous state. With $J=1$ and $K=0$ and $\bar{Q}=1$, the output of G_1 falls as C rises, causing Q to fall setting the master flipflop to 1. If $\bar{Q}$ had been 0, Q would already have been 1 and no change would result. Likewise with $K=1$ and $J=0$, the output of G_2 falls, setting the master flipflop to 0. Now with $J=K=1$, the state of G_1 and G_2 is controlled by the output of the slave as C rises. For example, since G_1 is controlled by $\bar{Q}$, the master is set if the slave is in the reset state ($\bar{Q}=1$); while $\bar{Q}$ is high, Q is low, inhibiting G_2 as C rises.

(6.12) Consider the 4-tuple (Q_3, Q_2, Q_1, Q_0) as a binary number using a weighted binary positional notation with Q_0 the LSB. N_7 is intended to be 1 only at the count of 7, ie when $(Q_3Q_2Q_1Q_0)$ reaches the state (0111) representing $(0\times2^3+1\times2^2+1\times2^1+1\times2^0)$ or 7 for which the corresponding minterm is $\bar{Q}_3Q_2Q_1Q_0$ ie $N_7=\bar{Q}_3Q_2Q_1Q_0$. Likewise N_{13} corresponds to a count of 13 such that

$$N_{13}=Q_3Q_2\bar{Q}_1Q_0$$

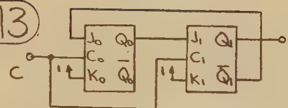
(6.16) 

The resistance to the right of node A is $R_1 = 2R$. The resistance to the right of the resistor connected to the left of node A is $R_1 \| R_2 = 2R\|2R = R$. The resistance to the right of node B is $R_3 + R_2\|R_1 = R+R = 2R$, the same as to the right of node A. Thus the resistance to the right of any node X is 2R. Thus the total resistance loading any resistor R is $2R\|2R = R$. Accordingly the voltage at any node X is half that on the node at its immediate left. Thus each current (to ground) from node X is half that from the node to the left. Thus $I_1 = 2I_2 = 4I_3 = \text{--}\,2^{N-1}I_N = V_{ref}/2R$. Thus I_o consists of:

$$\frac{V_{ref}}{2R}\left(b_1+\frac{b_2}{2}+\text{--}\ \frac{b_N}{2^{N-1}}\right)$$
$$= V_{ref}/R\left(\frac{b_1}{2}+\frac{b_2}{2^2}+\text{--}\ \frac{b_N}{2^N}\right)$$
$$= \frac{V_{ref}}{R}\cdot D$$

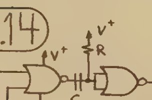
(6.17) In Figure 6.60 an analog input of 0 volts is assumed to be represented by a count of zero and the largest input by a count of 2^N-1. This level is reached after 2^N-1 clock pulses.

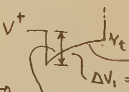
(6.18) The digital output of an A/D converter is constant for inputs which vary by amounts corresponding to ½ of the least significant bit (LSB) below to ½ LSB above the apparent value. Thus the maximum quantization error is ± ½ LSB. Since V_{FS} corresponds to a count of 2^N-1, the LSB corresponds to $1/(2^N-1)$ of full scale, ie $\frac{V_{FS}}{2^N-1}$. Thus the maximum error is $\frac{1}{2}\frac{V_{FS}}{2^N-1}$ or $V_{FS}/(2(2^N-1))$

(6.13) 

The following table summarizes the action following successive clock pulses. The present state of the counter, represented by (Q_1, Q_0) provides (J_0, J_1) which in turn leads to the next state (Q_1', Q_0') entered in the same row and recopied as the initial state (Q_1, Q_0) prior to the next clock. Thus the count sequence, 0,1,2,0,1,2 etc is that of a mod 3 counter.

Count	Q_1	Q_0	$J_0=\bar{Q}_1$	Q_0'	$J_1=Q_0$	Q_1'
0	0	0	1	1	0	0
1	0	1	1	0	1	1
2	1	0	0	0	0	0
0	0	0	1	1	0	0

(6.14) 



Time Const, $\tau = C(R+R_{on})$

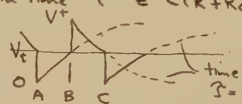
$\Delta V_1 = \frac{V^+R}{R+R_{on}}$

$v = V^+\left(1-\frac{R}{R+R_{on}}e^{-t/\tau}\right)$

$$V^+ - \frac{R}{R+R_{on}}V^+e^{-t/\tau} = V_t \text{ when } e^{-t/\tau} = (V^+-V_t)\left(\frac{R+R_o}{R}\right)$$

$$\text{or } -t = \tau\ln\left(\frac{V^+-V_t}{V^+}\right)\left(\frac{R+R_{on}}{R}\right) \text{ or } t = \tau\ln\left(\frac{V^+}{V^+-V_t}\cdot\frac{R}{R+R_{on}}\right)$$

ie at the time $t = C(R+R_{on})\ln\left(\frac{R}{R+R_{on}}\cdot\frac{V^+}{V^+-V_t}\right)$

(6.15) 

Period $T = T_{AB} + T_{BC}$. The waveform from A to B is represented by $v_{AB}=V^+(1-e^{-t/\tau})$ which reaches V_t when $V_t = V^+(1-e^{-T_{AB}/\tau})$ for which $T_{AB} = -\tau\ln\left(1-\frac{V_t}{V^+}\right)$ or $T_{AB} = \tau\ln\left(\frac{V^+}{V^+-V_t}\right)$. The waveform from B to C is represented by $v_{BC} = V^+(e^{-t/\tau})$ which reaches V_t when $V_t = V^+(e^{-T_{BC}/\tau})$ or $T_{BC} = \tau\ln\left(\frac{V^+}{V_t}\right)$

Thus $T = T_{AB}+T_{BC} = \tau\left(\ln\frac{V^+}{V^+-V_t}+\ln\frac{V^+}{V_t}\right)$

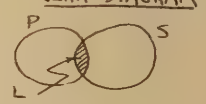
CHAPTER 6 - PROBLEMS

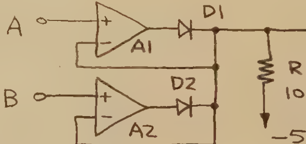
(6.1) PLUG IN ($P=1$); SWITCH ON ($S=1$);
LIGHT ON ($L=1$); DARKNESS ($D=\bar{L}$)

P	S	L	D
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

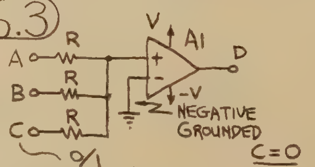
CONCLUDE:
$L = P.S$
$D = \bar{P}+\bar{S}$

VENN DIAGRAM



(6.2) 

$C = A+B$

(6.3) 

GROUNDED INPUT:

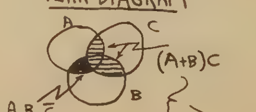
C=0

NEGATIVE A B D		POSITIVE A B D	
0 0 0	$D=A.B$	0 0 1	$D=\overline{A.B}$
0 1 0		0 1 1	
1 0 0		1 0 1	
1 1 1		1 1 0	

C=1

NEGATIVE A B D		POSITIVE A B D	
0 0 0	$D=A+B$	0 0 1	$D=\overline{A+B}$
0 1 1		0 1 0	
1 0 1		1 0 0	
1 1 1		1 1 0	

VENN DIAGRAM



$D = AB\bar{C} + (A+B)C$
$D = AB + BC + CA$

from which

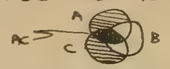
$\bar{D} = \overline{AB+BC+CA} = \overline{AB}\cdot\overline{BC}\cdot\overline{AC}$
$\equiv (\bar{A}+\bar{B})(\bar{B}+\bar{C})(\bar{A}+\bar{C})$ which can be reduced to

$D = \overline{A.B\bar{C}} + (\overline{A+B})C$
$D = \bar{A}\bar{C}+\bar{B}\bar{C}+\bar{A}\bar{B}C$
$D = \bar{A}\bar{C}+\bar{B}\bar{C}+\bar{A}\bar{B}$

6.4 $f_1 = (A+B)(B+\bar{A}) = AB + A\bar{A} + BB + \bar{A}B = AB + B + \bar{A}B$
$= B(A+1+\bar{A}) = B$

$f_2 = ABC + A\bar{C}B + BA = AB(C+\bar{C}+1) = AB$

$f_3 = (A+B+C)(A+\bar{B})(\bar{B}+C) = (AA\bar{B} + AAC + A\bar{B}\bar{B} + A\bar{B}C + BA\bar{B} +$
$BAC + B\bar{B}\bar{B} + B\bar{B}C + CA\bar{B} + CAC + C\bar{B}\bar{B} + C\bar{B}C)$
$= A\bar{B} + AC + A\bar{B} + \underbrace{A\bar{B}C + ABC}_{AC} + \bar{B}C = A\bar{B} + AC + \bar{B}C$
$= A(\bar{B}+C) + \bar{B}C$
$= \bar{B}(A+C) + AC$



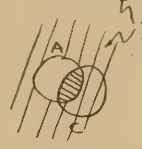
C	B	A	A+B	B+Ā	f_1	ABC	AC̄B	BA	f_2	A+B+C	A+B̄	B̄+C	f_3
0	0	0	0	1	0	0	0	0	0	0	1	1	0
0	0	1	1	0	0	0	0	0	0	1	1	1	1
0	1	0	1	1	1	0	0	0	0	1	0	0	0
0	1	1	1	1	1	0	1	1	1	1	1	0	0
1	0	0				0	0	0	0	1	1	1	1
1	0	1				0	0	0	0	1	1	1	1
1	1	0				0	0	0	0	1	0	1	0
1	1	1			B	1	0	1	1	1	1	1	1

AB — f_2; $\bar{B}(A+C)+AC$ — f_3

6.5

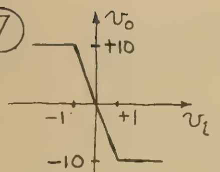
C	B	A	D
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$D = \bar{A}\bar{B}\bar{C} + C(\bar{A}\bar{B} + A\bar{B} + B\bar{A}) = \bar{A}\bar{B} + A\bar{B}C + \bar{A}BC$
$= \bar{B}(\bar{A}+AC) + \bar{A}(\bar{B}+BC) = \bar{B}(\bar{A}+C) + \bar{A}(\bar{B}+C)$
$= \bar{A}\bar{B} + \bar{B}C + \bar{A}\bar{B} + \bar{A}C$
$= \bar{A}\bar{B} + \bar{B}C + \bar{A}C$
$= \bar{A}\bar{B} + C(\bar{A}+\bar{B})$
$= \overline{A+B} + C\overline{AB}$



6.6 THE CIRCUIT CONSISTS OF A LINEAR ADDER FEEDING A COMPARATOR REFERENCED TO THE MIDPOINT OF THE 0/1 LEVELS. THE OUTPUT OF THE ADDER ATTAINS THE POLARITY OF THE MAJORITY OF THE INPUTS. WITH TWO INPUTS AT LOGIC 0 (INDEPENDENT OF LOGIC CONVENTION, THE REMAINING 3 MUST BE LOGIC 1 FOR THE THRESHOLD TO BE CROSSED: $f_1 \Rightarrow D = \overline{A.B.C}$; $f_2 \Rightarrow D = \overline{A.B.C}$; $f_3 \Rightarrow D = \overline{A+B+C}$; $f_4 \Rightarrow \bar{D} = ABC + ABX + ABY + ACX + ACY + AXY + BCX + BCY + BXY + CXY$

6.7

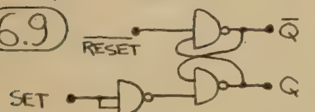


IN THE LINEAR GAIN REGION
$$v_o = 10^4 \frac{100}{10^5+100} v_i$$
$$= 10^4/1001 \; v_i \approx 10 v_i$$

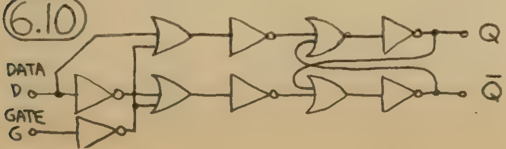
$V_{OL} = -10V$; $V_{OH} = +10V$
$V_{IL} = -1V$; $V_{IL} = +1V$
$\Delta 1 = 9V$; $\Delta 0 = 9V$

6.8 TOTAL CAPACITANCE = 30M × 30pF/M = 900pF
TWO TRANSITIONS OF (10+10) = 20 V AMPLITUDE OCCUR IN 10^{-4} SEC.
METHOD 1: ENERGY LOST IN CHARGING AND DISCHARGING THE CAPACITANCE IN 10^{-4} SEC. IS $(\frac{1}{2} CV^2) \times 2$
POWER REQUIRED $= CV^2/T = 900 \times 10^{-12} \times 20^2/10^{-4} = 3.6$ mW
METHOD 2: CHARGE EXCHANGED TWICE IN 10^{-4} SEC IS Q = CV
CURRENT FROM +10 IS CV/T AND FROM −10 IS CV/T
POWER FROM BOTH SUPPLIES IS $2(CV/T \cdot V/2)$
OR $2 \times 900 \times 10^{-12} \times 20 \times 10^4 \times 10 = 3.6$ mW.
CV = IT → AT 10mA, $T = \frac{900 \times 10^{-12} \times 20}{10 \times 10^{-3}} = 1.8\,\mu$sec.
THUS THE AMPLIFIER MUST BE SLEW RATE LIMITED, AND RISE AND FALL TIMES AT 2V/μsec MUST EACH BE 20/2 = 10μsec.

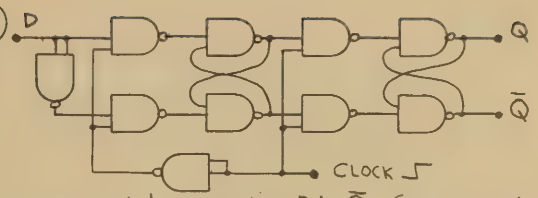
6.9



6.10



6.11



Counting arranged by connecting D to $\bar{Q}$. Same connection applies to circuit of Fig 6.37 since also edge triggered, but not to Fig 6.36 which is level triggered. For this case a closed negative feedback path is formed with clock high.

6.12 Clock Pulse $Q_1 \rightarrow Q_2 \rightarrow Q_3$

6.13 $Q_1 \rightarrow Q_2 \rightarrow Q_3$

Clock Pulse	Q_1	Q_2	Q_3
Initial State	1	0	0
1	0	1	0
2	0	0	1
3	1	0	0
4	0	1	0
5	0	0	1
6	1	0	0
7	0	1	0

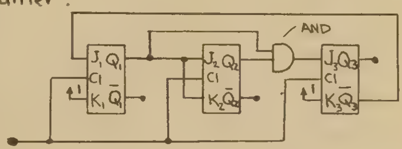
3 distinct states: Counts to 3

	Q_1	Q_2	Q_3
0	0	0	0
1	1	0	0
2	1	1	0
3	1	1	1
4	0	1	1
5	0	0	1
6	0	0	0
7	1	0	0

6 distinct states: Counts to 6

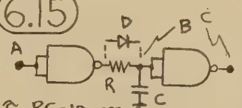
6.14 Modulo 5 counter:

STATE	Q_3	Q_2	Q_1
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0



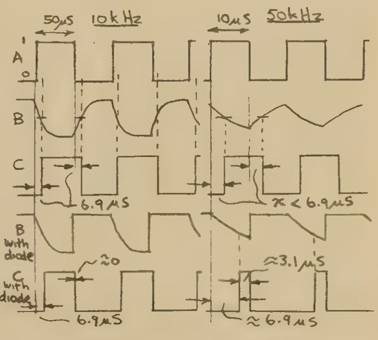
Note: Q_1 reverses while $J_1 = K_1 = 1$, ie until $\bar{Q}_3$ goes to 0.
Q_2 reverses only when $Q_1 = 1$
Q_3 reverses when $Q_1 \cdot Q_2 = K_3 = 1$

6.15

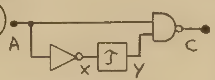


τ = RC = 10μsec.
The logic threshold is half way between the logic levels:
$e^{-t/\tau} = 0.5$ where
$t = 0.693\tau$

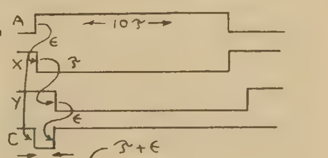
? What is x? At what frequency does operation cease with the diode?



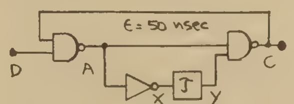
6.16

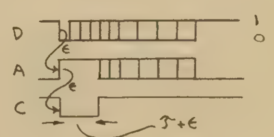


Note: ε represents one gate delay



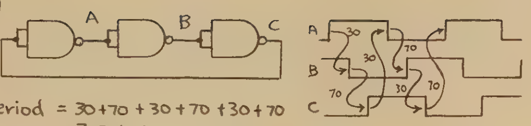
6.17





This is a one-shot.
It requires a logic 0 input of at least two gate delays (100 nsec) duration.
The delay element τ can be formed of an even number (n) of NAND gates. For example if n = 2, ε = 50nsec, a 150nsec pulse results.

6.18



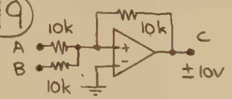
Total Period $= 30+70+30+70+30+70 = 300$ nsec

Frequency $= 1/300 = 3.3$ MHz

Average propagation delay $= \frac{1}{2\times 3}\cdot\frac{1}{3.3\times 10^6} = 50$ nsec

At 12.5 MHz, period is 80 nsec and prop. delay is $\frac{80}{6} = 13.3$ ns

6.19



$C' = AB + (A\bar{B} + \bar{A}B)C$

new output · inputs agree · inputs disagree · old output

Resistors (3) form a majority logic gate: 2 of 3 control the output

6.20

Logic Levels ±10V

C	B	A	C'
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	?
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	?

B acts as SET

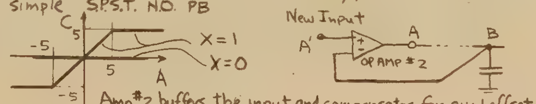
A acts as RESET

C is unknown when A = B = 1

A	B	C'
0	0	C
0	1	1
1	0	0
1	1	?

6.21 The circuit has positive DC feedback, hence is bistable with C at ±10V. Label the op amp + input as A, and the open switch contact as B. When C changes, B follows with a time constant $\tau = .01\mu F(10k + 10k\|10k)$ or $\tau = 150\mu sec$ to ±10V. When PB grounds B, A changes by 10V from ±5V to ∓5V, ie it reverses, causing C to reverse establishing a new state opposite the old one. Circuit can be used to form a push-on/push off switch from a simple S.P.S.T. N.O. PB

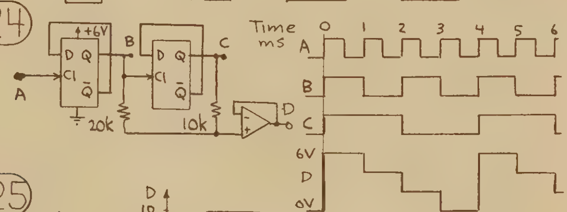
6.22



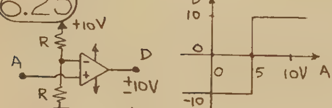
Amp#2 buffers the input and compensates for quad offset

6.23

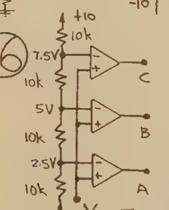
6.24



6.25

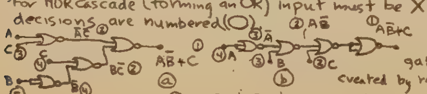


6.26



V Volts	A	B	C	X_2	X_1
0-2.5	0	0	0	0	0
2.5-5	1	0	0	0	1
5-7.5	1	1	0	1	0
7.5-10	1	1	1	1	1

Note that this truth table is not complete, having only 4 entries of the 2^3 or 8 possible. The remaining 4 are said to be "don't cares": their output does not matter since their inputs cannot be produced by the comparators. Proceed as follows: Note that X_1, line 2, is represented uniquely by $A\bar{B}\bar{C}$ but adequately by $A\bar{B}$ since A = B when C = 1. Likewise X_1, line 4, is represented uniquely by ABC but adequately by C; Thus $X_1 = A\bar{B} + C$. Correspondingly but directly, $X_2 = B$. For NOR realization at least two approaches exist which begin at the output: For single NOR, input must be $\bar{X}_1 = \overline{A\bar{B}+C} = \overline{A\bar{B}}\cdot\bar{C} = (\bar{A}+B)\bar{C} = \bar{A}\bar{C} + B\bar{C}$. For NOR cascade (forming an OR) input must be $X_1 = A\bar{B} + C$. Subsequent decisions are numbered (○)



Note: $\overline{P+Q} = \bar{P}\cdot\bar{Q}$

For case (b) number of gates reduces to 3 if $\bar{A}$ is created by reversing A comparator. For case (a) no such saving is possible since X2 = B is also required.

6.27 Using H for high (1) and L for low (0) and D for the output of the circuit as shown and C as the output when the ± inputs are exchanged; D is seen to be high when the inputs are the same and C high when the inputs differ: ie $D = A\cdot B + \bar{A}\bar{B}$ (Com.) and $C = A\cdot\bar{B} + \bar{A}B$ (ExOr)

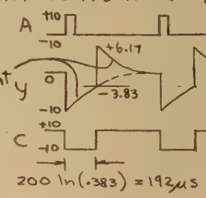
A	B	D	$C = \bar{D}$
H	H	H	L
L	L	H	L
H	L	L	H
L	H	L	H

6.28 Call the op amp negative input X and the positive input Y:

The voltage at X is normally $\frac{62k}{162k}(-10)$ or -3.827 V

The circuit operates as a one-shot provided A goes high infrequently and stays high for less than 192μs. For longer inputs, C becomes an inverted version of A

Time Constant 1nF × 200k = 200 μs



200 ln(.383) = 192μs

CHAPTER 7—EXERCISES

7.1 $v_{DS\,min} = v_{DG\,min} + v_{GS} = |V_P| - 2 = \underline{2V}$

$$i_D = I_{DSS}\left(1 - \frac{v_{GS}}{V_P}\right)^2 = 10\left(1 - \frac{-2}{-4}\right)^2 = \underline{2.5\,mA}$$

7.2 $i_{D1} = 2.5$ mA $i_{D2} = 10\left(1 - \frac{-1.6}{-4}\right)^2 = 3.6$ mA

$$\Delta i_D = i_{D2} - i_{D1} = \underline{1.1\,mA}$$

7.3 $i_D = I_{DSS}\left[2\left(1 - \frac{v_{GS}}{V_P}\right)\left(\frac{v_{DS}}{-V_P}\right) - \left(\frac{v_{DS}}{V_P}\right)^2\right]$

For small v_{DS}:

$$i_D \simeq 2I_{DSS}\left(1 - \frac{v_{GS}}{V_P}\right)\left(\frac{v_{DS}}{-V_P}\right)$$

$$r_{DS} \equiv \frac{v_{DS}}{i_D} = 1\Big/\left[\frac{2I_{DSS}}{-V_P}\left(1 - \frac{v_{GS}}{V_P}\right)\right]$$

$v_{GS} = 0$ V, $r_{DS} = 1\Big/\left[\left(\frac{20}{4}\right)(1-0)\right] = \underline{200\,\Omega}$

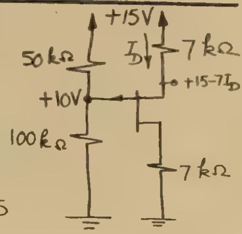
$v_{GS} = -3$ V, $r_{DS} = 1\Big/\left[\left(\frac{20}{4}\right)\left(1 - \frac{-3}{-4}\right)\right] = \underline{800\,\Omega}$

7.4 $v_{SD} = 1$ V, $v_{GD} = 4$ V → Triode region:

$$i_D = 10\left[2\left(1 - \frac{3}{5}\right)\left(\frac{-1}{-5}\right) - \left(\frac{1}{5}\right)^2\right] = \underline{1.2\,mA}$$

$v_{SD} = 2$ V, $v_{GD} = 5$ V → Pinch-off: $i_D = 10\left(1 - \frac{3}{5}\right)^2 = \underline{1.6\,mA}$

7.5 $V_G = +10\text{ V}$.



Assume operation in the pinch-off region:

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$V_{GS} = +10 - (+15 - 7I_D) = 7I_D - 5$$

Thus, $I_D = 9\left(1 - \frac{7I_D - 5}{3}\right)^2$

$$49I_D^2 - 113I_D + 64 = 0$$

$I_D = 1\text{ mA}$ or 1.3 mA.

For $I_D = 1\text{ mA}$, $V_{GS} = 2\text{ V}$

For $I_D = 1.3\text{ mA}$, $V_{GS} = 4.14\text{ V}$, which is impossible for the device would be in cut-off. Thus

$\underline{I_D = 1\text{ mA}}$ and $V_{GS} = 2\text{ V}$

Thus, $V_S = 10 - 2 = \underline{+8\text{ V}}$

$V_D = 7I_D = \underline{+7\text{ V}}$

Since $V_{DG} = -3\text{ V}$, then the device is in pinch-off, as already assumed.

7.6 Assume pinch-off operation:

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 12\left(1 - \frac{-2}{-4}\right)^2 = 3\text{ mA}$$

$$V_D = V_{DD} - R_D I_D = 15 - 3\times 3 = +6\text{ V}$$

Thus, $V_{DG} = 8\text{ V}$ which means operation in pinch-off, as already assumed. Thus,

$\underline{I_D = 3\text{ mA}}$ and $\underline{V_D = +6\text{ V}}$

7.7 Obviously operation will be in the triode region. Thus,

$$i_D = I_{DSS}\left[2\left(1 - \frac{V_{GS}}{V_P}\right)\left(\frac{V_{DS}}{-V_P}\right) - \left(\frac{V_{DS}}{V_P}\right)^2\right]$$

$$= 12\left[2(1-0)\left(\frac{0.1}{4}\right) - \left(\frac{0.1}{4}\right)^2\right]$$

$$= 0.5925\text{ mA}$$

$$R_D = \frac{V_{DD} - v_D}{i_D} = \frac{15 - 0.1}{0.5925} \simeq \underline{25\text{ k}\Omega}$$

7.8 The maximum possible value of R_D is determined by the "high" device:

$$R_{D\max} = \frac{4}{5.85} \simeq \underline{684\ \Omega}$$

With this value of R_D:

the "low" device has $V_D = 20 - 0.684\times 4 = \underline{17.3\text{ V}}$

& the "high" device has $V_D = 20 - 0.684\times 5.85 = \underline{16\text{ V}}$

7.9 $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 9\left(1 - \frac{-2}{-3}\right)^2 = 1\text{ mA}$

$$g_m = \left(\frac{2I_{DSS}}{-V_P}\right)\sqrt{\frac{I_D}{I_{DSS}}} = \frac{2\times 9}{3}\times\frac{1}{3} = \underline{2\text{ mA/V}}$$

Voltage Gain $= -g_m R_d = -2\times 10 = -20\text{ V/V}$

The signal at the drain will be a triangular waveform with $0.2\times 20 = 4\text{ V}$ peak-to-peak amplitude. This signal will be superimposed on the dc drain voltage $V_D = 15 - 1\times 10 = +5\text{ V}$. Thus the minimum drain voltage will be 3 V and the maximum will be 7 V. It is easy to show that even when the drain voltage is at its minimum the device is still in pinch-off, as assumed.

7.10 The circuit is similar to that in Fig. 7.27 except that R_S is split into two resistances: a 2.4 kΩ bypassed by C_S, in series with an unbypassed 300 Ω resistance. From the results of Example 7.5 we have $I_D = 2.96\text{ mA}$, $g_m = 2.98\text{ mA/V}$, and $R_{in} = 420\ \Omega$. For our case here we have

$$\text{Gain} \equiv \frac{v_d}{v_i} = \frac{v_g}{v_i}\times\frac{v_d}{v_g} = \frac{420}{420+100}\times\frac{-(2.7//2.7)\text{k}\Omega}{\left(0.3 + \frac{1}{g_m}\right)\text{k}\Omega}$$

$$= -0.808\times 2.12 = \underline{-1.7\text{ V/V}}$$

Corresponding to a v_{gs} of 0.4 V we have

$v_g = 0.4\times\dfrac{\frac{1}{g_m} + 0.3}{\frac{1}{g_m}} = 0.76\text{ V}$. This corresponds to an input signal v_i of $\dfrac{0.76}{0.808} = \underline{0.94\text{ V}}$.

7.11 Refer to the circuit in Fig. E7.11.

$V_G = 0 \quad V_{GS} = 10 - I_D\times 4 = 10 - 4I_D$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$= 12\,(3.5 + I_D)^2 \Rightarrow I_D = \quad 3\text{ mA}$$

$$g_m = \frac{2I_{DSS}}{-V_P}\sqrt{\frac{I_D}{I_{DSS}}} = \frac{2\times 12}{4}\sqrt{\frac{3}{12}} = 3\text{ mA/V}$$

$R_{in} = \underline{1\ M\Omega}$

$$\text{Voltage Gain} \equiv \frac{v_o}{v_i} = \frac{v_g}{v_i} \times \frac{v_o}{v_g}$$

$$= \frac{1}{1+0.1} \times \frac{(4 /\!/ 4)\ k\Omega}{(4/\!/4)\ k\Omega + \frac{1}{g_m}}$$

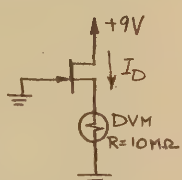
$$= \underline{\underline{0.78\ V/V}}$$

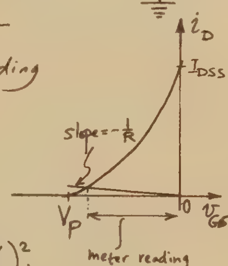
$$R_{out} = 4\ k\Omega /\!/ \left(\frac{1}{g_m}\right)$$

$$= \frac{4 \times \frac{1}{3}}{4 + \frac{1}{3}} = \underline{\underline{307.7\ \Omega}}$$

CHAPTER 7 — PROBLEMS

7.1 Since $v_{DG} = 9V$ ($> |V_P|$) then the JFET is operating in pinch-off. As illustrated graphically, the meter reading $I_D R$ is approximately equal to $|V_P|$. To find an expression for the meter reading $V_M = I_D R$ we assume that $V_M = |V_P| - \epsilon$ where ϵ is small and use the relationship $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$.





$$|V_P| - \epsilon = I_D R = I_{DSS} R \left(1 - \frac{V_M}{|V_P|}\right)^2$$

$$|V_P| - \epsilon = (I_{DSS} R)\left(1 - \frac{|V_P| - \epsilon}{|V_P|}\right)^2$$

$$= (I_{DSS} R)\ \frac{\epsilon^2}{|V_P|^2}$$

$$\epsilon^2\ \frac{I_{DSS} R}{|V_P|^2} + \epsilon - |V_P| = 0$$

The meaningful (physically) solution of this quadratic is

$$\epsilon = \frac{-1 + \sqrt{1 + 4\frac{I_{DSS}R}{|V_P|}}}{[2 I_{DSS} R / |V_P|^2]}$$

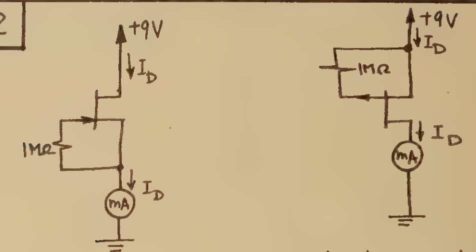
Assuming that $I_{DSS} R \gg |V_P|$ we have

$$\epsilon \simeq \frac{-1 + 2\sqrt{\frac{I_{DSS}R}{|V_P|}}}{2 I_{DSS} R / |V_P|^2} \simeq |V_P| \Big/ \sqrt{\frac{I_{DSS}R}{|V_P|}}$$

Thus,

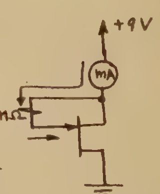
$$\underline{\underline{V_M = |V_P| \left[1 - \sqrt{\frac{|V_P|}{I_{DSS} R}}\right]}}$$

7.2



n-channel device　　　p-channel device

In both case the device is operating in pinch-off with $v_{GS} = 0V$. Thus the milliameter reads I_{DSS}. If the terminals are accidentally interchanged the situation shown in the Figure below results. (for the n-channel case.) In this case the gate-drain pn junction will become forward biased. The 1-MΩ resistor will limit the current through this forward biased junction to less than 9 μA. For instance if the forward voltage drop across the junction is about 0.5 V then the current read by the meter will be approximately 8.5 μA.



7.3 $$i_D = I_{DSS}\left[2\left(1 - \frac{v_{GS}}{V_P}\right)\left(\frac{v_{DS}}{-V_P}\right) - \left(\frac{v_{DS}}{V_P}\right)^2\right]$$

For small v_{DS}:

$$i_D \simeq 2 I_{DSS}\left(1 - \frac{v_{GS}}{V_P}\right)\left(\frac{v_{DS}}{-V_P}\right)$$

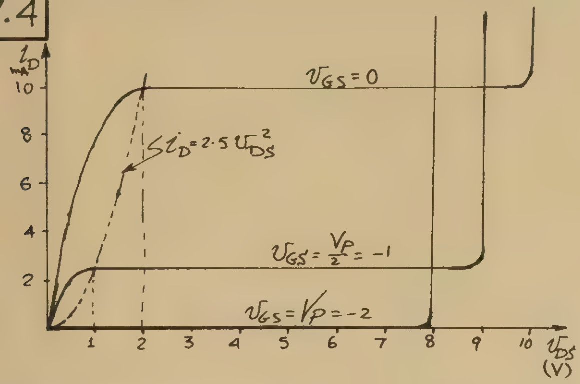
(a) For $V_{GS} = 0V$

$$r_{DS} \equiv \frac{v_{DS}}{i_D} = 100\ \Omega$$

(b) For $V_{GS} = -1V$

$$r_{DS} \equiv \frac{v_{DS}}{i_D} = 200\ \Omega$$

7.4



7.5 $v_{DS\,min} = v_{DG\,min} + v_{GS}$

$= |V_P| + v_{GS} = 2 - 1.5 = 0.5\,V$

7.6 $i_D = I_{DSS}\left(1-\frac{v_{GS}}{V_P}\right)^2 = 10\left(1-\frac{v_{GS}}{-2}\right)^2$

$v_{GS} = -1\,V$ $\quad i_D = I_D = 10\left(1-\frac{1}{2}\right)^2 = 2.5$ mA

$v_{GS} = -1.1\,V$ $\quad i_D = I_{D1} = 2.025$ mA

$v_{GS} = -0.9\,V$ $\quad i_D = I_{D2} = 3.025$ mA

Thus: $\Delta v_{GS} = -0.1\,V \rightarrow \Delta i_D = -0.475$

& $\Delta v_{GS} = +0.1\,V \rightarrow \Delta i_D = +0.525$

$$\frac{\partial i_D}{\partial v_{GS}} = 2\,I_{DSS}\left(1-\frac{v_{GS}}{V_P}\right)\left(\frac{-1}{V_P}\right)$$

$$\left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS}=-1V} = \frac{2\times10}{2}\left(1-\frac{1}{2}\right) = 5\,\text{mA/V}$$

For $\Delta v_{GS} = -0.1\,V \rightarrow \Delta i_D \simeq -0.5$ mA

& for $\Delta v_{GS} = +0.1\,V \rightarrow \Delta i_D \simeq +0.5$ mA

Thus the linearization based on the derivative results in a values that is the average of the actual values.

7.7 $i_D = I_{DSS}\left[2\left(1-\frac{v_{GS}}{V_P}\right)\left(\frac{v_{DS}}{-V_P}\right) - \left(\frac{v_{DS}}{V_P}\right)^2\right]$

For small v_{DS}:

$$i_D \simeq 2I_{DSS}\left(1-\frac{v_{GS}}{V_P}\right)\left(\frac{v_{DS}}{-V_P}\right)$$

$$r_{DS} \equiv \frac{v_{DS}}{i_D} = \left(\frac{-V_P}{2I_{DSS}}\right)\Big/\left(1-\frac{v_{GS}}{V_P}\right)$$

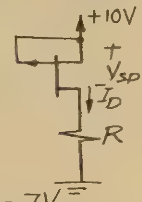
$$= \frac{2}{2\times10}\;\frac{1}{1-\frac{v_{GS}}{-2}} = \frac{100\,\Omega}{1+\frac{v_{GS}}{2}}$$

$v_{GS} = 0$, $\quad r_{DS} = 100\,\Omega$

$v_{GS} = -1V$, $\quad r_{DS} = 200\,\Omega$

$v_{GS} = -1.9V$, $\quad r_{DS} = 2000\,\Omega$

7.8 (a) Assume the FET is in pinch-off. $I_D = I_{DSS} = 3$ mA



$V_D = 3\times1 = +3V$. Thus D is lower than G by 7V ($>|V_P|$) and the device is indeed in pinch-off. $V_{SD} = 7V$

(b) The FET cannot be in pinch-off for the drain voltage would have to be +30V! Thus the FET is in the triode region.

$$I_D = I_{DSS}\left[2\left(1-\frac{V_{GS}}{V_P}\right)\frac{V_{DS}}{-V_P} - \left(\frac{V_{DS}}{V_P}\right)^2\right]$$

$$= 3\left[2\times\frac{V_{DS}}{-3} - \frac{V_{DS}^2}{9}\right]$$

$$= 2V_{SD} - \frac{1}{3}V_{SD}^2$$

Also, $V_{SD} = 10 - I_D R = 10 - 10I_D \rightarrow I_D = 1 - 0.1V_{SD}$

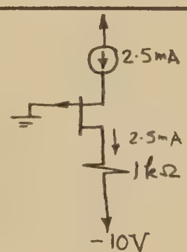
Solving these two equations yield $V_{SD} = 0.52V$ and $I_D = 0.95$ mA

(c) Procedure same as in (b),

$I_D = 2V_{SD} - \frac{1}{3}V_{SD}^2$ and $I_D = 0.1 - 0.01V_{SD}$

Thus $V_{SD} = 0.05V$ and $I_D \simeq 0.1$ mA

7.9 $V_D = -10 + 2.5\times1.0 = -7.5\,V$



Thus the FET is in pinch-off and $I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$

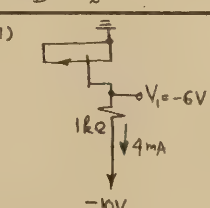
$2.5 = 10\left(1-\frac{V_{GS}}{3}\right)^2$

$V_{GS} = 1.5V \rightarrow V_S = -1.5V$

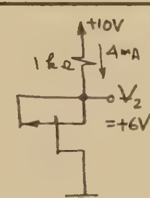
$V_{GD} = 7.5V$

$V_{DS} = V_D - V_S = -7.5 + 1.5 = -6\,V$

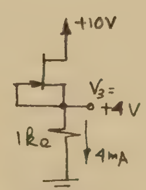
7.10 (1)



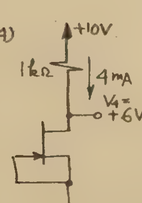
(2)



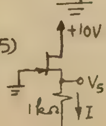
(3)



(4)



(5)



$V_5 = I\times1 = I$

$V_{GS} = -V_5$

$I = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$

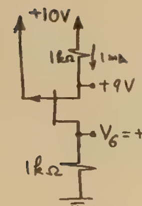
$V_5 = 4\left(1-\frac{V_5}{2}\right)^2$

$V_5 = 1\,V$ or $4V$

4V is impossible for it means cut-off.

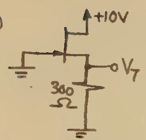
Thus $V_5 = +1V$

(6)



Bias is similar to problem (5) above. Thus $I_D = 1\,mA$. $V_6 = 1 \times 1 = +1\,V$

(7)

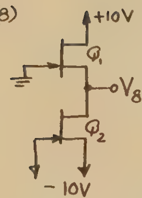


$V_{GS} = -V_7 \quad I_D = \frac{V_7}{0.3\,k\Omega}$

$I_D = I_{DSS}(1 - \frac{V_{GS}}{V_P})^2$

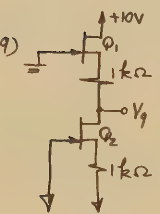
$\frac{V_7}{0.3} = 4\left(1 - \frac{V_7}{2}\right)^2 \Rightarrow V_7 = 0.6\,V$

(8)



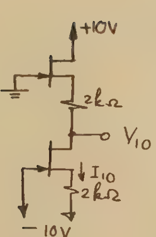
Q_2 carries a current equal to I_{DSS}, i.e. 4 mA. Q_1 carries the same current and thus its V_{GS} will be equal to zero. Thus $V_8 = 0\,V$. Both FETs are in pinch-off.

(9)



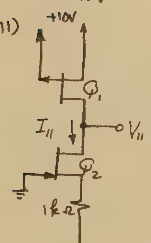
Q_2 will conduct a current determined by the 1 kΩ self-bias resistor. Q_1 is forced to conduct the same current and since it also has a 1 kΩ self bias resistor, V_9 will be equal to zero volts. We can easily show that $I_D = 1\,mA$ and both devices are indeed in pinch-off.

(10)



Following the reasoning of (9) above $V_{10} = 0\,V$. The current I_{10} is determined from: $V_{GS} = -I_{10} \times 2 = -2I_{10}$ and $I_{10} = I_{DSS}(1 - \frac{V_{GS}}{V_P})^2$. The result is $I_{10} = 0.61\,mA$.

(11)



Because of the self bias arrangement of Q_2, if Q_2 is in pinch-off then its drain current will be less than I_{DSS}. In fact we can easily show that I_{11} will be $\underline{1\,mA}$. Now since $V_{GS1} = 0$, it follows that Q_1 cannot be in pinch-off but in the triode region. Assuming this to be the case we can write for Q_1:

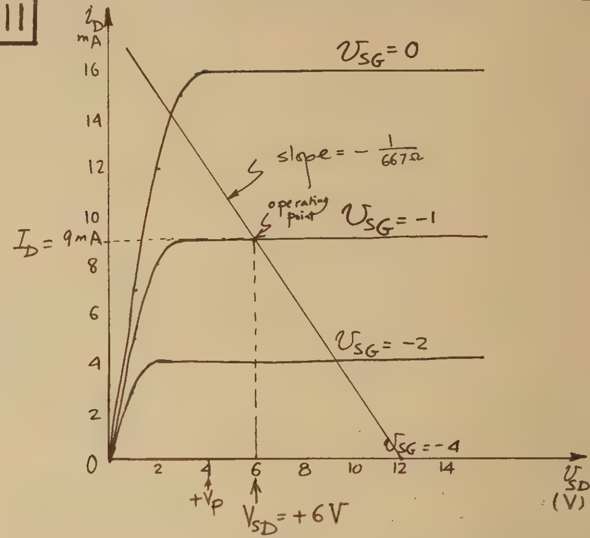
$$I_{11} = I_{DSS}\left[2\left(1 - \frac{V_{GS1}}{V_P}\right)\left(\frac{V_{DS1}}{-V_P}\right) - \left(\frac{V_{DS1}}{V_P}\right)^2\right]$$

Thus:

$$1 = 4\left[2 \times \frac{V_{SD1}}{2} - \frac{V_{SD1}^2}{4}\right] \Longrightarrow V_{SD1} = 0.27V$$

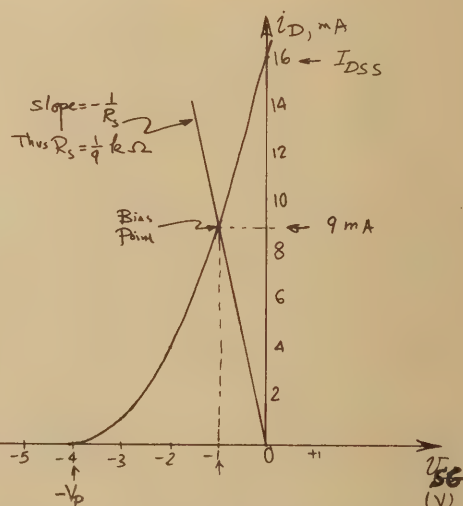
Thus $V_{11} = \underline{+9.73\,V}$ which confirms that Q_2 is in pinch-off as assumed.

$\boxed{7.11}$



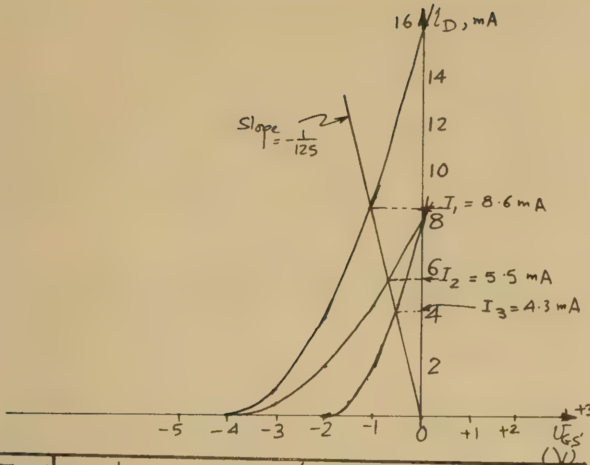
$\boxed{7.12}$ Please note that in the first printing of the Text an error exists in the problem statement. The current level should be $\underline{\underline{9}}$ mA.

The graphical construction is shown below. Such a construction is not necessary since the device is well described by equations.



$\boxed{7.13}$ The graphical construction is illustrated below.

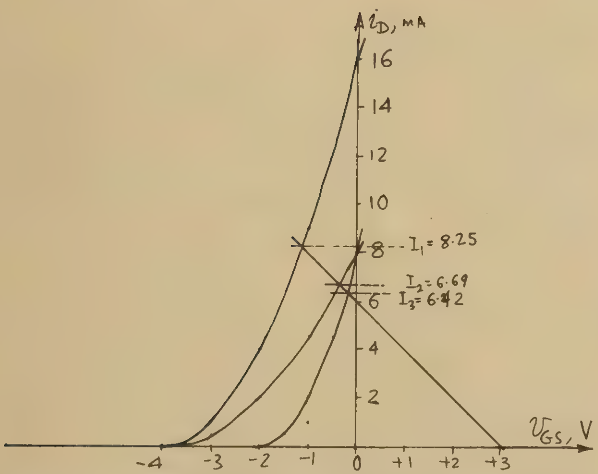
* For 50% reduction in I_{DSS} and no change in V_P, I_D decreases from 8.6 mA to 5.5 mA, i.e -36% change.
* For a 50% reduction in I_{DSS} and $|V_P|$, I_D changes from 8.6 mA to 4.3 mA, a -50% change.



7.14 The graphical construction is indicated below.

* For a 50% reduction in I_{DSS}, I_D decreases from 8.25 mA to 6.69 mA, a −19% change.

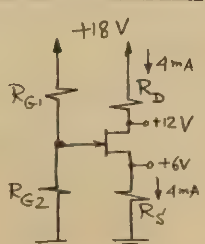
* For a 50% reduction in both I_{DSS} and $|V_P|$, I_D decreases from 8.25 mA to 6.42 mA, a −22% change.



7.15 $R_D = \frac{6V}{4mA} = \underline{\underline{1.5\ k\Omega}}$

$R_S = \frac{6V}{4mA} = \underline{\underline{1.5\ k\Omega}}$

$R_{G1} = 10\ M\Omega$



$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$4 = 8\left(1 + \frac{V_{GS}}{4}\right)^2$

$V_{GS} = -1.17V$

$V_G = 6 - 1.17 = 4.83V$

$4.83 = 18\ \frac{R_{G2}}{R_{G1} + R_{G2}}$

$1 + \frac{10}{R_{G2}} = \frac{18}{4.83} \Rightarrow R_{G2} = \underline{\underline{3.67\ M\Omega}}$

* For a device with $I_{DSS} = 16$ mA and $V_P = -4V$:

$V_{GS} = 4.83 - 1.5 I_D$

$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 16\left(1 - \frac{V_{GS}}{-4}\right)^2$

Solving these two equations results in

$I_D = 4.5$ mA & $V_{GS} = -1.9$ V

Thus $V_S = 4.83 + 1.9 = \underline{\underline{+6.73\ V}}$

$V_D = 18 - 4.5 \times 1.5 = +11.25$

$V_{DS} = 11.25 - 6.73 = \underline{\underline{4.52V}}$

** For a device with $I_{DSS} = 8$ mA and $V_P = -2V$:

$V_{GS} = 4.83 - 1.5\ I_D$

$I_D = 8\left(1 - \frac{V_{GS}}{-2}\right)^2$

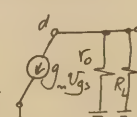
Solving these two equations results in

$I_D = 3.6$ mA $\quad V_{GS} = -0.62V$

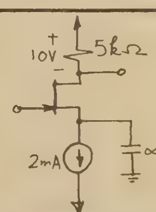
$V_S = 4.83 + 0.62 = \underline{\underline{+5.45\ V}}$

$V_D = 18 - 3.6 \times 1.5 = 12.6V \qquad V_{DS} = 12.6 - 5.45 = \underline{\underline{+7.15V}}$

7.16 The largest possible gain is obtained with $R_L = \infty$ and is equal to $g_m r_o = \mu = \underline{\underline{100}}$. For a gain of 50, $R_L = r_o = \underline{\underline{100\ k\Omega}}$.



7.17 Gain $= -g_m R_L = -g_m \times 5k\Omega$



$g_m = \frac{2 I_{DSS}}{-V_P}\sqrt{\frac{I_D}{I_{DSS}}}$

$= \frac{2 \times 8}{4}\sqrt{\frac{2}{8}} = 2$ mA/V

Thus, Gain $= -2 \times 5 = \underline{\underline{-10\ V/V}}$

7.18 $g_m = \frac{2 I_{DSS}}{-V_P}\sqrt{\frac{I_D}{I_{DSS}}}$

$I_D = 16\left(1 - \frac{3}{4}\right)^2 = 1$ mA

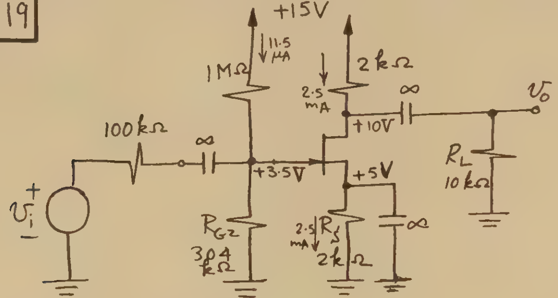
$g_m = \frac{2 \times 16}{4}\sqrt{\frac{1}{16}} = 2$ mA/V

Voltage gain $= -g_m R_d = -2 \times 10 = -20$ V/V

Amplitude of output signal at drain = 2V

$v_{D\min} = 8V$

7.19



$I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$

$2.5 = 10\left(1-\frac{V_{GS}}{-3}\right)^2 \Rightarrow V_{GS} = -1.5V \Rightarrow V_G = +3.5V$

$R_{G2} = \frac{3.5V}{11.5\mu A} = 304\ k\Omega$

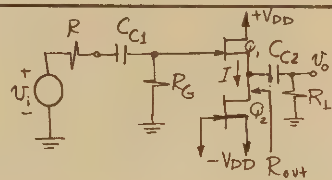
$R_{in} = R_{G1} // R_{G2} = 233\ k\Omega$

Voltage Gain $= -\frac{233}{100+233}\, g_m\, (2k\Omega // 10k\Omega)$

$g_m = \frac{2\times 10}{3}\sqrt{\frac{2.5}{10}} = 3.33$ mA/V

Voltage Gain $= -0.7 \times 3.33 \times \frac{20}{12} = \underline{-3.9 V/V}$

7.20 Since V_{GS} of Q_2 is zero it conducts a dc current $I = I_{DSS}$.



Because the two FETs are matched, V_{GS} of Q_1 will be zero. Thus the dc voltage at the source of Q_1 will be zero and the dc offset voltage of the follower will be zero. (i.e. the dc voltage between input and output). If R_L is referenced to ground we can dispense with C_{C2} with no effect on dc bias.

Neglecting r_o of Q_1 and Q_2, the output resistance of the follower is $1/g_{m1}$.

7.21 $R_{out} = \left(\frac{1}{g_m}\right) // R_S = 0.5k\Omega // 10k\Omega$

$= \underline{476\ \Omega}$

Gain $= \frac{(10k\Omega // 10k\Omega)}{(10k\Omega//10k\Omega) + \left(\frac{1}{g_m}\right)} = \frac{5}{5+0.5}$

$= \underline{0.91\ V/V}$

Alternatively: Gain = Open-Circuit Gain $\times \frac{R_L}{R_L + R_{out}}$

$= \frac{10}{10+\frac{1}{g_m}} \times \frac{10}{10+0.476}$

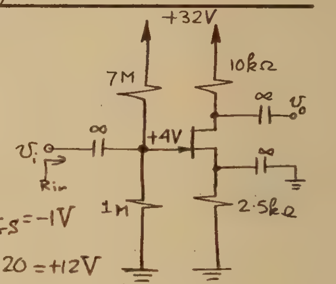
$= \underline{0.91\ V/V}$

7.22 If the source resistance (R_S) is totally bypassed, gain $= -g_m (R_d // R_L)$

$= -2\,(10//10) = -10 V/V$

To reduce the gain by a factor of 2, a source resistance equal to $\frac{1}{g_m} = 0.5k\Omega$ must be left unbypassed.

7.23 (a) dc Analysis:



$V_G = +4V$

$V_{GS} = 4 - 2.5 I_D$

$I_D = 8\left[1 - \frac{4-2.5I_D}{-2}\right]^2$

$\Rightarrow I_D = 2mA$ and $V_{GS} = -1V$

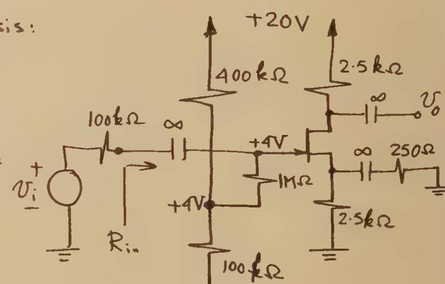
$V_S = +5V \quad V_D = 32-20 = +12V$

$R_{in} = 1M\Omega // 7M\Omega = 875\ k\Omega$

Gain $= -g_m \times 10\ k\Omega \qquad g_m = \frac{2\times 8}{2}\sqrt{\frac{2}{8}} = 4mA/V$

Gain $= -40$ V/V

(b) dc analysis:



$V_G = +4V$

$V_{GS} = 4 - 2.5 I_D$

$I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$

$\Rightarrow I_D = 2mA$

$V_{GS} = -1V$

$V_S = +5V \quad V_D = 20 - 2.5\times 2 = +15V$

$R_{in} = 1M\Omega + (100k\Omega // 400k\Omega) = 1.08\ M\Omega$

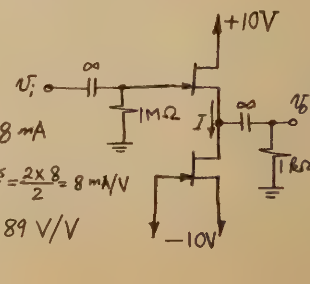
& $g_m = \frac{2I_{DSS}}{-V_P}\sqrt{\frac{I_D}{I_{DSS}}} = \frac{2\times 8}{2}\sqrt{\frac{2}{8}} = 4$ mA/V

Gain $= \frac{1.08}{1.08+0.1} \times - \frac{2.5\ k\Omega}{\frac{1}{g_m} + (2.5k\Omega // 250\Omega)}$

$= 0.915 \times -\frac{2.5}{0.25+0.227}$

$= \underline{-4.8\ V/V}$

(c) dc analysis



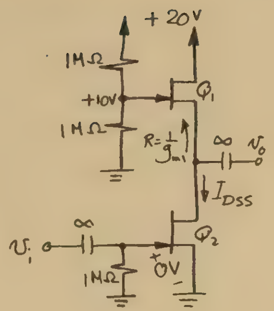
Bias current $I = I_{DSS} = 8$ mA

ac analysis $g_m = \frac{2I_{DSS}}{-V_P} = \frac{2\times 8}{2} = 8$ mA/V

Gain $= \frac{1\ k\Omega}{(1/g_m) + 1\ k\Omega} = 0.89$ V/V

(d) The load resistance of Q_2 is the resistance looking into the source of Q_1 which is equal to $1/g_{m1}$
Thus the voltage gain is given by

$$\text{Gain} = -g_{m2} \times \frac{1}{g_{m1}} = -1\ \text{V/V}$$

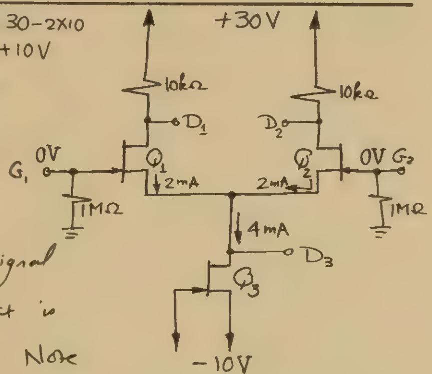


7.24 (a) $V_{D1} = V_{D2} = 30 - 2\times10 = +10\text{V}$

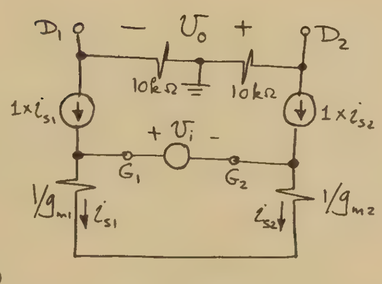
$$2 = 4\left(1 - \frac{V_{GS1}}{V_P}\right)^2$$

$V_{GS1} = -0.59\ \text{V}$

$V_{D3} = +0.59\text{V}$



(b) The small-signal equivalent circuit is shown below. Note that since Q_3 is operating as a constant-current source, its incremental resistance is considered infinite (i.e. an open circuit) in the equivalent circuit.



Since

$$1/g_{m1} = 1/g_{m2} = \frac{1}{g_m}$$

where $g_m = \frac{2\times4}{2}\sqrt{\frac{2}{4}} = 2.83\ \text{mA/V}$

then, $i_{s1} = -i_{s2} = \frac{v_i}{2/g_m} = \frac{g_m v_i}{2}$

and, $v_o = v_{d2} - v_{d1}$

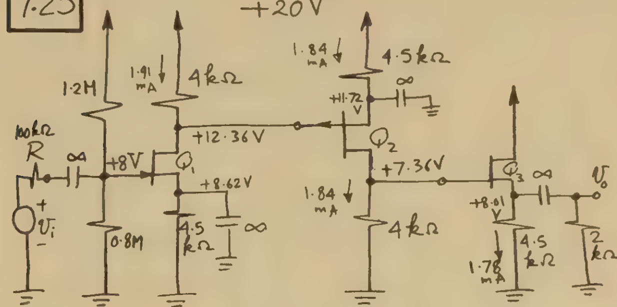
$$= -i_{s2}\times10\,\text{k}\Omega - (-i_{s1}\times10\,\text{k}\Omega)$$

$$= i_{s1}\times10 + i_{s1}\times10$$

$$= 20\times i_{s1} = 20\times\frac{g_m v_i}{2}$$

Thus, $\frac{v_o}{v_i} = 28.3\ \text{V/V}$

7.25



dc analysis: $V_{G1} = +8\text{V}$ $\quad V_{GS1} = 8 - I_{D1}\times4.5$ (1)

$I_{D1} = 4\left(1 - \frac{V_{GS1}}{-2}\right)^2$ (2). Solving (1) and (2) yields: $I_{D1} = 1.91\text{mA}$ and $V_{GS1} = -0.62$

$V_{S1} = +8.62$ $\qquad V_{D1} = 20 - 4\times1.91 = +12.36\text{V}$

For Q_2 we have: $V_{SG2} = 7.64 - I_{D2}\times4.5$ (3)

and, $I_{D2} = 4\left(1 - \frac{V_{GS2}}{2}\right)^2$ (4)

Solving (3) and (4) yields:

$I_{D2} = 1.84\ \text{mA}$ and $V_{SG2} = -0.64\text{V}$

Thus $V_{S2} = 12.36 - 0.64 = +11.72\text{V}$

$V_{D2} = 4\times1.84 = +7.36\ \text{V}$

For Q_3 we have:

$V_{GS3} = 7.36 - I_{D3}\times4.5$ (5)

$I_{D3} = 4\left(1 - \frac{V_{GS3}}{-2}\right)^2$ (6)

Solving (5) & (6) results in

$I_{D3} = 1.78\ \text{mA}$ $\qquad V_{GS3} = -0.65\text{V}$

Thus, $V_{S3} = 7.36 + 0.65 = +8.01\ \text{V}$

Small-signal analysis

$$g_{m1} = \frac{2\times4}{2}\sqrt{\frac{1.91}{4}} = 2.76\ \text{mA/V}$$

$$g_{m2} = \frac{2\times4}{2}\sqrt{\frac{1.84}{4}} = 2.71\ \text{mA/V}$$

$$g_{m3} = \frac{2\times4}{2}\sqrt{\frac{1.78}{4}} = 2.67\ \text{mA/V}$$

$$\text{Gain} = \frac{(1.2//0.8)\,\text{M}\Omega}{(1.2//0.8)\,\text{M}\Omega + 100\,\text{k}\Omega} \times -g_{m1}\times4\times -g_{m2}\times4\times\frac{(4.5//2)\,\text{k}\Omega}{(4.5//2)\,\text{k}\Omega + \frac{1}{g_{m3}}}$$

$$= \underline{78}\ \text{V/V}$$

$R_{in} = \underline{480}\ \text{k}\Omega$ $\qquad R_{out} = 4.5\,\text{k}\Omega \,//\, \frac{1}{g_{m3}} = \underline{\underline{346\,\Omega}}$

Signal Swing:

Limitation imposed by Q_1: For Q_1 to remain in pinch-off $v_{D1} \geq v_{G1} + |V_P|$. Thus,

$$12.36 - A_1 v_{g1} \geq 8 + v_{g1} + 2$$

$$v_{g1} \leq \frac{2.36}{1+A_1} \quad \text{where } A_1 = g_{m1}\times4 = 11.04$$

$\hat{v}_{g1} \simeq 0.2\,V$

At this value of ~~the~~ signal at the gate of Q_1 we have a signal of 2.2V amplitude at the gate of Q_2. We can easily show that such a signal drives Q_2 out of pinch-off. In fact for Q_2 to remain in pinch-off we should have:

$$v_{D2} \leq v_{G2} - |V_P|$$

$$7.36 + A_2 v_{g2} \leq 12.36 - v_{g2} - 2$$

$$v_{g2} \leq \frac{3}{1 + A_2} \qquad \text{where} \quad A_2 = g_{m2} \times 4 = 10.84$$

$$v_{g2} \leq 0.21\,V$$

At this value of signal at the gate of Q_2 we have at the gate of Q_3 a signal of 2.3V amplitude. This signal can be accomodated by Q_3 while remaining in pinch-off. Here we should also check that Q_3 is not driven into

into cut-off. Q_3 will be driven into cut off if v_o is negative of value $\hat{V}$ where

$$\frac{\hat{V}}{2k\Omega} = \frac{8.01 - \hat{V}}{4.5}$$

$$\hat{V} = 2.5\,V$$

At this value of negative output signal swing, the current through Q_3 is reduced to zero.

From the above we conclude that Q_2 limits the signal swing of this multistage amplifier to a maximum output signal ~~of~~ swing of $2.3 \times A_3$

$= 2.3 \times 0.787 = 1.81V$ peak or 3.62V peak-to-peak.

$\boxed{7.26}$ DC Analysis:

$V_{G1} = +8\,V \qquad V_{GS1} = 8 - 4.5\, I_{D1}, \quad (1)$

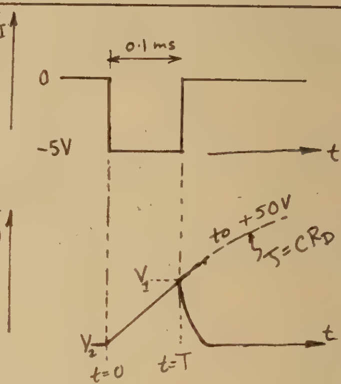
$$I_{D1} = 16\left(1 - \frac{V_{GS1}}{-4}\right)^2 \quad (2)$$

Solving (1) and (2) yields: $I_{D1} = 2.33$ mA & $V_{GS1} = -2.5V$

$V_{S1} = 8 + 2.5 = 10.5\,V$ and $V_{D1} = 20 - 2.33 \times 4 = +10.68\,V$

$V_{D1} - V_{G1} = 2.68 < |V_P|$. Thus Q_1 will not be operating in the active mode and the circuit will not operate as a linear amplifier. We shall not concern ourselves with its analysis any further.

$\boxed{7.27}$ Refer to Figures 7.34, 7.35 and 7.36. To find V_2:



$$i_D = I_{DSS}\left[2(1-0)\frac{V_{DS}}{-V_P} - \left(\frac{V_{DS}}{V_P}\right)^2\right]$$

$$i_D = 8\left[V_2 - \frac{V_2^2}{4}\right] \quad (1)$$

But: $i_D = \dfrac{V_{DD} - V_{DS}}{R_D}$

$$i_D = \frac{50 - V_2}{50} \quad (2)$$

Solving (1) and (2) yields

$V_2 = 0.13V$

To find V_1:

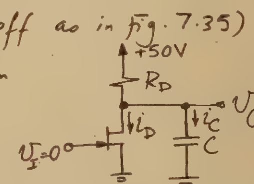
$$v(t) = +50 - (50 - V_2)\, e^{-t/\tau}$$

$$= 50 - 49.87\, e^{-t/\tau}$$

$$V_1 = 50 - 49.87\, e^{-10^{-4}/(10^{-7} \times 50 \times 10^3)}$$

$$= 1.12\,V$$

At time $t = T+$, the FET enters the triode (no the pinch-off as in Fig. 7.35) region. From circuit shown



Circuit for $t \geq T+$

$$i_C = C\frac{dv_O}{dt} = \frac{50 - v_O}{R_D} - i_D$$

where

$$i_D = I_{DSS}\left[2 \times (1-0) \times \left(\frac{v_O}{-V_P}\right) - \left(\frac{v_O}{V_P}\right)^2\right]$$

$$= 8\, v_O - 2\, v_O^2, \text{ mA}$$

Thus $$C\frac{dv_O}{dt} = \frac{50 - v_O}{50\,k\Omega} - 8\,v_O - 2\,v_O^2$$

Solution of this nonlinear differential equations gives the details of the falling edge of the

output waveform.

7.28 (a) When v_C is negative the FET is cut off and the capacitor is charged by a constant current of $\frac{15\,V}{150\,k\Omega} = 0.1\,mA$. Thus v_O rises linearly with a slope of $\frac{0.1\,mA}{0.1\,\mu F} = 10^3\,V/s$. For this ramp signal to reach 5V we must allow a time of $5\times10^{-3}\,s$ or 5 ms.

(b) When v_C goes positive the FET operates with $v_{GS} = 0$. Since initially it will have $v_{DG} = 5V$ it will be operating in pinch-off and discharging the capacitor at a constant current of I_{DSS} (10 mA). This continues until the output voltage falls to +2 V at which point the FET enters the ~~pinc~~ triode region (still operating with $v_{GS} = 0$ V.

$$-C\frac{dv_O}{dt} = (10\,v_O - 2.5\,v_O^2)\times10^{-3}$$

$$\frac{dv_O}{2.5v_O^2 - 10v_O} = 10^4\,dt$$

$$\int_{+2V}^{0.05V}\frac{dv_O}{0.25v_O^2 - v_O} = \int_0^{t_2} 10^5\,dt$$

where t_2 is the interval for the output voltage to fall from +2V to +0.05V (note that for the capacitor to lose 99 % of its charge its voltage has to fall to $0.01\times5 = 0.05V$).

$$\left[\ln\left(1 - \frac{1}{0.25v_O}\right)\right]_2^{0.05} = 10^5\,t_2$$

$$t_2 = 10^{-5}\times\ln 79 = 43.7\,\mu s$$

Thus the time required to discharge the capacitor is $t_1 + t_2 = \underline{73.7\,\mu s}$

We shall now consider the discharge process quantitatively. For the period of pinch-off operation the output voltage falls linearly from +5V to +2V. Since the discharge current is approx. 10 mA (neglecting the 0.1 mA current through the 150 kΩ resistor) then this interval lasts for t_1 seconds,

$$t_1 = \frac{C\times3V}{10\,mA} = \frac{0.1\times10^{-6}\times3}{10\times10^{-3}} = 30\,\mu s$$

Next we have the interval during which the FET is operating in the triode region. Again, neglecting the 0.1 mA through the 150-kΩ resistor the discharge current of the capacitor is

$$i_D = 10\left[2(1-0)\frac{v_{DS}}{-2} - \left(\frac{v_{DS}}{-2}\right)^2\right],\ mA$$
$$= 10\,v_O - 2.5\,v_O^2$$

Thus we can write

7.29 With $v_I = 0\,V$, $v_{GS_n} = -v_O$ and $v_{GS_p} = -v_O$. Thus

$$i_{D_n} = I_{DSS}\left(1 - \frac{v_O}{|V_P|}\right)^2$$

$$i_{D_p} = I_{DSS}\left(1 + \frac{v_O}{|V_P|}\right)^2$$

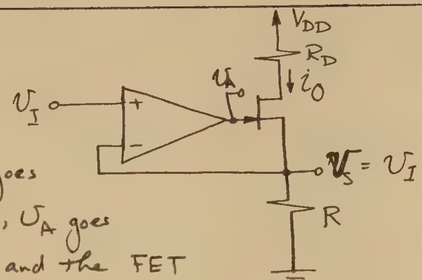
But $i_{D_n} = i_{D_p} + \frac{v_O}{R_L}$

$$I_{DSS}\left(1 - \frac{v_O}{|V_P|}\right)^2 = I_{DSS}\left(1 + \frac{v_O}{|V_P|}\right)^2 + \frac{v_O}{R_L}$$

The only physically-meaningful solution to this equation is $v_O = 0V$. Thus the quiescent current is equal to I_{DSS}. The output resistance of the follower is then the parallel equivalent of $1/g_m$ of the n-channel device and $1/g_m$ of the p-channel device. Thus

$$R_{out} = \frac{1}{2g_m} = \frac{1}{2}\frac{|V_P|}{2I_{DSS}} = \frac{|V_P|}{4I_{DSS}}$$

7.30



As v_I goes positive, v_A goes positive and the FET conducts, thus closing the negative-feedback loop around the op amp. Thus v_S is forced to equal v_I and thus the drain current of the FET, i_O, is given by

$$i_O = \frac{v_I}{R}$$

This assumes that the device is in pinch-off. Note that the circuit operates as a voltage-controlled current source having a precise conversion factor (equal to $1/R$). The output voltage of the op amp, v_A, will be

$$v_A = v_I + v_{GS}$$

where v_{GS} is determined from

$$i_O = \frac{v_I}{R} = I_{DSS}\left(1 - \frac{v_{GS}}{V_P}\right)^2$$

For the numerical values given,

$$i_O = \frac{4}{1} = \underline{\underline{4\ \text{mA}}}$$

$$v_{GS} = -0.88\ \text{V}$$

$$v_A = -0.88 + 4 = \underline{\underline{+3.12\ \text{V}}}$$

CHAPTER 8 — EXERCISES

8.1 For the depletion device to operate in pinch-off, $v_{DG} \geq |V_P|$. Thus,

$$v_{DS\,\text{minimum}} = v_{DG\,\text{min.}} + v_{GS}$$
$$= 2 + 1 = \underline{\underline{3V}}$$

$$i_D = I_{DSS}\left(1 - \frac{v_{GS}}{V_P}\right)^2$$
$$= 8\left(1 - \frac{1}{-2}\right)^2 = \underline{\underline{18\ \text{mA}}}$$

8.2 For $v_{GS} = 4V$ and $v_{DS} = 5V$, $v_{DG} = 1V$ and thus the device is in pinch-off. Its current can be found from $i_D = \frac{1}{2}\beta(v_{GS} - V_T)^2$. The value of β can be determined from the given data: $i_D = 1\ \text{mA}$ at $v_{GS} = v_{DS} = 3V$

$$1 = \tfrac{1}{2}\beta\,(3-2)^2$$

Thus, $\beta = 2\ \text{mA/V}^2$

Now, $i_D = \frac{1}{2} \times 2\,(4-2)^2 = \underline{\underline{4\ \text{mA}}}$.

In the triode region:

$$i_D = \beta\left[(v_{GS} - V_T)\,v_{DS} - \tfrac{1}{2}v_{DS}^2\right]$$

For small v_{DS} we have

$$i_D \simeq \beta\,(v_{GS} - V_T)\,v_{DS}$$

Thus $r_{DS} \equiv \dfrac{v_{DS}}{i_D} = \dfrac{1}{\beta\,(v_{GS} - V_T)}$

For $v_{GS} = 4V$ we have

$$r_{DS} = \frac{1}{2\,(4-2)} = \frac{1}{4}\ \text{k}\Omega = \underline{\underline{250\Omega}}$$

8.3 To find the new value of I_D we solve the equation

$$I_D = \tfrac{1}{2} \times 0.5\,(V_{GS} - 3)^2$$

together with

$$V_{GS} = V_G - I_D R_S$$

i.e. $V_{GS} = 8 - 4I_D$

Thus $I_D = 0.25\,(5 - 4I_D)^2$

$\Rightarrow$ $I_D = 0.8\ \text{mA}$ (the other solution is not physically meaningful)

Thus I_D changes by $\underline{\underline{-20\ \%}}$

8.4 $V_{GS} = V_{DS}$

Thus, $I_D = \frac{1}{2}\beta\,(V_{DS} - V_T)^2$

$$1 = \tfrac{1}{2} \times 0.5\,(V_{DS} - 2)^2$$

$V_{DS} = 4\text{ V}$

$$R_d = \frac{V_{DD}-V_{DS}}{I_D} = \frac{20-4}{1} = \underline{\underline{16\text{ k}\Omega}}$$

To find the new value of I_D, obtained when the device is replaced by another with $V_T = 3\text{V}$, we solve the equation

$$I_D = \tfrac{1}{2}\beta\,(V_{DS}-V_T)^2$$

together with

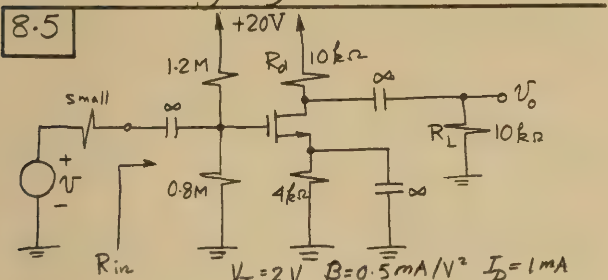
$$V_{DS} = V_{DD} - R_d I_D$$

Thus $I_D = \frac{1}{2}\times 0.5\,(20-16I_D-3)^2$

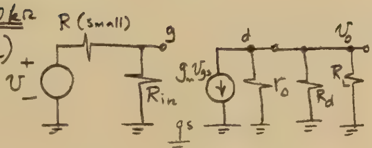
$\Rightarrow I_D = 0.94$ mA (the other solution is not physically meaningful)

Thus I_D changes by -6%.

8.5



$R_{in} = 1.2\text{M}\Omega // 0.8\text{M}\Omega = \underline{480\text{k}\Omega}$



$\text{Gain} = -g_m(r_o // R_d // R_L)$

$g_m = \beta(V_{GS}-V_T)$

$= 0.5(4-2)$

$= 1\text{ mA/V}$

$r_o = \frac{50}{I_D} = 50\text{ k}\Omega$

Thus, Gain $= -1\,(50 // 10 // 10) = -4.54$ V/V.

8.6 $I = \frac{1}{2}\beta_1(15-V_1-1.5)^2 = \frac{1}{2}\beta_2(V_1-V_2-1.5)^2$

$$= \tfrac{1}{2}\beta_3\,(V_2+15-1.5)^2$$

Since $\beta_1 = \beta_3$,

$$(13.5-V_1)^2 = (V_2+13.5)^2$$

Thus, $V_2 = -V_1$ (This should be also obvious from the symmetry of the circuit)

Now, $\frac{1}{2}\beta_1(13.5-V_1)^2 = \frac{1}{2}\beta_2(2V_1-1.5)^2$

$$\sqrt{\frac{\beta_1}{\beta_2}}\,(13.5-V_1) = 2V_1 - 1.5$$

$\Rightarrow V_1 = \underline{+11.18\text{ V}}$ $\quad V_2 = \underline{-11.18\text{ V}}$

$I = \frac{1}{2}\times 70\,(13.5-11.18)^2 = \underline{188\,\mu\text{A}}$

8.7 The current I in transistors Q_1, Q_2 and Q_3 was calculated in the previous (8.6) Exercise to be 0.188 mA $\simeq \underline{\underline{0.2\text{ mA}}}$. From the results of the previous Exercise we have

$V_{GS1} = 15 - 11.18 = \underline{\underline{3.82\text{V}}}$

$V_{GS2} = V_1 - V_1 = 11.18-(-11.18) = \underline{\underline{22.36\text{ V}}}$

$V_{GS3} = -11.18-(-15) = \underline{\underline{3.82\text{V}}}$

Now for Q_8 we have:

$I_{D8} = I_{D3} = I = \underline{\underline{0.2\text{ mA}}}$

$V_{GS8} = V_{GS3} = 3.82\text{V}$

Next consider Q_7 and Q_{10}:

$I_{D7} = I_{D10} = \frac{I}{2} = \underline{\underline{0.1\text{ mA}}}$

$0.1 = \frac{1}{2}\times 72\,(V_{GS7}-1.5)^2$

$V_{GS7} = V_{GS10} = \underline{\underline{1.87\text{ V}}}$

For Q_6 and Q_9 we have:

$I_{D6} = I_{D7} = \underline{\underline{0.1\text{mA}}}$

$I_{D9} = I_{D10} = \underline{\underline{0.1\text{mA}}}$

and since $\beta_6 = \beta_9 = \frac{1}{2}\beta_8$ then

$V_{GS6} = V_{GS9} = V_{GS8} = \underline{\underline{3.82\text{V}}}$

Thus, $V_B = V_A = +11.18\text{ V}$

For Q_4 and Q_5 we can write:

$I_{D4} = I_{D5} = \frac{1}{2}\beta_4\,(11.18-V_{G5}-1.5)^2 = \frac{1}{2}\beta_5\,(V_{G5}+15-1.5)^2$

Thus, $V_{G5}+13.5 = \sqrt{\frac{\beta_4}{\beta_5}}\,(9.68-V_{G5})$

and for Q_2 and Q_3 we can obtain

$V_{G3}+13.5 = \sqrt{\frac{\beta_2}{\beta_3}}\,(9.68-V_{G3})$

But $\frac{\beta_4}{\beta_5} = \frac{\beta_2}{\beta_3}$,

Thus $V_{G5} = V_{G3} = -11.18\text{ V}$.

Hence, $V_{GS4} = V_{GS2} = \underline{\underline{22.36\text{V}}}$

& $V_{GS5} = V_{GS3} = \underline{\underline{3.82\text{ V}}}$

$I_{D4} = I_{D5} = \frac{1}{2}\times 1.88\,(22.36-1.5)^2$

$\simeq \underline{\underline{0.4\text{ mA}}}$

Finally consider Q_{11} and Q_{12}:

Since Q_5 and Q_{12} are identical and have the same V_{GS}, $V_{GS12} = V_{GS5} = \underline{\underline{3.82\text{V}}}$, then

$I_{D12} = I_{D5} = \underline{\underline{0.4 \text{ mA}}}$

$I_{D11} = I_{D12} = \underline{\underline{0.4 \text{ mA}}}$

Since Q_{11} is identical to Q_4, ~~and~~ $V_C = V_B$, and $I_{D11} = I_{D4}$, then

$V_{GS11} = V_{GS4} = \underline{\underline{22.36\text{ V}}}$.

8.8 Refer to Fig. 8.34. To find the logic 0 level (V_{on}) we note that at point A (Fig. 8.34c) Q_1 is operating in the triode region with $v_{GS1} = V_{DD} - V_T = 10 - 1 = 9\text{V}$. Thus

$$i_{D1} = \beta_1 \left[(9-1) V_{on} - \tfrac{1}{2} V_{on}^2 \right]$$
$$= 30 \left[8 V_{on} - \tfrac{1}{2} V_{on}^2 \right]$$

But $i_{D1} = i_{D2} = \frac{1}{2} \beta_2 \left[V_{DD} - V_{on} - V_T \right]^2$

$$= \tfrac{1}{2} \times 3 \left[9 - V_{ON} \right]^2$$

Thus $30\left(8V_{on} - \frac{1}{2} V_{on}^2\right) = 1.5\,(9 - V_{on})^2$

$\Rightarrow V_{on} = \underline{\underline{0.47\text{V}}}$

The logic 1 level is $V_{DD} - V_T = \underline{\underline{9\text{ V}}}$

8.9 From Eq. (8.30)

$$V_t = V_{T1} + \frac{|V_{T2}|}{\sqrt{\beta_1/\beta_2}} = 1 + \frac{3}{\sqrt{\beta_1/\beta_2}}$$
$$= 1.95\text{V}$$

To find the logic-0 level, V_{on}, refer to Fig. 8.35c. At the operating point labeled D we have for Q_1:

$$i_{D1} = \beta_1 \left[(V_{DD} - V_{T1}) V_{on} - \tfrac{1}{2} V_{on}^2 \right],$$

and for Q_2:

$$i_{D2} = \tfrac{1}{2} \beta_2 V_{T2}^2 .$$

Equating i_{D1} and i_{D2} results in

$$\beta_1 \left(9 V_{on} - \tfrac{1}{2} V_{on}^2\right) = \tfrac{1}{2} \beta_2 \times 9$$

$9 V_{on} - \frac{1}{2} V_{on}^2 = \frac{9}{2} \times \frac{3}{30} \Rightarrow V_{on} = \underline{\underline{0.05\text{V}}}$

8.10 Voltage Gain $= -g_{m1} \left[r_{o1} // r_{o2} \right]$

where $g_{m1} = \beta_1 \left[V_{GS1} - V_{T1} \right] = 30 \left[2 - 1 \right] = 30\mu\text{A/V}$

To find r_{o1} and r_{o2} we have to know the value of I_D. An approximate value is obtained from $I_{D1} \simeq I_{D2} \simeq \frac{1}{2} \beta_2 V_{T2}^2 = 13.5\mu\text{A}$

Thus $r_{o1} = r_{o2} = \dfrac{50}{13.5\mu\text{A}} = 3.7\text{ M}\Omega$

Gain $= -30 \times \frac{3.7}{2} = -55.5$ V/V

8.11 Refer to Figures 8.39 through 8.42. For $v_I < V_T$ i.e. $v_I < 2$ V, Q_1 is off and as can be seen from the graphical construction of Fig. 8.41, $v_O = V_{DD}$ (10V). This situation persists until v_I exceeds 2V at which point Q_1 turns on and v_O begins to decrease. Over a range of v_I, Q_1 operates in the pinch-off region while Q_2 operates in the triode region. This range of v_I is: $2\text{V} \le v_I \le v_O - 2$; the upper limit being the point at which Q_2 enters the pinch-off region. Note that Q_1 does not leave the pinch-off region until $v_I \geq v_O + 2$, a point beyond that the limits the above mentioned range. This range of v_I gives rise to the segment AB of the transfer characteristic in Fig. 8.42. The equation describing this segment can be derived as follows

$$i_{D1} = \tfrac{1}{2} \beta \left(v_I - V_T\right)^2 = \tfrac{1}{2} \beta \left(v_I - 2\right)^2$$
$$i_{D2} = \beta \left[(V_{DD} - v_I - V_T)(V_{DD} - v_O) - \tfrac{1}{2}(V_{DD} - v_O)^2 \right]$$
$$= \beta \left[(8 - v_I)(10 - v_O) - \tfrac{1}{2}(10 - v_O)^2 \right]$$

But $i_{D1} = i_{D2}$, thus

$$(v_I - 2)^2 = 2(8 - v_I)(10 - v_O) - (10 - v_O)^2$$

$$\Rightarrow \boxed{v_O = 5 + \sqrt{25 - (v_I - 2)^2}} \quad (1)$$

We can use this equation to evaluate v_O for $2 \le v_I \le v_O - 2$. The upper limit can be easily determined by substituting $v_I = v_O - 2$ in equation (1) to be $v_I = 5$ and $v_O = 7$. Thus eqn. (1) applies for $2 \le v_I \le 5$.

Some points are:

v_I	v_O
2	10
3	9.9
4	9.46
5	7

As v_I reaches 5 V (and v_O correspondingly reaches 7V) Q_2 enters the pinch-off region. Q_1 will still be in the pinch-off region. In fact

Q_1 will remain in pinch-off for $v_O \geq 3$ V which defines point C on the transfer characteristic. The slope of the segment BC is very high and is ideally vertical (when we assume that in pinch-off the devices behave as constant-current sources).

For $v_I > 5$V, Q_1 will be in the triode region and Q_2 will remain in the pinch-off region. The segment CD of the transfer characteristic can be easily shown to be described by

$$\boxed{v_O = (v_I - 2) - \sqrt{(v_I-2)^2 - (8-v_I)^2}\,,\ 5 \leq v_I \leq 8}$$

The upper limit on v_I is determined by Q_2 becoming cut-off. This equation can be used to determine the following points.

v_I	v_O
6	0.54
7	0.1
8	0

Finally, for $v_I > 8$ V, Q_2 turns off and $v_O = 0$ V, as can be seen from Fig. 8.40.

But $i = C_L \frac{dv}{dt}$ and thus $i\,dt = C\,dv$ and we can write

$$\text{Energy from Supply} = \int_0^{V_{DD}} V_{DD} C_L\, dv$$

where we have assumed that at $t = T_1$, $v = V_{DD}$.

Thus, Energy from Supply $= C_L V_{DD}^2$.

Since at the end of the interval T_1, the energy stored in C_L is $\frac{1}{2} C_L V_{DD}^2$ it follows that the energy dissipated in R_2 during T_1 is $\frac{1}{2} C_L V_{DD}^2$.

Consider next the second part of the cycle obtained when S_2 opens and S_1 closes. C_L discharges through R_1 and eventually (at end of interval T_2 during which S_1 is closed, $T_1 + T_2 = T = 1/f$) its voltage reaches zero. Thus the energy lost by C_L is $\frac{1}{2} C_L V_{DD}^2$ which is dissipated in R_1.

From the above we see that in one cycle the total energy dissipated in R_1 and R_2 is $C_L V_{DD}^2$. The dynamic power dissipation is therefore given by $P = C_L V_{DD}^2 f$.

8.12 We shall consider one complete cycle and thus determine the ~~per~~ energy dissipated in the gate (i.e. in R_1 and R_2) during each cycle. The dynamic power dissipation can then be determined by multiplying the energy dissipated per cycle by the frequency of switching f.

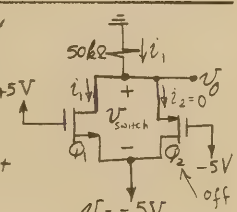
When S_1 is closed and S_2 is open C_L will eventually be fully discharged and the voltage across it will be zero. Consider now the beginning of a cycle as S_1 open and S_2 closes. C_L will begin to charge up and its voltage eventually reaches V_{DD}. If the charging current is denoted by i and the capacitor voltage is denoted by v then we can write

Power Drawn from $V_{DD} = V_{DD}\, i$

Energy From the Supply $V_{DD} = \int_0^{T_1} V_{DD}\, i\, dt$

where T_1 is the interval during which S_2 is closed.

8.13 First please note that some of the answers to this Exercise as printed in the first printing of the Text were in error. Secondly, note that the voltage across the switch, $v_{switch} = v_{DS1} = v_{SD2}$



(a) $v_A = -5$V

Q_2 will be off. Q_1 will conduct i_1,

$$i_1 \simeq \beta\,(v_{GS1} - V_T)\, v_{DS1}$$

$$= 0.1 \times (10-2) \times v_{DS1}$$

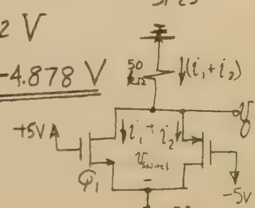
$$r_{DS1} \equiv \frac{v_{DS1}}{i_1} = \frac{1}{0.8} = 1.25\,\text{k}\Omega$$

Thus $R_{switch} = \underline{\underline{1.25\ \text{k}\Omega}}$

Also, $i_1 \times 50 + i_1 \times 1.25 = 5 \Rightarrow i_1 = \frac{5}{51.25}$ mA

$$v_{switch} = i_1 \times 1.25 = 0.122\ \text{V}$$

$$v_O = -5 + 0.122 = \underline{\underline{-4.878\ \text{V}}}$$



(b) $v_A = -2$V

$$i_1 = 0.1 \times (7-2) \times v_{DS1}$$

$$r_{DS1} \equiv \frac{v_{DS1}}{i_1} = \frac{1}{0.5} = 2\ \text{k}\Omega$$

$$i_1 = 0.5\, v_{switch} = 0.5\,(v_O + 2) \quad (1)$$
$$i_2 = 0.1\,(v_O + 5 - 2)\, v_{SD2}$$
$$r_{DS2} \equiv \frac{v_{SD2}}{i_2} = \frac{1}{0.1(v_O+3)} \quad (2)$$
$$i_2 = 0.1\,(v_O+3)(v_O+2) \quad (3)$$

From (1) and (3)

$$i_1 + i_2 = 0.5(v_O+2) + 0.1\,(v_O+3)(v_O+2)$$

But $(i_1 + i_2) \times 50\ k\Omega = -v_O$

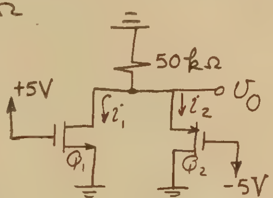
Thus $-\frac{v_O}{50} = 0.5\,(v_O+2) + 0.1(v_O+3)(v_O+2)$

$$\Rightarrow v_O = -1.936\ V \quad \& \ v_{switch} = 0.064\ V$$

From (2) $\quad r_{DS2} = \frac{1}{0.1(-1.936+3)} = 9.4\ k\Omega$

$$R_{switch} = r_{DS1} // r_{DS2} = 2\ k\Omega // 9.4\ k\Omega = 1.649\ k\Omega$$

(c) $v_A = 0\ V$

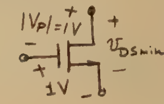


It should be obvious that $i_1 = i_2 = 0$ and $v_O = 0\ V$. To find the switch resistance we note that

$$i_1 = 0.1 \times 3 \times v_{DS1} \Rightarrow r_{DS1} = \frac{1}{0.3} = 3.333\ k\Omega$$
$$i_2 = 0.1 \times 3 \times v_{SD2} \Rightarrow r_{DS2} = 3.333\ k\Omega$$
$$R_{switch} = r_{DS1} // r_{DS2} = 1.667\ k\Omega$$

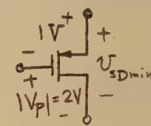
CHAPTER 8 — PROBLEMS

8.1



$$v_{DS\,min} = v_{DG\,min} + v_{GS}$$
$$= |V_P| + v_{GS}$$
$$= 1 + 1 = 2V$$
$$i_D = I_{DSS}\left(1 - \frac{v_{GS}}{V_P}\right)^2$$
$$= 1\left(1 - \frac{1}{-1}\right)^2 = 4\ mA$$

8.2

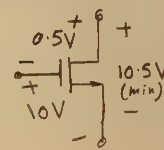


$$v_{GDmin} = |V_P| = 2V$$
$$v_{SDmin} = v_{SG} + v_{GDmin}$$
$$= 1 + 2 = 3V$$

or, $v_{DS\,max} = -3\ V$

$$i_D = I_{DSS}\left(1 - \frac{v_{GS}}{V_P}\right)^2$$
$$= 8\left(1 - \frac{-1}{2}\right)^2 = 18\ mA$$

8.3



$$i_D = I_{DSS}\left(1 - \frac{v_{GS}}{V_P}\right)^2$$
$$= 0.5\left(1 - \frac{10}{-0.5}\right)^2$$
$$= 220.5\ mA$$
$$v_{DS\,min} = |V_P| + v_{GS} = 10.5\ V$$

8.4 $V_P = V_T = V_0$

$$i_D = I_{DSS}\left(1 - \frac{v_{GS}}{V_P}\right)^2 \quad (7.16)$$
$$i_D = \frac{1}{2}\beta\,(v_{GS} - V_T)^2 \quad (8.4)$$
$$= \frac{1}{2}\beta V_T^2\left(1 - \frac{v_{GS}}{V_T}\right)^2$$

Thus, $I_{DSS} = \frac{1}{2}\beta V_T^2$

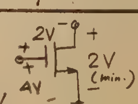
or, $\beta = \frac{2 I_{DSS}}{V_T^2} = \frac{2 I_{DSS}}{V_0^2}$

Consider an enhancement MOSFET. For $v_{GS} = V_T$ volts above the threshold voltage V_T we have

$$i_D = \frac{1}{2}\beta\,(2V_T - V_T) = \frac{1}{2}\beta V_T^2 = I_{DSS}$$

Thus the same definition of I_{DSS} [i.e. the current obtained when v_{GS} is $|V_P|$ volts above the threshold voltage (V_P)] extends to enhancement devices as well (with V_P replaced by V_T).

8.5



$$v_{DGmin} = -V_T = -2V$$
$$v_{DSmin} = v_{DGmin} + v_{GS} = -2 + 4 = +2V$$

8.6 $2=\frac{1}{2}\times\beta\,(4-2)^2 \Rightarrow \beta = 1\ \text{mA/V}^2$

$i_D = \frac{1}{2}\times 1\,(8-2)^2 = \underline{\underline{18\ \text{mA}}}$

8.7 $2 = \frac{1}{2}\times\beta(4-1)^2 \Rightarrow \beta = \frac{4}{9}\ \text{mA/V}^2$

For small v_{DS} in the triode region:

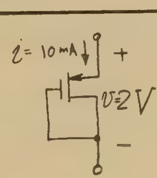
$$i_D \simeq \beta\,(v_{GS}-V_T)\,v_{DS}$$

Thus $r_{DS} \equiv \dfrac{v_{DS}}{i_D} = \dfrac{1}{\beta(v_{GS}-V_T)}$

For $v_{GS} = 4V$ we have

$$r_{DS} = \frac{1}{\frac{4}{9}(4-1)} = \underline{\underline{750\ \Omega}}$$

8.8 $i = \frac{1}{2}\beta\,(v-V_T)^2$



$10 = \frac{1}{2}\times\beta\times(2-1)^2$

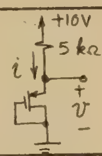
$\beta = 20\ \text{mA/V}^2$

For $i = 1\text{mA}$ $\quad v = V_T + \sqrt{\frac{2i}{\beta}}$

$= 1 + \sqrt{\frac{2}{20}} = \underline{\underline{1.3\,V}}$

For $i = 0.1\text{mA}$ $\quad v = 1 + \sqrt{\frac{0.2}{20}} = \underline{\underline{1.1\,V}}$

8.9 $i = 10\,(v-1)^2$ mA

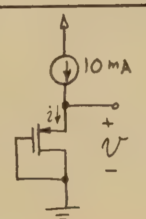


$i = \frac{10-v}{5} = 2-0.2v$ mA

$10(v-1)^2 = 2-0.2v$

$\Rightarrow v = \underline{1.4\text{V}}$

8.10 $i = 10\,(v-\ \ )^2$



$v = 1 + \sqrt{i/10}$

For $i = 10\text{mA}$ $\quad v = \underline{+2V}$

$r = \left.\dfrac{\partial v}{\partial i}\right|_{v=2V\ (\text{or } i=10\text{mA})}$

$= \left.\dfrac{1}{2\sqrt{i/10}}\times\dfrac{1}{10}\right|_{i=10\text{mA}} = \dfrac{1}{20}\,k\Omega = \underline{\underline{50\,\Omega}}$

If 1mA of load current is taken from the regulator the output voltage drops by 1mA X 50Ω

$= \underline{\underline{50\ \text{mV}}}$

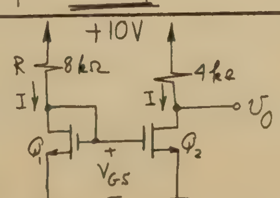
8.11 $2 = \frac{1}{2}\beta\,(2V_T - V_T)^2 \Rightarrow \beta = \dfrac{4}{V_T^2}$

$1 = \frac{1}{2}\times\frac{4}{V_T^2}\,(v_{GS}-V_T)^2$

$v_{GS} = V_T + 0.707V_T \simeq \underline{\underline{1.7\,V_T}}$

$v_{DS\,min} = -V_T + 1.7V_T = \underline{\underline{0.7\,V_T}}$

8.12 $1 = \frac{1}{2}\beta\,(2-1)^2$



$\beta = 2\ \text{mA/V}^2$

Because the devices are identical and have equal V_{GS}, their i_D's will be equal, (denoted I). To find the value of v_O we first determine I as follows:

$$I = \frac{1}{2}\times 2\times(V_{GS}-1)^2$$

But, $V_{GS} = 10 - 8I$

Thus, $I = (9-8I)^2 \Rightarrow I = 1\ \text{mA}$ and $V_{GS} = 2V$

$v_O = 10 - 4\times 1 = \underline{\underline{+6\,V}}$

With R reduced to 4kΩ

$V_{GS} = 10-4I$

Thus, $I = (9-4I)^2 \Rightarrow I = 1.9\ \text{mA}$ & $V_{GS} = 2.4\,V$

$v_O = 10 - 4\times 1.9 = \underline{\underline{+2.4\,V}}$

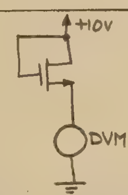
With R = 8kΩ and a third matched FET connected in parallel with Q_2:

The third FET will carry a current equal to that in Q_1 and Q_2, i.e. 1 mA. Thus the total current in the 4kΩ load resistor will be 2 mA and v_O becomes

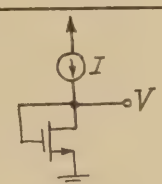
$v_O = 10 - 4\times 2 = \underline{\underline{+2V}}$

This voltage is sufficient to keep the FETs in pinch-off.

8.13 The MOSFET will be barely conducting with a voltage drop equal to V_T. The DVM will read $(10-V_T)$ volts.



8.14



$i_D = \frac{1}{2}\beta\,(v_{GS}-V_T)^2$

$I = \frac{1}{2}\beta\,(V-V_T)^2$

$4 = \frac{1}{2}\beta\,(4-V_T)^2 \quad (1)$

$1 = \frac{1}{2}\beta\,(3-V_T)^2 \quad (2)$

From (1) and (2)

$$\frac{4}{1} = \frac{(4-V_T)^2}{(3-V_T)^2}$$

$$2 = \frac{4-V_T}{3-V_T} \Rightarrow \underline{\underline{V_T = 2V}}$$

Substituting in (2) results in

$$\beta = \underline{\underline{2\ \text{mA/V}^2}}$$

For operation in pinch-off with $v_{GS} = 2V_T$,

$i_D = \frac{1}{2}\times 2\times 4 = \underline{\underline{4\ \text{mA}}}$

8.15 With the switch closed, most of the current I will still flow through the drain of

of the MOSFET. Thus the MOSFET will be operating in pinch-off with a drain current of approximately 1 mA. It follows that its V_{GS} will be 2V and from the voltage divider made up of the two 1-MΩ resistors we see that V_{DS} will be 4V. Thus the DVM will read 4V.

8.16 With the switch open: $i_D = \underline{6.5\text{ mA}}$, $v_{GS} = v_D = 10 - 6.5\times1 = \underline{3.5\text{V}}$

With the switch closed: $i_D = \underline{4\text{mA}}$, $v_{GS} = \frac{1}{2} v_D = \frac{1}{2}(10 - 4\times1) = \underline{3\text{V}}$

$$i_D = \frac{1}{2}\beta(v_{GS} - V_T)^2$$

$$6.5 = \frac{1}{2}\beta(3.5 - V_T)^2 \quad (1)$$

$$4 = \frac{1}{2}\beta(3 - V_T)^2 \quad (2)$$

Thus $\frac{6.5}{4} = \left(\frac{3.5 - V_T}{3 - V_T}\right)^2 \Rightarrow V_T = \underline{1.18V}$

$$\beta = \frac{8}{(3 - 1.18)^2} = \underline{2.4\text{ mA/V}^2}$$

8.17 With the switch open: $i_D = 4\text{mA}$, $v_{GS} = 10 - 4\times1 = 6V$

With the switch closed: $i_D = 6\text{ mA}$ (because another 3 mA will be flowing through the other 1-kΩ resistor), $v_{GS} = 10 - 3\times1 = 7V$

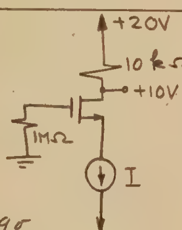
$$4 = \frac{1}{2}\beta(6 - V_T)^2 \quad (1)$$

$$6 = \frac{1}{2}\beta(7 - V_T)^2 \quad (2)$$

Thus $1.5 = \left(\frac{7 - V_T}{6 - V_T}\right)^2 \Rightarrow V_T = \underline{1.55V}$

$$\beta = \frac{8}{(6 - 1.55)^2} = 0.4\text{ mA/V}^2.$$

8.18 $I = \frac{20-10}{10} = \underline{1\text{ mA}}$



$$1 = \frac{1}{2}\times0.5(V_{GS} - 2)^2$$

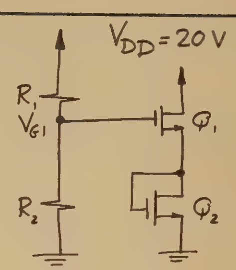
$V_{GS} = 4V$

$V_S = \underline{-4V}$

The drain voltage can go down to −2V without the device leaving pinch-off. The upper limit is the supply voltage (i.e. +20V) at which point the device cuts off.

Substituting a device with $V_T = 4V$ will result in $V_{GS} = 6V$ and thus $V_S = -6V$. The lower limit of the drain signal swing will now be −4V.

8.19 To obtain a current of 1 mA, V_{GS} is determined from



$$1 = \frac{1}{2}\times0.5(V_{GS} - 2)^2$$

$V_{GS} = 4V$

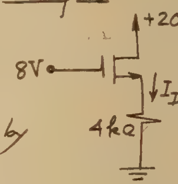
Thus $V_{G1} = 8V$

Choose $R_1 = 10\text{ MΩ}$ then $R_2 = \underline{6.67\text{MΩ}}$

If V_T is reduced to 1.5V: V_{GS1} remains equal to V_{GS2} and to half V_{G1} which is 8V.

Thus $I_D = \frac{1}{2}\times0.5(4 - 1.5)^2 = \underline{1.56\text{ mA}}$

If a resistor had been used in place of Q_2:



The value of this resistor must be $\frac{4V}{1\text{mA}} = 4\text{ kΩ}$. The new value of I_D is determined by solving the two equations:

$$I_D = \frac{1}{2}\times0.5(V_{GS} - 1.5)^2$$

$$8 = V_{GS} + 4I_D$$

The result is $V_{GS} = 3.6V$ and $I_D = \underline{1.1\text{ mA}}$

8.20 In the triode region at low v_{DS} we have

$$i_D \simeq \beta(v_{GS} - V_T)v_{DS}$$

Thus at $v_{GS} = 5V$ the switch resistance $r_{DS} \equiv \frac{v_{DS}}{i_D}$ is

$$r_{DS} = \frac{1}{\beta(v_{GS} - V_T)} = \frac{1}{\beta(5 - V_T)} \quad (1)$$

$$r_{DS}\big|_{Typical} = \frac{1}{\beta_{typ.}(5 - V_{T_{typ}})}$$

But, $\beta_{typ.} = \frac{2\times2.0}{(10 - 1.2)^2} = 51.65\text{ mA/V}^2$

Thus, $r_{DS_{typ.}} = \frac{1}{51.65(5 - 1.2)} = \underline{5.1\ Ω}$

The voltage drop of the switch when $i_D = 0.1A$ is $\underline{0.51V}$

To find the maximum r_{DS} we use equation (1) and substitute for β from

$$I = \frac{1}{2}\beta(10 - V_T)^2$$

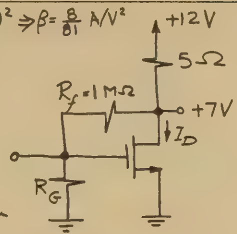
where I is the pinch-off current at $v_{GS} = 10V$, to obtain

$$r_{DS} = \frac{(10 - V_T)^2}{2I(5 - V_T)}$$

The value of r_{DS} will be maximum (worst case) when I is at its minimum (1.0A) and V_T is at its typical value (1.2V),

$$r_{DS}\big|_{max.} = \frac{(10-1.2)^2}{2\times 1\times(5-1.2)} = \underline{10.2\ \Omega}$$

8.21 $V_T = 1V \qquad 4 = \frac{1}{2}\beta(10-1)^2 \Rightarrow \beta = \frac{8}{81}\ A/V^2$



$$I_D = \frac{7W}{7V} = 1A$$

$$R_L = \frac{12-7}{1} = 5\Omega$$

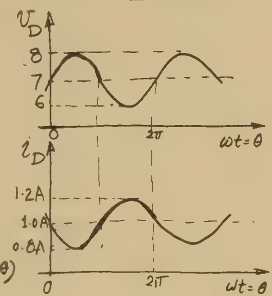
To obtain $I_D = 1A$, V_{GS} should be as obtained from

$$1 = \frac{1}{2}\times\frac{8}{81}\,(V_{GS}-1)^2$$

$$V_{GS} = 5.5V$$

Thus, $\frac{R_G}{R_G+1}\times 7 = 5.5 \Longrightarrow R_G = \underline{\underline{3.67\ M\Omega}}$

The instantaneous power dissipation in the FET is



$$p = v_D i_D$$

$$= (7+1\sin\theta)(1-0.2\sin\theta)$$

$$= 7-1.4\sin\theta+\sin\theta-0.2\sin^2\theta$$

$$= 7-0.4\sin\theta-0.2\left(\tfrac{1}{2}-\tfrac{1}{2}\cos 2\theta\right)$$

$$= 6.9-0.4\sin\theta+0.1\cos 2\theta$$

Thus, $P_{average} = \underline{\underline{6.9\ W}}$

$$P_{Load} = \frac{V_{rms}^2}{5\Omega} = \frac{(1/\sqrt{2})^2}{5} = \underline{0.1\ W}$$

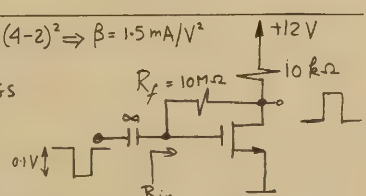
Voltage Gain $\simeq -g_m R_L$ (Neglecting the effect of R_f and R_G)

where $g_m = \beta(V_{GS}-V_T) = \frac{8}{81}(5.5-1) = 0.44\ A/V$

Thus, voltage gain $= -0.44\times 5 = -2.22\ V/V$

To produce 2V peak-to-peak output we need a gate signal of $\frac{2}{2.22} = \underline{0.9\ V\ \text{peak-to-peak}}$.

8.22 $V_T = 2V,\ 3 = \frac{1}{2}\beta(4-2)^2 \Rightarrow \beta = 1.5\ mA/V^2$



$V_{DS} = V_{GS}$ where V_{GS} is determined by solving the two equations: $I_D = \frac{1}{2}\times 1.5\,(V_{GS}-2)^2$

& $\quad I_D = \frac{12-V_{GS}}{10}$

$$\Longrightarrow V_{GS} \simeq 3.1V \quad \text{and } I_D \simeq 0.9\ mA$$

Thus $V_{DS} \simeq \underline{3.1V}$.

$$g_m = \beta(V_{GS}-V_T) = 1.5\times(3.1-2) = 1.65\ mA/V$$

Voltage Gain $= -g_m \times 10\ k\Omega = -16.5$

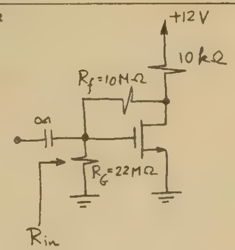
Thus at the drain we obtain a positive pulse of 1.65-V amplitude.

The input resistance R_{in} is obtained by applying Miller's theorem: (see Chapter 2)

$$R_{in} = \frac{R_f}{1-\text{Gain}} = \frac{10\ M\Omega}{1-(-16.5)} = 571\ k\Omega$$

Thus the pulse input current is $\frac{0.1\,V}{571\,k\Omega} = \underline{\underline{0.175\ \mu A}}$

8.23 $V_T = 2V,\quad \beta = 1.5\ mA/V^2$



$$I_D = \tfrac{1}{2}\times 1.5\,(V_{GS}-2)^2 \quad (1)$$

$$V_{GS} = \frac{22}{22+10}\,V_{DS}$$

$$V_{DS} = 12 - I_D\times 10$$

Thus, $V_{GS} = \frac{22}{32}(12-10I_D) \quad (2)$

Solution of (1) and (2) yields $V_{GS} \simeq 3V$ and $I_D = 0.75\ mA$

Thus, $V_{DS} = \underline{4.36\ V}$

$$g_m = \beta(V_{GS}-V_T) = 1.5(3-2) = 1.5\ mA/V$$

Voltage Gain (Neglecting R_f & R_G) $\simeq -1.5\times 10 = -15\ V/V$

Output-pulse amplitude $= 0.1\times 15 = \underline{1.5\ V}$

$$R_{in} = R_G \,\Big/\!\!\Big/\, \frac{R_f}{1-A} = 22M\Omega \,/\!/\, \frac{10}{16}M\Omega = \underline{\underline{608k\Omega}}$$

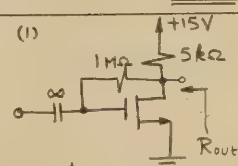
8.24 At $V_{GS} = 2V$: $\quad I_D = \frac{1}{2}\times 0.5\times(2-1)^2 = \underline{0.25\ mA}$

$\qquad g_m = 0.5\times(2-1) = \underline{0.5\ mA/V}$

At $V_{GS} = 3V$: $\quad I_D = \frac{1}{2}\times 0.5(3-1)^2 = \underline{\underline{1\ mA}}$

$\qquad g_m = 0.5(3-1) = \underline{\underline{1\ mA/V}}$

8.25 $I_D = \frac{1}{2}\times 0.5\,(V_{GS}-1)^2 \quad (1)$

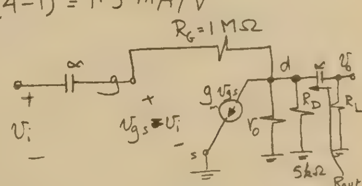


$$I_D = \frac{15-V_{DS}}{5} = \frac{15-V_{GS}}{5} \quad (2)$$

Solving (1) together with (2) yields $V_{GS} \simeq 4V$ and $I_D \simeq 2.2\ mA$

$$g_m = 0.5\,(4-1) = 1.5\ mA/V$$

The small-signal equivalent circuit, including r_o and R_L, is shown in the Figure.



It can be verified that the effect of R_G on the value of gain is negligible. Thus in the most general case (i.e. with r_o and R_L included) we have

$$\text{Voltage Gain} \equiv \frac{v_o}{v_i} \simeq -g_m(R_D /\!/ r_o /\!/ R_L)$$

To find the output resistance R_{out} we short circuit

the input terminal (i.e. make $v_i = 0$) with the result that $v_{gs} = 0$ and

$$R_{out} = R_D \,//\, r_o \,//\, R_G$$

(a) no load and r_o ignored:

$$\text{Gain} = -1.5 \times 5 = -7.5 \text{ V/V}$$

$$R_{out} = 5\,k\Omega \,//\, 1\,M\Omega \simeq 5\,k\Omega$$

(b) $5\,k\Omega$ load and r_o ignored:

$$\text{Gain} = -1.5 \times 2.5 = -3.75 \text{ V/V}$$

$$R_{out} \simeq 5\,k\Omega$$

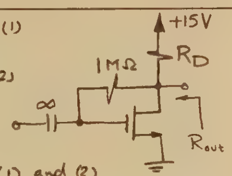
(c) no load and $r_o = \mu/g_m = 100/1.5 = 66.7\,k\Omega$:

$$\text{Gain} = -1.5 \times 4.65 \simeq -7 \text{ V/V}$$

$$R_{out} \simeq 5\,k\Omega \,//\, 66.7\,k\Omega = 4.65\,k\Omega$$

$\boxed{8.26}$ $I_D = \frac{1}{2} \times 0.5 \times (V_{GS} - 1)^2$ (1)

$I_D = \dfrac{15 - V_{DS}}{R_D} = \dfrac{15 - V_{GS}}{R_D}$ (2)



(a) For $R_D = 0.5\,k\Omega$:

Simultaneous solution of (1) and (2) results in $V_{GS} = 8.31$ V and $I_D = 13.37$ mA

$$g_m = 0.5\,(8.31 - 1) = 3.66 \text{ mA/V}$$

$$\text{Gain} \simeq -g_m R_D = -3.66 \times 0.5 = -1.83 \text{ V/V}$$

$$R_{out} \simeq 5\,k\Omega$$

(b) For $R_D = 50\,k\Omega$

Solution of (1) together with (2) yields

$V_{GS} \simeq 2$V and $I_D \simeq 0.25$ mA

$$g_m = 0.5\,(2 - 1) = 0.5 \text{ mA/V}$$

$$\text{Voltage Gain} \simeq -g_m R_D = -0.5 \times 50 = -25 \text{ V/V}$$

$$R_{out} \simeq 50\,k\Omega$$

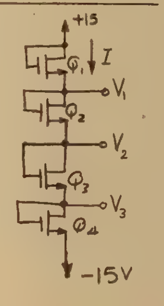
$\boxed{8.27}$ Refer to Figures 8.28, 8.29 and 8.30. Equation (8.23) describes the operation of the amplifier circuit in Fig. 8.28 for the range of v_I over which Q_1 remains in the pinch-off (active) region (Q_2 is always in pinch-off) The coordinates of one end of the linear characteristic (which is the part of the transfer characteristic that is described by eqn. 8.23) are $v_I = V_{T1}$ and $v_O = V_{DD} - V_T$. The coordinates of the other end of this straight line are obtained as follows:

Substitute in 8.22: $v_O = v_I - V_T$ (the point at which Q_1 enters the triode region) and solve the resulting equation together with 8.22. The result is

$$v_I = \frac{V_{DD} + V_T\sqrt{\beta_1/\beta_2}}{1 + \sqrt{\beta_1/\beta_2}} \quad \& \quad v_O = \frac{V_{DD} - V_T}{1 + \sqrt{\beta_1/\beta_2}}$$

$\boxed{8.28}$ $\text{Gain} = -\sqrt{\dfrac{\beta_1}{\beta_2}} = -\sqrt{\dfrac{0.36}{0.04}} = -3 \text{ V/V}$

$\boxed{8.29}$ Since Q_1 and Q_4 are identical and Q_2 and Q_3 are identical we see that $V_3 = -V_1$



$$I = \tfrac{1}{2}\beta_1\,(15 - V_1 - V_T)^2 = \tfrac{1}{2}\beta_2\,(V_1 - V_2 - V_T)^2$$
$$= \tfrac{1}{2}\beta_3\,(V_2 - V_3 - V_T)^2$$

Thus:

$$\frac{V_1 - V_2 - 2}{13 - V_1} = 10 \qquad (1)$$

$$\& \quad V_1 - V_2 - 2 = V_2 + V_1 - 2$$

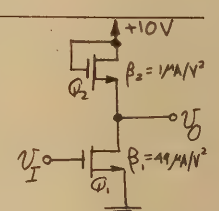
i.e. $\underline{\underline{V_2 = 0\text{ V}}}$ (which should be obvious from symmetry)

Substituting in (1) $\Rightarrow$ $V_1 = \underline{+12\text{ V}}$

Thus $V_3 = \underline{\underline{-12\text{ V}}}$ & $I = \frac{1}{2} \times 100\,(15 - 12 - 2)^2 = \underline{\underline{50\,\mu A}}$

$\boxed{8.30}$ $v_I = 0\text{V} \Rightarrow v_O = +9\text{V}$

$v_I = +10\text{V} \Rightarrow v_O = V_{OL}$

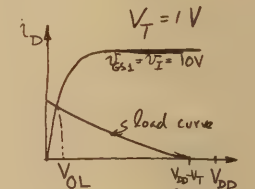


where V_{OL} is determined as follows:

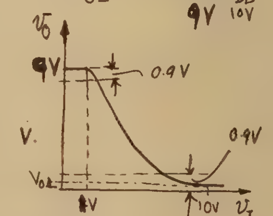
$$\beta_1\left[(10 - 1)\,V_{OL} - \tfrac{1}{2}V_{OL}^2\right]$$
$$= \tfrac{1}{2}\beta_2\,(10 - V_{OL} - 1)^2$$

$$\Rightarrow V_{OL} = \underline{0.09\text{ V}}$$

$V_T = 1$ V



The full output voltage range is $0.09\text{V} \Rightarrow 9\text{V}$, say 9V.



(a) We wish to find the value of v_I for which $v_O = 0.9 \times 9 =$ V.

$$i_D = \tfrac{1}{2}\beta_2\,(10 - 8.1 - 1)^2$$
$$= \tfrac{1}{2} \times 1 \times 0.81 = 0.405\ \mu A$$

$$0.405 = \tfrac{1}{2}\beta_1\,(v_I - 1)^2 \Longrightarrow v_I = \underline{\underline{1.13\text{ V}}}$$

(b) We wish to find the value of v_I at which $v_O = 0.9$V

$$i_D = \tfrac{1}{2} \times 1 \times (10 - 0.9 - 1)^2 = 32.8\ \mu A$$

Q_1 will be in triode region, thus

$$32.8 = 49\left[(v_I - 1) \times 0.9 - (0.81/2)\right] \Rightarrow v_I = \underline{2.19\text{V}}$$

In the middle of the switching range, the slope of the transfer characteristic is

$$-\sqrt{\frac{\beta_1}{\beta_2}} = \underline{-7 \ V/V}.$$

At the middle of the output swing, i.e. at $v_O \simeq 4.5\,V$ the equivalent resistance of the load is $\frac{1}{g_{m2}}$ where

$$g_{m2} = \beta_2 \,(v_{GS2} - V_T) = 1\,(10 - 4.5 - 1) = 4.5\,\mu A/V$$

Thus $R_{eq} = \frac{1}{4.5}\,M\Omega = \underline{222\ k\Omega}$

Using this value an estimate of the 10% to 90% inverter rise-time can be obtained as follows:

$$t_r \simeq 2.2\,\tau = 2.2 \times 222 \times 10^{3} \times 1 \times 10^{-12} = \underline{0.49\,\mu s}$$

Using equation (8.26) a much more precise value of t_r can be obtained as

$$t_r = \frac{17.8\ C}{\beta_2 (V_{DD} - V_T)} = \frac{17.8 \times 1 \times 10^{-12}}{1 \times 10^{-6} \times 9} = \underline{1.98\,\mu s}$$

$\boxed{8.31}$ Refer to Fig. 8.35. $V_{T2} = -1V$ (i.e. $V_P = -1V$) and $\beta_2 = 1\mu A/V^2$. Thus $I_{DSS} = \frac{1}{2}\beta_2 V_{T2}^2 = 0.5\,\mu A$

$V_{OH} = V_{DD} = \underline{5\,V}$ $\quad V_{OL}$ (or V_{on} in Fig. 8.35d) is found as follows:

Q_1 is in the triode region with $v_{GS} = +5V$ and is conducting a current equal to I_{DSS} of Q_2, thus

$$0.5\,\mu A = 10\left[(5-1)\,V_{on} - \tfrac{1}{2}V_{on}^2\right]$$

$$\Rightarrow V_{on} = \underline{0.013\,V}$$

The point $v_O = 4.5V$ lies on the A'B' segment of the transfer characteristic of Fig 8.35d. Thus to find the corresponding value of v_I we assume Q_1 to be in pinch-off and Q_2 to be in triode region and equate their i_D's to obtain

$$\tfrac{1}{2} \times 10 \times (v_I - 1)^2 = 0.5\left[2 \times \frac{0.5}{1} - \left(\frac{0.5}{1}\right)^2\right]$$

$$\Rightarrow v_I = \underline{1.27V}$$

For this inverter, the segment B'C' of the transfer characteristic is a vertical straight line at $v_I = V_t$ where V_t is determined using Eqn. (8.30) to be

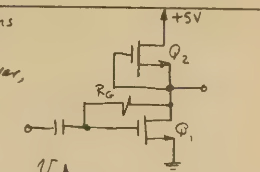
$$V_t = 1 + \frac{1}{\sqrt{10/1}} = 1.316\,V$$

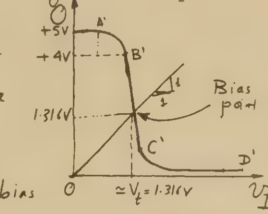
At C' Q_1 leaves pinch-off, thus

$$v_O\Big|_{C'} = V_t - V_{T1} = 0.316\,V$$

Thus the point $v_O = 2.5V$ lies on the B'C' segment and the corresponding $v_I = V_t = \underline{1.316\,V}$

$\boxed{8.32}$ Refer to the calculations of Problem 8.31. Note, however, that the segment B'C' will no longer be a vertical ~~straig~~ straight line since r_o of Q_1 and Q_2 are finite. Nevertheless, for the purpose of calculating the bias point obtained due to R_G we shall assume B'C' to be almost vertical. Thus as shown on the sketch for v_O vs. v_I, the bias point coordinate are





$$V_{GS}\Big|_{Q_1} = V_{DS}\Big|_{Q_1} = \underline{\underline{1.316\,V}} \quad \& \quad V_{DS}\Big|_{Q_2} = 5 - 1.316 = \underline{3.684V}$$

The amplifier gain is given by

$$\text{Gain} = -g_{m1}\left[r_{o1} /\!/ r_{o2}\right]$$

where
$$g_{m1} = 10\,(1.316 - 1) = 3.16\,\mu A/V$$

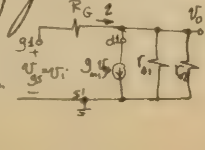
$$r_{o1} = \frac{\mu}{g_{m1}} = \frac{100}{3.16} = 31.6\,M\Omega$$

$$g_{m2} = \frac{2 \times 0.5}{1} = 1\,\mu A/V$$

$$r_{o2} = \frac{100}{1} = 100\,M\Omega$$

Thus, Gain $= -3.16\,[31.6 /\!/ 100] = \underline{-76\ V/V}$

This value was obtained neglecting the effect of R_G. Since r_{o1} and r_{o2} are quite large, the effect of R_G may not be negligible and should be taken into account by analyzing the equivalent circuit shown



$i = \frac{v_{gs} - v_o}{R_G}$

$v_o = (i - g_m v_{gs})(r_{o1} // r_{o2})$

$= \left(\frac{1}{R_G} - g_m\right)(r_{o1}//r_{o2})\, v_{gs} - v_o \frac{r_{o1}//r_{o2}}{R_G}$

$$\text{Gain} \equiv \frac{v_o}{v_i} = \frac{v_o}{v_{gs}} = \frac{\left(\frac{1}{R_G} - g_m\right)(r_{o1}//r_{o2})}{1 + \left(\frac{r_{o1}//r_{o2}}{R_G}\right)}$$

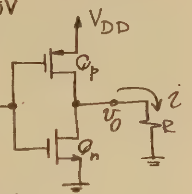
$$= -g_m (r_{o1}//r_{o2}) \left\{ \frac{1 - \frac{1}{g_m R_G}}{1 + \left(\frac{r_{o1}//r_{o2}}{R_G}\right)} \right\}$$

For $R_G = 10\,M\Omega$ we obtain

$$\text{Gain} = -76 \left\{ \frac{1 - \frac{1}{3.16 \times 10}}{1 + \frac{24}{10}} \right\} \simeq \underline{-22.4 \text{ V/V}}$$

8.33 (a) $V_{DD} = +5V$: $V_{OH} = +5V$, $V_{OL} = 0V$

(b) $V_{DD} = +15V$: $V_{OH} = +15V$, $V_{OL} = 0V$



To find the maximum output current that is obtained while limiting the change in output voltage to $0.1\,V_{DD}$ consider the case $v_I = 0\,V$, then Q_n will be off and Q_p will be operating in the triode region with $v_{SD} = V_{DD} - v_O = 0.1\,V_{DD}$. Thus

(a) For $V_{DD} = 5V$

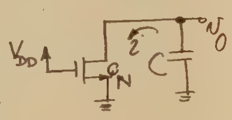
$$i = 60\left[(5-1)\,0.5 - \frac{(0.5)^2}{2}\right] = \underline{112.5\,\mu A}$$

(b) For $V_{DD} = 15V$

$$i = 60\left[(15-1)\times 1.5 - \frac{(1.5)^2}{2}\right] \simeq \underline{1.2\text{ mA}}$$

Identical results apply for the maximum current that the gate can sink while $v_O \le 0.1 V_{DD}$.

8.34 Refer to Fig. 8.43.



During t_{f1}, Q_N will be in the pinch-off region and i will be constant, $i = \frac{1}{2}\beta(V_{DD} - V_T)^2$. Thus v_O will fall linearly.

$$\frac{0.9\,V_{DD} - (V_{DD} - V_T)}{t_{f1}} = \frac{\frac{1}{2}\beta(V_{DD} - V_T)^2}{C}$$

$$\boxed{t_{f1} = \frac{2C\,(V_T - 0.1\,V_{DD})}{\beta\,(V_{DD} - V_T)^2}}$$

Note: We have assumed that $0.9\,V_{DD} > V_{DD} - V_T$. If this is not the case then $t_{f1} = 0$.

Beyond t_{f1}, Q_N enters the triode region and the discharge current i becomes

$$i = \beta\left[(V_{DD} - V_T)v_O - \frac{1}{2}v_O^2\right]$$

Thus

$$-C\frac{dv_O}{dt} = \beta(V_{DD} - V_T)\,v_O - \frac{1}{2}\beta v_O^2$$

$$\int_{v_O = V_{DD} - V_T}^{v_O = 0.1V_{DD}} \frac{-dv_O}{(V_{DD} - V_T)v_O - \frac{1}{2}v_O^2} = \int_{t=0}^{t=t_{f2}} \frac{\beta}{C}\,dt \qquad (1)$$

Note: In Eqn. (1) we assume that $t = 0$ is the beginning of the second discharge interval. We also assume that at the beginning of this interval (t_{f2}), $v_O = V_{DD} - V_T$, otherwise the lower limit of the integral on the left-hand-side should be changed.

Now

$$\frac{1}{V_{DD} - V_T}\int_{V_{DD} - V_T}^{0.1V_{DD}} \frac{dv_O}{v_O^2\left[\frac{1}{2(V_{DD} - V_T)}\right] - v_O} = \frac{\beta}{C}\int_0^{t_{f2}} dt \qquad (2)$$

$$\frac{1}{V_{DD} - V_T}\left\{\ln\left[1 - \frac{2(V_{DD} - V_T)}{v_O}\right]\right\}_{V_{DD} - V_T}^{0.1V_{DD}} = \frac{\beta}{C}\,t_{f2} \qquad (3)$$

$$\boxed{t_{f2} = \frac{C}{\beta(V_{DD} - V_T)}\ln\left[20\left(\frac{V_{DD} - V_T}{V_{DD}}\right) - 1\right]}$$

For $C = 10\,pF$, $V_T = 1$ and $\beta = 60\,\mu A/V^2$

(a) $V_{DD} = 5V$

$t_{f1} = 10.4$ ns

$t_{f2} = 112.8$ ns

Thus $t_f = t_{f1} + t_{f2} = \underline{123.2 \text{ ns}}$

(b) $V_{DD} = 15V$

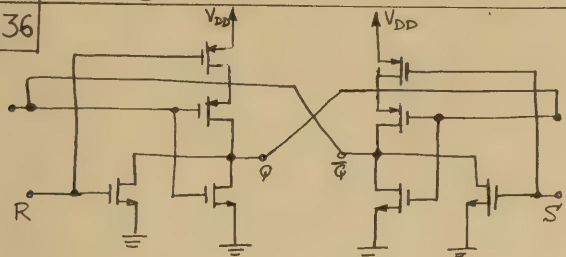
$t_{f1} = 0$ because $0.9\,V_{DD} = 13.5V < \underset{14V}{V_{DD} - V_T}$

Thus to start with the device will be in the triode region. The expression derived above for t_{f2} gives the discharge time from $v_O = 14\,V$ which is slightly greater than that the $0.9\,V_{DD}$ point (13.5V). Thus t_{f2} evaluated will be slightly greater than the actual 90% to 10% fall time.

$t_f = t_{f2} = \underline{34.2 \text{ ns}}$

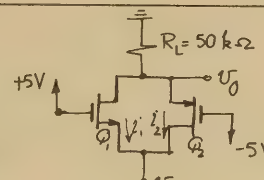
8.35 See Fig. 15.70

8.36



In the rest position, R and S should be at 0V. To set the flip/flop S should ~~both~~ be raised to V_{DD}. To reset, R should be raised to V_{DD}.

8.37 (a) $v_A = -5V$



Q_2 is off and Q is in the triode region with $v_{GS} = 10V$. For small v_{DS} we can write

$$i_D \simeq \beta(v_{GS} - V_T)v_{DS}$$

$$= 30 \times 9\ (v_0 + 5)$$

But, $i_D = -v_0/R_L = -0.02 \times 10^{+3}\, v_0$, μA

Solving these two equations yields

$v_0 = \underline{-4.655\ V}$ & $v_{DS} = 0.345V$, $i_D = 0.093$ mA

The switch resistance is

$$R_{switch} = \frac{0.345}{0.093} = \underline{\underline{3.706\ k\Omega}}$$

(b) $v_A = -2V$

$$i_1 = 30 \times 6 \times (v_0 + 2) = 180(v_0 + 2),\ \mu A$$

$$i_2 = 30\ (v_0 + 5 - 1)(v_0 + 2) = 30(v_0 + 4)(v_0 + 2),\ \mu A$$

$$v_0 = -(i_1 + i_2)R_L = -0.05\,(v_0 + 2)\,[30(v_0 + 4) + 180]$$

$$\Rightarrow v_0 = \underline{-1.849\,V}\ \&\ v_{DS1} = v_{SD2} = 0.151\ V$$

$$i_1 + i_2 = \frac{+1.849}{50} = 0.037\ mA$$

$$R_{switch} = \frac{0.151}{0.037} = \underline{\underline{4.083\ k\Omega}}$$

(c) $v_A = 0\,V$

$\underline{\underline{v_0 = 0}}$, $i_1 = i_2 = 0$

To find the switch resistance we note that

$$R_{switch} = r_{DS1} \,//\, r_{DS2}$$

$$= \frac{1}{30 \times 10^{-3} \times 4}\,k\Omega \,//\, \frac{1}{30 \times 10^{-3} \times 4}\,k\Omega$$

$$= \underline{\underline{4.167\ k\Omega}}$$

8.38 (a) Q_1 and Q_2 are identical, thus $\underline{\underline{V_0 = +5V}}$.

(b) $i_{D1} = i_{D2} \Rightarrow \frac{1}{2}\beta_1\,(V_0 - V_{T1})^2 = \frac{1}{2}\beta_2\,(10 - V_0 - V_{T2})^2$

$$\sqrt{\frac{\beta_2}{4}}\,(V_0 - V_T) = \sqrt{\beta_2}\,(10 - V_0 - V_T)$$

$$V_0 - V_T = 20 - 2V_0 - 2V_T$$

$$3V_0 = 20 - V_T$$

$$V_0 = \underline{\underline{\tfrac{1}{3}(20 - V_T)}}$$

(c) $\frac{1}{2}\beta_1\,(V_0 - V_{T1})^2 = \frac{1}{2}\beta_2\,(10 - V_0 - V_{T2})$

$\beta_1 = \beta_2$, thus

$$V_0 - V_{T1} = 10 - V_0 - V_{T2}$$

$$V_0 - 4V_{T2} = 10 - V_0 - V_{T2}$$

$$V_0 = \underline{\underline{\tfrac{1}{2}(10 + 3V_{T2})}}$$

8.39 $I_2 = I_1 = 1\,mA$ $\quad I_3 = 2I_1 = 2\,mA$

8.40 Since Q_1, Q_2 and Q_3 are matched then

$$V_{GS1} = V_{GS2} = V_{GS3} = 5\,V$$

$$I_{D1} = I_{D2} = I_{D3} = \tfrac{1}{2}\beta\,(5 - V_T)^2$$

Because Q_3 and Q_4 have their gates joined and their sources joined, then $V_{GS4} = 5\,V$

Now since Q_4 is also matched to Q_1, Q_2 and Q_3 then $I_{D4} = \frac{1}{2}\beta\,(5 - V_T)^2$.

Transistor Q_5 has

$$I_{D5} = I_{D4} = \tfrac{1}{2}\beta(5 - V_T)^2$$

Since Q_5 is matched to all other transistors then $I_{D5} = \frac{1}{2}\beta\,(V_{GS5} - V_T)^2$. Thus $V_{GS5} = 5\,V$ and $\underline{\underline{V_0 = +5V}}$

8.41 $I_1 = 1\,mA = \frac{1}{2}\beta_1(V_{GS1} - V_{T1})^2$

$$1 = \tfrac{1}{2} \times 2\,(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 3\,V$$

Thus, $\underline{\underline{V_1 = +3\,V}}$.

$$I_2 = I_{D2} = \tfrac{1}{2}\beta_2\,(V_{GS2} - V_{T2})^2 = \tfrac{1}{2} \times 1 \times (3 - 2)^2$$

$$= \underline{\underline{1\ mA}}$$

$$I_{D3} = I_2 = 1 = \tfrac{1}{2} \times \beta_3\,(V_{GS3} - V_{T3})^2$$

$$\Rightarrow V_{GS3} = 3\,V$$

$$\underline{\underline{V_2 = +6\,V}}$$

8.42 $1 = \frac{1}{2} \times 0.5 \times (4 - V_0 - 2)^2 \Rightarrow V_0 = 0\,V$

$$g_m = 0.5\,(4 - 0 - 2) = 1\ mA/V$$

$$R_o = 1/g_m = \underline{\underline{1\ k\Omega}}$$

8.43 $1 = \frac{1}{2} \times 0.5 (V_{GS} - 2)^2 \Rightarrow V_{GS} = 4V$

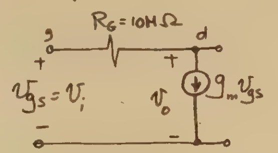
$V_O = 9 - 5\times 1 - 4 = \underline{0V}$

$R_o = \frac{1}{g_m} + 5 = \frac{1}{0.5(4-2)} + 5 = \underline{6\ k\Omega}$

8.44 $1 = \frac{1}{2} \times 0.5 (V_{GS} - 2)^2 \Rightarrow V_{GS} = 4V$

DC voltage at output = 4V

The small-signal equivalent circuit is shown.



$v_o = v_{gs} - g_m v_{gs} R_G \Rightarrow \frac{v_o}{v_i} = \frac{v_o}{v_{gs}} = -(g_m R_G - 1)$

$g_m = 0.5\ (4-2) = 1mA/V$

Thus, $\frac{v_o}{v_i} \simeq -1 \times 10^{-3} \times 10 \times 10^6 = -10{,}000\ V/V$

Using Miller's theorem,

$R_i = \frac{R_G}{1 - \frac{v_o}{v_i}} = \frac{R_G}{g_m R_G} = \frac{1}{g_m} = \underline{1\ k\Omega}$

To find the output resistance with we short-circuit the input signal source v_i. Thus $v_{gs} = 0$ and $g_m v_{gs} = 0$. It follows that $R_o = R_G = \underline{\underline{10M\Omega}}$

8.45 $\frac{1}{2} \times 0.5 (V_{GS1} - 2)^2 = \frac{1}{2} \times \frac{0.5}{36} (10 - V_{GS1} - 2)^2$

$6(V_{GS1} - 2) = 8 - V_{GS1} \Rightarrow V_{GS1} = \frac{20}{7} = 2.86V$

Thus the dc voltage at the output is $\underline{2.86V}$.

Ignoring the effect of the $10M\Omega$ resistor on the voltage gain we have

$\text{voltage gain} \equiv \frac{v_o}{v_i} = -g_{m1} \times \left(\frac{1}{g_{m2}}\right) = -\frac{g_{m1}}{g_{m2}}$

where $g_{m1} = 0.5\ (2.86 - 2) = 0.43\ mA/V$

$g_{m2} = \frac{0.5}{36} (10 - 2.86 - 2) = 0.071\ mA/V$

Thus, gain = $\underline{\underline{-6\ V/V}}$

To obtain R_i we use Miller's theorem,

$R_i = \frac{10M\Omega}{1 - \text{gain}} = \frac{10}{7} = \underline{1.43\ M\Omega}$.

To obtain the output resistance R_o we short-circuit the input signal source v_i. Thus $v_{gs1} = 0$ and $g_m v_{gs1} = 0$. It follows that

$R_o = 10M\Omega \,//\, \frac{1}{g_{m2}} = 10M\Omega // 14k\Omega \simeq \underline{14\ k\Omega}$

The lower 3dB frequency is given by

$f_{3dB} = \frac{1}{2\pi \times 0.1\mu F \times R_i} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 1.43 \times 10^6} = \underline{\underline{1.11\ Hz}}$

8.46 __DC Calculations__ From a dc point of view the three stages have identical conditions. Thus all transistors will be carrying equal dc currents. Also $V_{GS1A} = V_{GS1B} = V_{GS1C}$.

But the two 10-MΩ resistors cause V_D of Q_{1C} to equal to V_G of Q_{1A}. Thus V_{DG} of $Q_{1C} = 0$ and we can write of the third stage (and for each of the other two stages),

$\frac{1}{2} \times 0.5\ (V_{GS1} - 2)^2 = \frac{1}{2} \times \frac{0.5}{4} (10 - V_{GS1} - 2)^2$

Thus, $2(V_{GS1} - 2) = 8 - V_{GS1}$

$\Rightarrow V_{GS1} = 4V$

Thus the dc voltage at the output is $\underline{4V}$

__Small-Signal Calculations__

If we neglect the effect of the $10M\Omega$ load resistance at the output then we have an amplifier consisting of three identical stage each of gain $= -g_{m1} \times \frac{1}{g_{m2}}$ where

$g_{m1} = 0.5\ (4-2) = 1\ mA/V$

& $g_{m2} = \frac{0.5}{4} (6-2) = 0.5\ mA/V$.

Thus each stage has a voltage gain of −2 and the total gain is $(-2)^3 = \underline{-8\ V/V}$.

With $C_2 = \infty$, the input resistance is $10M\Omega$.

With $C_2 = 0$, the gain remains unchanged but the input resistance becomes (using Miller's theorem),

$R_i = \frac{20M\Omega}{1 - \text{gain}} = \frac{20}{9} = \underline{\underline{2.22\ M\Omega}}$

8.47 C will be high if A is high or B is high. Thus the circuit can perform the OR logic function. If $V_A = +5V$ and $V_B = 0V$ then transistor A will be operating in pinch-off and transistor B will be off.

$1 = \frac{1}{2} \times 0.5 (V_{GSA} - 1)^2 \Rightarrow V_{GSA} = 3V$

Thus, $V_C = 5 - 3 = \underline{+2V}$

8.48 The voltage at C will be high (+5V) in only one case: when V_A and V_B are simultaneously low (0V). Thus, in a positive logic system we can write

$C = \overline{A}\,\overline{B}$

or, using DeMorgan's law (Chapter 6)

$C = \overline{A+B}$

Thus the circuit is a NOR gate.

To find the output voltage when $v_A = +5V$ and $v_B = 0\,V$ we assume that Q_A is in the triode region, thus

$$i_{DA} = 0.5\left[(5-1)v_C - \tfrac{1}{2}v_C^2\right] = \frac{5-v_C}{5}$$

$$\Rightarrow \quad v_C = \underline{\underline{0.48V}}$$

To find the output voltage when $v_A = v_B = +5V$, we assume that both transistors are in the triode region. The total current through the 5-kΩ load resistor will be $2 \times 0.5\left[4v_C - \tfrac{1}{2}v_C^2\right]$. Thus

$$4v_C - \tfrac{1}{2}v_C^2 = \frac{5-v_C}{5}$$

$$\Rightarrow \quad v_C = \underline{\underline{0.25\,V}}$$

CHAPTER 9 — EXERCISES

$\boxed{9.1}$ $i_C = I_S\, e^{v_{BE}/V_T}$

Thus, $v_{BE2} - v_{BE1} = V_T \ln(i_{C2}/i_{C1})$

For $i_C = 0.1$ mA, $v_{BE} = 0.7 + 0.025 \ln(0.1/1) = \underline{\underline{0.64V}}$

For $i_C = 10$ mA, $v_{BE} = 0.7 + 0.025 \ln(10/1) = \underline{\underline{0.76V}}$

$\boxed{9.2}$ $\beta = 50 \Rightarrow \alpha = \frac{\beta}{\beta+1} = \frac{50}{51} = \underline{\underline{0.980}}$

$\beta = 150 \Rightarrow \alpha = \frac{\beta}{\beta+1} = \frac{150}{151} = \underline{\underline{0.993}}$

$\boxed{9.3}$ $i_C = I_S\, e^{v_{EB}/V_T} = 10^{-15} \times e^{0.7/0.025}$

$= \underline{\underline{1.446 \text{ mA}}}$

$i_B = i_C/\beta = \frac{1.446\text{ mA}}{100} = \underline{\underline{14.46\ \mu A}}$

$i_E = i_C + i_B = 1.446 + 0.01446 = \underline{\underline{1.460 \text{ mA}}}$

$\boxed{9.4}$ $I_E = \frac{V_E - (-10)}{10\text{ k}\Omega} = \frac{-0.7+10}{10} = \underline{\underline{0.93 \text{ mA}}}$

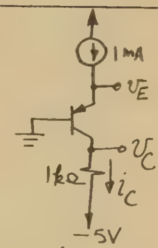
$I_B = \frac{I_E}{\beta+1} = \frac{0.93}{51} = \underline{\underline{18.2\ \mu A}}$

$I_C = \frac{\beta}{\beta+1} I_E = \frac{50}{51} \times 0.93 = \underline{\underline{0.91 \text{ mA}}}$

$V_C = 10 - I_C R_C = 10 - 0.91 \times 5 = \underline{\underline{+5.45V}}$

Thus the transistor is in the active mode, as assumed.

$\boxed{9.5}$ Assuming operation in the active mode, then $i_C = \alpha \times 1 \simeq 1$ mA. Thus $v_C = -5 + 1 \times 1 = -4V$. Thus the transistor is indeed in the active mode. Since the emitter current is constant then v_{EB}, and thus v_E, decreases by 2 mV for every °C rise in temperature. For a 30°C increase in temperature, $\Delta v_E = \underline{\underline{-60\text{mV}}}$. Since i_E remains constant then i_C will remain constant and so will v_C; $\Delta v_C = \underline{\underline{0}}$.



$\boxed{9.6}$ If β is very large then the base current can be neglected and the base voltage V_B will be given approximately by

$$V_B \simeq 15\,\frac{100}{100+100} = +7.5V$$

$$V_E \simeq V_B - 0.7 = \underline{\underline{+6.8V}}$$

$$I_E = \frac{V_E}{6.8\text{k}\Omega} = \frac{6.8}{6.8} = \underline{\underline{1\text{mA}}}$$

$$V_C = +15 - 4.3 \times I_C \simeq 15 - 4.3 \times I_E = \underline{\underline{+10.7V}}$$

Since $V_C > V_B$, the transistor is in the active mode, as assumed.

$\boxed{9.7}$ $g_m = I_C/V_T = \frac{0.5\text{mA}}{0.025V} = \underline{\underline{20 \text{ mA/V}}}$

$r_e = V_T/I_E \simeq \frac{0.025V}{0.5\text{ mA}} = \underline{\underline{50\,\Omega}}$

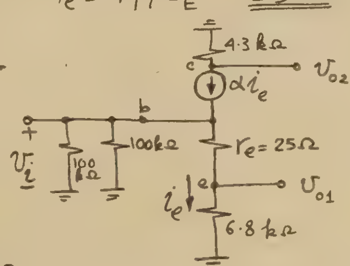
$r_\pi = \beta/g_m = \frac{100}{20\text{ mA/V}} = \underline{\underline{5\text{ k}\Omega}}$

$\boxed{9.8}$ $I_E = 1$mA. $r_e = V_T/I_E = \underline{\underline{25\,\Omega}}$.

$v_{o1} = v_i\,\frac{6.8}{6.8+0.025}$

$\frac{v_{o1}}{v_i} = \underline{\underline{0.996 \text{ V/V}}}$

$i_e = \frac{v_i}{6.8+0.025}$ mA

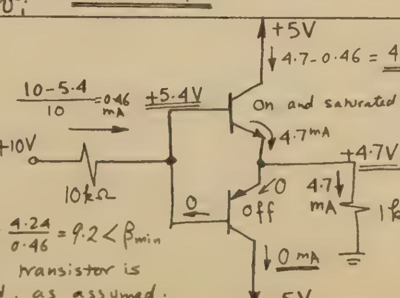


$v_{o2} = -\alpha i_e \times 4.3$

$\simeq -\frac{v_i \times 4.3}{6.825}$

Thus, $\frac{v_{o2}}{v_i} = \underline{\underline{-0.63 \text{ V/V}}}$.

$\boxed{9.9}$



Since $\frac{I_C}{I_B} = \frac{4.24}{0.46} = 9.2 < \beta_{min}$ then the transistor is saturated, as assumed.

CHAPTER 9 — PROBLEMS

9.1 $i_C = I_S \, e^{v_{BE}/V_T}$

Thus, $v_{BE2} - v_{BE1} = V_T \ln (i_{C2}/i_{C1})$

At $i_C = 1\,mA$, $v_{BE} = 0.64 + 0.025 \ln (1/0.1) = \underline{0.7\,V}$

At $i_C = 10\,mA$, $v_{BE} = 0.64 + 0.025 \ln (10/0.1) = \underline{0.76\,V}$

At $i_C = 100\,mA$, $v_{BE} = 0.64 + 0.025 \ln (100/0.1) = \underline{0.81\,V}$

9.2 $\beta = i_C / i_B = 0.37 / 2.7 \times 10^{-3} = \underline{137}$.

9.3 $\alpha = 0.900$; $\beta = \frac{\alpha}{1-\alpha} = \frac{0.9}{1-0.9} = \underline{9}$.

$\alpha = 0.999$; $\beta = \frac{\alpha}{1-\alpha} = \frac{0.999}{1-0.999} = \underline{999}$.

9.4 $i_B = 14.46\,\mu A$, $i_E = 1.460\,mA$

$i_C = i_E - i_B = 1.446\,mA$

$\beta = \frac{i_C}{i_B} = \frac{1.446}{14.46} = \underline{100}$.

$\alpha = \frac{i_C}{i_E} = \underline{0.99}$

$I_S = i_C \, e^{-v_{BE}/V_T}$

$= 1.446 \times 10^{-3} \, e^{-0.7/0.025}$

$= \underline{10^{-15}\,A}$.

9.5 $i_C = I_S \, e^{v_{BE}/nV_T}$

Thus, $v_{BE2} - v_{BE1} = nV_T \ln (i_{C2}/i_{C1})$

$$n = \frac{0.9 - 0.8}{0.025 \ln (100/10)} = 1.737$$

$$I_S = i_C \, e^{-v_{BE}/nV_T}$$

$$= 10 \times 10^{-3} \, e^{-0.8/(1.737 \times 0.025)}$$

$$= \underline{10^{-10}\,A}.$$

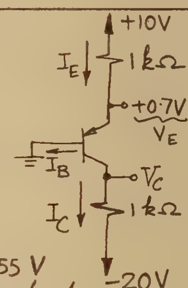
9.6 Transistor 1 : Active mode

Transistor 2 : Cut-off

Transistor 3 : Saturation

9.7 $I_E = \frac{10 - V_E}{1\,k\Omega} = \frac{10 - 0.7}{1}$

$= \underline{9.3\,mA}$.



$\beta = 10 \Rightarrow \alpha = \frac{\beta}{\beta+1} = \frac{10}{11} = 0.909$

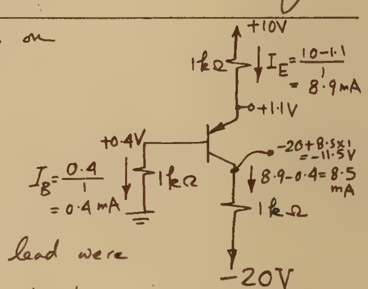
$I_B = \frac{I_E}{\beta+1} = \frac{9.3}{11} = \underline{0.845\,mA}$

$I_C = \alpha I_E = 8.45\,mA$

$V_C = -20 + 8.45 \times 1 = -11.55\,V$

If instead V_C was measured and found to be -10.7 then we conclude that I_C must be greater than $8.45\,mA$; in fact I_C must be $9.3\,mA$. Since I_E is also $9.3\,mA$ then we conclude that α must be very close to unity and thus β must be very large.

9.8 See analysis on circuit diagram.

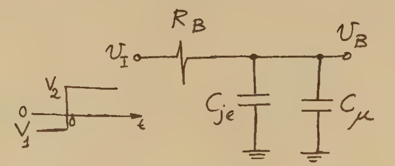


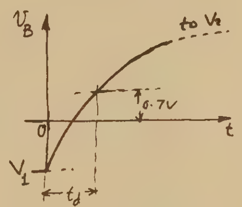
$\beta = \frac{8.5}{0.4} = \underline{21.25}$.

$V_C = \underline{-11.5\,V}$.

If the collector lead were opened then we simply have a forward conducting diode (the base-emitter junction) connected in series with two 1-kΩ resistors across a 10-V supply. Thus we should have a voltage drop of $\frac{10-0.7}{2} = 4.65\,V$ across each of the two resistor. Thus $V_B = +4.65\,V$ and $V_E = 4.65 + 0.7 = +5.35\,V$. (This assumes that $V_{EB} \simeq 0.7\,V$)

9.10 See the equivalent input circuit of the inverter and the waveform of v_B (or v_π).





$v_B = V_2 - (V_2 - V_1) \, e^{-t/\tau}$

where $\tau = (C_{je} + C_\mu) R_B$

$0.7 = V_2 - (V_2 - V_1) \, e^{-t_d/\tau}$

$(V_2 - V_1) \, e^{-t_d/\tau} = V_2 - 0.7$

$e^{t_d/\tau} = \frac{V_2 - V_1}{V_2 - 0.7}$

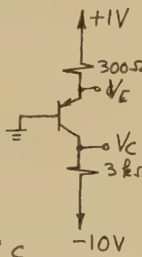
$t_d = \tau \ln [(V_2 - V_1)/(V_2 - 0.7)]$

Thus,

$$\underline{t_d = R_B (C_{je} + C_\mu) \ln \left(\frac{V_2 - V_1}{V_2 - 0.7} \right)}$$

9.9 Since V_{BE} changes by $-2\,mV/°C$ then the temperature at location A is $25°C - \frac{0.760-0.710}{0.002} = \underline{\underline{0°C}}$ at the temperature at location B is $25°C + \frac{0.710-0.560}{0.002}$ $= \underline{\underline{+100°C}}$.

9.10 At room temperature,



$V_{EB} = 0.700V$, $V_E = \underline{\underline{+0.7V}}$

$I_E = \frac{1-0.7}{0.3} = 1\,mA$,

$I_C \simeq I_E = 1\,mA$

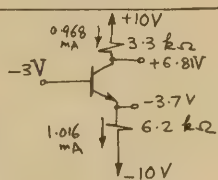
Thus, $V_C = -10 + 1 \times 3 = \underline{\underline{-7V}}$.

Raising the temperature by 50° C causes V_{EB} to decrease by $2 \times 50 = 100\,mV$.

Thus V_E becomes $\underline{+0.6\,V}$ and $I_E = \frac{1-0.6}{0.3} = 1.33\,mA$.

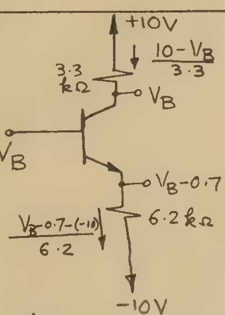
Thus V_C becomes $-10 + 1.33 \times 3 = \underline{\underline{-6\,V}}$.

9.11 See analysis on circuit diagram.



$V_E = \underline{\underline{-3.7V}}$, $V_C = \underline{\underline{+6.81V}}$

9.12 Let V_B be the highest voltage that can be applied at the base while the BJT remains in the active mode ($V_{CB} \geq 0$). From the circuit diagram we can write



$$\frac{10-V_B}{3.3} = \left(\frac{20}{21}\right) \frac{V_B - 0.7 + 10}{6.2}$$

$\Rightarrow V_B = \underline{\underline{3.5V}}$.

If β is very large the $\alpha = 1$ and

$$\frac{10-V_B}{3.3} = \frac{V_B - 0.7 + 10}{6.2}$$

$\Rightarrow V_B = \underline{\underline{3.3V}}$

9.13 (a) $V = 10.76V$, $I_E \simeq 10\,mA$ and $V_{BE} \simeq 0.76V$ then $V_E = V - V_{BE} = 10V$, $I_E = 10\,mA$ and $V_{BE} = 0.76\,V$. $I_C \simeq I_E = \underline{\underline{10\,mA}}$.

(b) $V = 1.70\,V \Rightarrow V_{BE} = 0.70V$, $I_E = \frac{1.7-0.7}{1} = 1\,mA$,

$I_C \simeq I_E = \underline{\underline{1\,mA}}$.

(c) $V = 0.74\,V$, $V_{BE} = 0.64\,V$, $V_E = 0.1\,V$, $I_E = 0.1\,mA$

$I_C \simeq I_E = \underline{\underline{0.1\,mA}}$

(d) $V = 0.59V$, $V_{BE} = 0.58V$, $V_E = 0.01\,V$,

$I_E = 0.01\,mA$, $I_C \simeq \underline{\underline{0.01\,mA}}$

9.14 Assume active-mode operation.

$V_B = 0.7V$ $\quad I_B = \frac{10-0.7}{1M\Omega} = 9.3\,\mu A$

$I_C = 9.3 \times 150 = \underline{\underline{1.395\,mA}}$

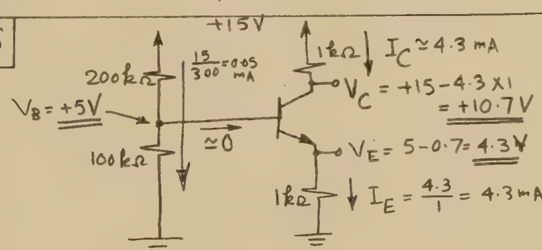
$V_C = 10 - 1.395 \times 3.3 = \underline{\underline{5.4V}}$

The device operates in the active mode as assumed.

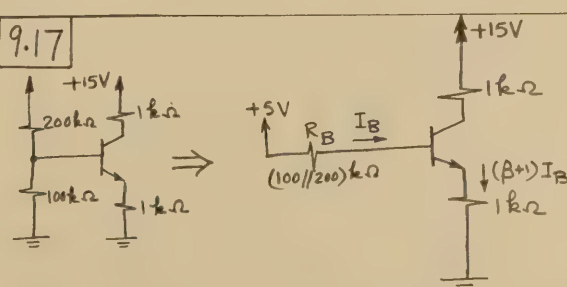
9.15 $V_B = 0.7V$ $\quad I_B = 9.3\,\mu A$.

If the transistor remains in the active mode then $I_C = 400 \times 9.3 = \underline{\underline{3.72\,mA}}$ which would imply that $V_C = \underline{-2.276}$ clearly an impossible situation and implies that the assumption of active-mode operation is incorrect. The lowest possible collector voltage is $\underline{0V}$ and is obtained as an extreme case of the saturation mode of operation.

9.16



9.17



$$I_B = \frac{5-0.7}{R_B + (\beta+1) \times 1k\Omega} = \frac{4.3}{66.7+51} = 0.0365\,mA$$

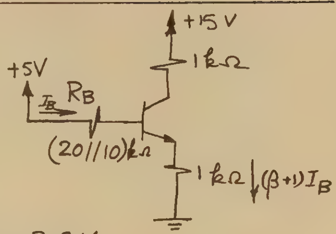
$V_B = 5 - I_B R_B = \underline{2.56\,V}$

$V_E = V_B - 0.7 = 2.56 - 0.7 = \underline{1.86\,V}$

$I_E = 1.86\,mA$ $\quad I_C = \frac{50}{51} \times 1.86 = 1.82\,mA$

$V_C = 15 - 1.82 \times 1 = \underline{\underline{13.18\,V}}$.

9.18



$I_B = \dfrac{4.3}{6.67+51}$

$= 0.07456\,mA$

$V_B = 5 - I_B R_B$

$= \underline{\underline{+4.5V}}$

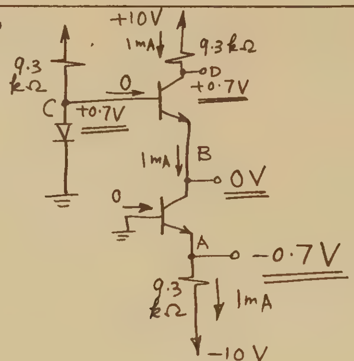
$V_E = 4.5 - 0.7 = \underline{\underline{3.8\,V}}$

$I_E = \dfrac{3.8}{1} = 3.8\ mA$

$I_C = \dfrac{50}{51} \times 3.8 = 3.73\,mA$

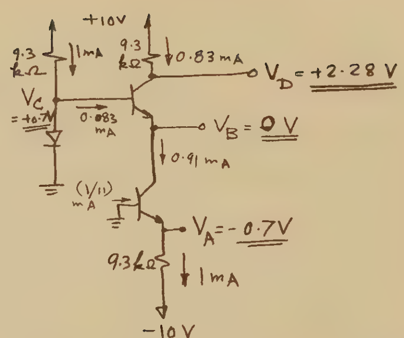
$V_C = 15 - 3.73 \times 1 = \underline{\underline{11.27\ V}}$.

9.19 (a) $\beta = \infty$



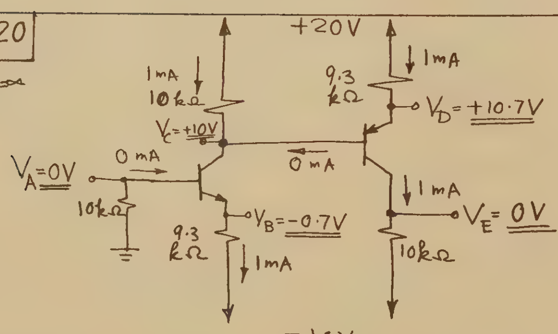
(b) $\beta = 10$

$\alpha = \frac{10}{11} = 0.91$

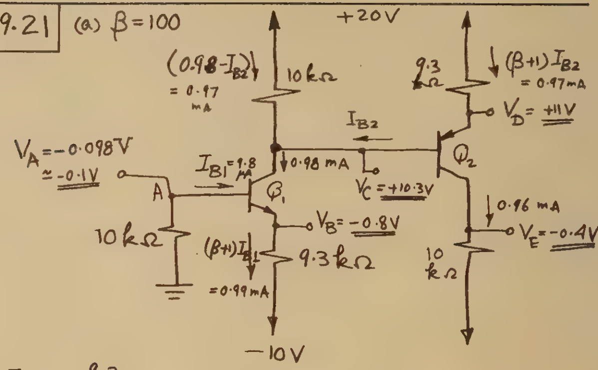


9.20

$\beta = \infty$



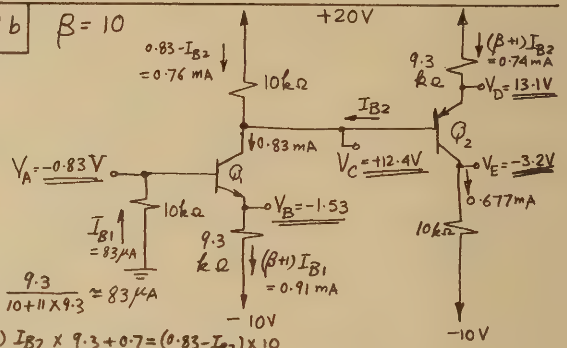
9.21 (a) $\beta = 100$



$I_{B1} = \dfrac{9.3}{10 + 101 \times 9.3} = 9.8\,\mu A$

$9.3 \times (\beta+1)\, I_{B2} + 0.7 = (0.98 - I_{B2}) \times 10 \Rightarrow I_{B2} = 9.6\,\mu A$

9.21 b $\beta = 10$

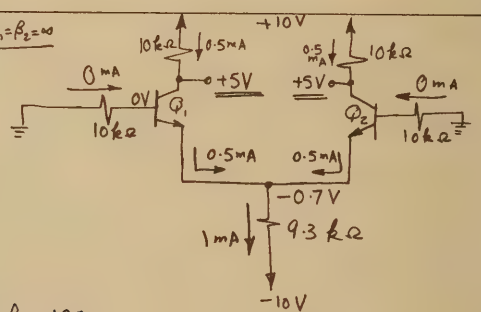


$I_{B1} = \dfrac{9.3}{10 + 11 \times 9.3} \simeq 83\,\mu A$

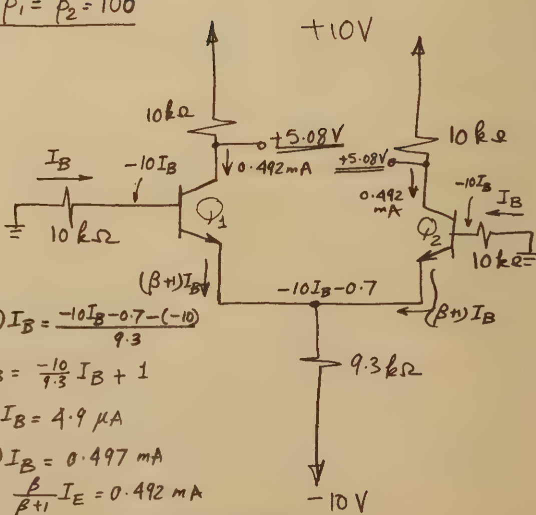
$(\beta+1)\, I_{B2} \times 9.3 + 0.7 = (0.83 - I_{B2}) \times 10$

$\Rightarrow I_{B2} = 67.7\,\mu A$

9.22 (a) $\beta_1 = \beta_2 = \infty$



(b) $\beta_1 = \beta_2 = 100$



$2(\beta+1) I_B = \dfrac{-10I_B - 0.7 - (-10)}{9.3}$

$202 I_B = \dfrac{-10}{9.3} I_B + 1$

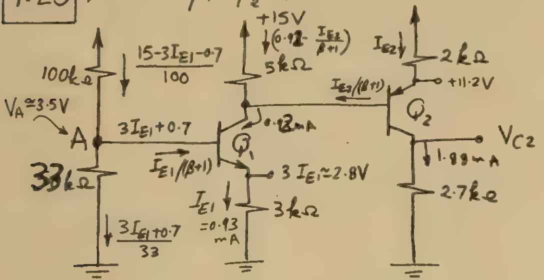
$\Rightarrow I_B = 4.9\,\mu A$

$(\beta+1) I_B = 0.497\ mA$

$I_C = \dfrac{\beta}{\beta+1} I_E = 0.492\ mA$

$V_C = 10 - 0.492 \times 10 = \underline{\underline{+5.08\,V}}$

9.23 Assume $\beta_1 = \beta_2 = 100$.



Node equation for node A ⇒

$$\frac{15-3I_{E1}-0.7}{100} = \frac{I_{E1}}{\beta+1} + \frac{3I_{E1}+0.7}{33}$$

$\Rightarrow I_{E1} = 0.93\ mA$

$$I_{E2} \times 2 + 0.7 = \left(0.92 - \frac{I_{E2}}{\beta+1}\right) \times 5$$

$\Rightarrow I_{E2} = 1.9\ mA$

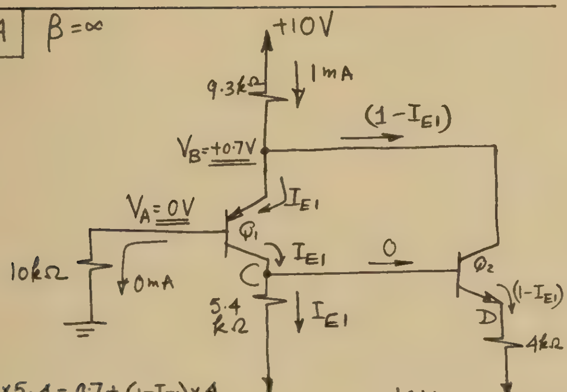
$V_{C1} = +10.5V$

$V_{E2} = +11.2V$

$I_{C2} = 1.88\ mA$

$V_{C2} = 1.88 \times 2.7 = +5.08\ V$

9.24 $\beta = \infty$

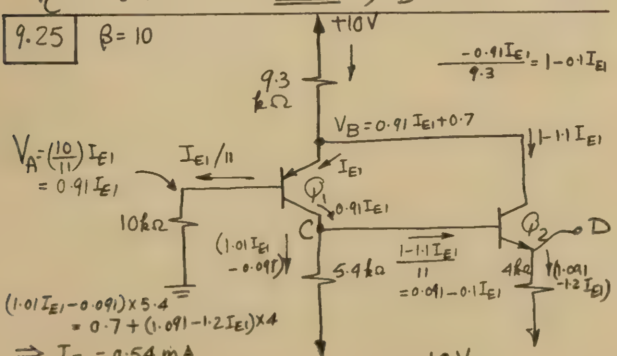


$I_{E1} \times 5.4 = 0.7 + (1-I_{E1}) \times 4$

$\Rightarrow I_{E1} = 0.5\ mA$

$V_C = -10 + 0.5 \times 5.4 = \underline{-7.3V}$, $V_D = -7.3 - 0.7 = \underline{-8V}$

9.25 $\beta = 10$



$(1.01I_{E1} - 0.091) \times 5.4$
$= 0.7 + (1.091 - 1.2I_{E1}) \times 4$

$\Rightarrow I_{E1} = 0.54\ mA$

$V_A = 0.91\,I_{E1} = \underline{0.49V}$

$V_B = 0.49 + 0.7 = \underline{1.19V}$

$V_C = -10 + (1.01 \times 0.54 - 0.091) \times 5.4$

$= \underline{-7.55V}$

$V_D = \underline{-8.25V}$

9.26

$$i_C = I_S\, e^{v_{BE}/V_T}$$

$$I_C = I_S\, e^{V_{BE}/V_T}$$

$$g_m = \frac{\partial i_C}{\partial v_{BE}}\bigg|_{i_C = I_C} = I_S \times \frac{1}{V_T} \times e^{v_{BE}/V_T}\bigg|_{I_C}$$

$$= \frac{I_S}{V_T}\, e^{V_{BE}/V_T}$$

Thus, $\underline{g_m = \frac{I_C}{V_T}}$.

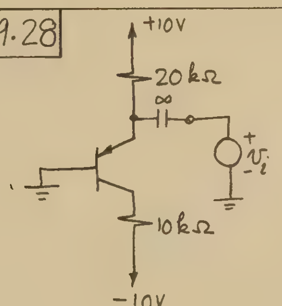
9.27

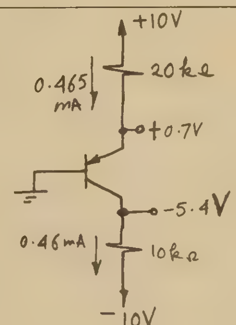
$$r_e = \frac{v_{be}}{i_e} = \frac{v_{be}}{(\beta+1)\,i_b} = \frac{1}{\beta+1}\left(\frac{v_{be}}{i_b}\right)$$

$$= \frac{1}{\beta+1}\, r_\pi$$

Thus, $\underline{r_\pi = (\beta+1)\, r_e}$

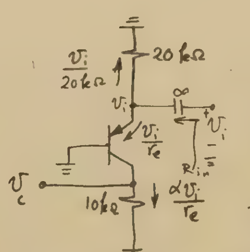
9.28





dc analysis ($\beta = 100$)

$r_e = \frac{V_T}{I_E} = \frac{25mV}{0.465mA} = 53.8\,\Omega$



Small-signal analysis

$v_c = \frac{\alpha v_i}{r_e} \times 10\,k\Omega$

$$\frac{v_c}{v_i} = +\frac{0.99 \times 10{,}000}{53.8} = \underline{184\ V/V}$$

To find the magnitude of the negative peaks of input signal that would cause the transistor to cut-off not that the ~~total~~ collector current can be expressed as

$$i_C = I_C\, e^{v_i/V_T}$$

Now if we define cut-off as $i_C = 0.01\, I_C$

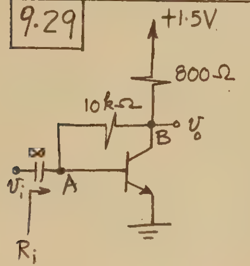
then $\hat{v}_i = \underline{-115 \text{ mV}}$. If on the other hand we define cut-off as $i_C = 0.05 I_C$ then $\hat{v}_i = \underline{-75 \text{ mV}}$.

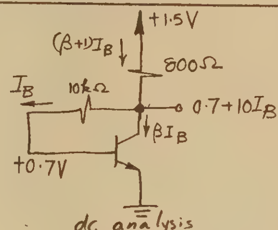
The input resistance R_{in} is given by

$$R_{in} = r_e // 20k\Omega \simeq r_e = \underline{53.8\ \Omega}.$$

If the voltage change across the junction, which is the amplitude of v_i, is limited to 10 mV then the peak output signal will be 10 X 184 mV = $\underline{1.84 \text{ V}}$.

9.29





dc analysis

$$(\beta+1) I_B = \frac{1.5-0.7-10 I_B}{0.8}$$

$$\Rightarrow I_B = \frac{1}{12.5+(\beta+1)}$$

$$I_C = \beta I_B = \frac{1}{\frac{12.5}{\beta}+1+\frac{1}{\beta}}$$

(a) $\beta = \infty$: I_C = 1 mA, $V_A = \underline{0.7V}$, $V_B = \underline{0.7V}$

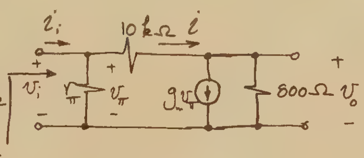
(b) $\beta = 100$: I_C = 0.88 mA, $V_A = \underline{0.7V}$, $V_B = \underline{0.788V}$

Signal Analysis

$v_\pi = v_i$

$i = \frac{v_\pi - v_o}{10}$

$v_o = (i - g_m v_\pi) \times 0.8k\Omega$



$$v_o = \left(\frac{v_\pi}{10} - \frac{v_o}{10} - g_m v_\pi\right) \times 0.8$$

$$v_o(1+0.08) = -v_\pi(g_m - 0.1) \times 0.8$$

$$\frac{v_o}{v_i} = \frac{v_o}{v_\pi} = -\frac{(g_m - 0.1)\times 0.8}{1.08}$$

(a) $\beta = \infty$: $g_m = 40$ mA/V, $\frac{v_o}{v_i} = \underline{-29.6 \text{ V/V}}$.

(b) $\beta = 100$: $g_m = 35.2$ mA/V, $\frac{v_o}{v_i} = -26$ V/V.

The input resistance R_i is given by

$$R_i \equiv \frac{v_i}{i_i} = \frac{v_i}{\frac{v_\pi}{r_\pi} + \frac{v_\pi - v_o}{10}} = \frac{1}{\frac{1}{r_\pi} + \frac{1}{10} - \frac{1}{10}\left(\frac{v_o}{v_i}\right)}$$

Thus:

(a) For $\beta = \infty$, $r_\pi = \infty$, $\frac{v_o}{v_i} = -29.6$, $R_i = \underline{326.8\ \Omega}$.

(b) For $\beta = 100$, $r_\pi = 2.87k\Omega$, $\frac{v_o}{v_i} = -26$, $R_i = \underline{328\ \Omega}$.

To find the largest peak-to-peak output at B we assume that it is allowed to forward bias the collector-base junction by as much as 0.3V. Thus:

(a) For $\beta = \infty$, the peak-to-peak output can be as high as 0.6 V. The corresponding peak-to-peak input is $\frac{600}{29.6} \simeq 20$ mV.

(b) For $\beta = 100$, the output can be allowed to vary from 0.4 to 1.176 V, thus the maximum allowed peak-to-peak output voltage is 776 mV. This corresponds to an input signal of $\frac{776}{26} \simeq 30$ mV. This signal swing is too large for the amplifier to remain linear. For this case linearity dictates that the peak of v_i should not exceed about 10mV (20 mV p-p) which results in an output signal of 520 mV peak-to-peak.

9.30

dc analysis:

$$\frac{3-1.4-10 I_B}{1.6} = (\beta+1) I_B + 0.07$$

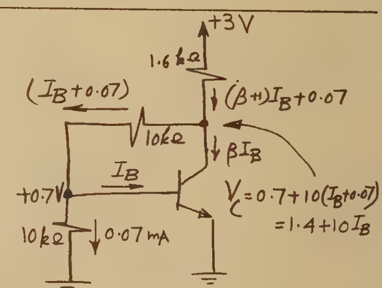
For $\beta = 100$,

$I_B = 8.67\ \mu A$

$V_C = 1.4 + 0.0867$

$\simeq \underline{1.49 \text{ V}}$

$I_C = \beta I_B = 0.87$ mA.

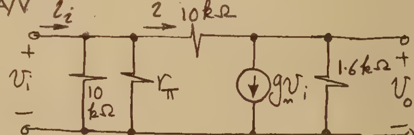


Small-signal Analysis

$g_m = \frac{I_C}{V_T} = 34.8$ mA/V

$r_\pi = \beta / g_m$

$= 2.87\ k\Omega$



$i = \frac{v_i - v_o}{10}$

$$v_o = (i - g_m v_i) \times 1.6 = (0.1 v_i - 0.1 v_o - g_m v_i) \times 1.6$$

$$v_o \times 1.16 = -v_i (g_m - 0.1) \times 1.6$$

$$\frac{v_o}{v_i} = -(34.7 \times 1.6)/1.16 = \underline{-47.9 \text{ V/V}}$$

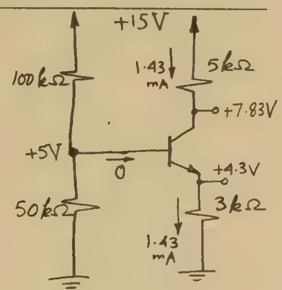
Using Miller's theorem we can write

$$R_i = 10\,k\Omega \,//\, 2.87\,k\Omega \,//\left(\frac{10\,k\Omega}{1 + \frac{v_o}{v_i}}\right)$$
$$= \left(10 // 2.87 // \frac{10}{48.9}\right) k\Omega$$
$$= \underline{\underline{187.3\,\Omega}}$$

The largest sine-wave input signal for which distortion is not great is about $\underline{10\text{-mV}}$ in amplitude. Such a signal results in an output sinusoid that is $479\,mV \simeq 0.5V$ peak. Thus the collector voltage will go from 1V to 2V.

9.31 $I_E = \underline{\underline{1.43\,mA}}$

$$r_e = \frac{V_T}{I_E} = \underline{\underline{17.5\,\Omega}}$$
$$\frac{v_e}{v_b} = \frac{3\,k\Omega}{3\,k\Omega + r_e}$$
$$= \frac{3000}{3017.5} = \underline{\underline{0.994\,V/V}}$$
$$\frac{v_c}{v_b} = \frac{v_e}{v_b}\,\frac{v_c}{v_e}$$
$$= 0.994 \times \frac{5\,k\Omega \times \alpha}{3\,k\Omega}$$
$$= \underline{1.66\ V/V}.$$



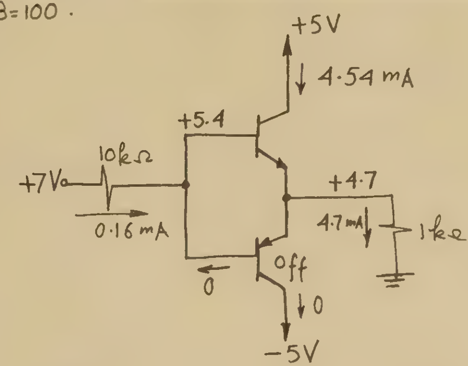
$V_E = \underline{\underline{-6.3\,V}}$ $V_B = \underline{\underline{-7V}}$ $V_C = \underline{\underline{-6.6\,V}}$

$I_B = 0.3\,mA$ $I_C = 0.34\,mA$

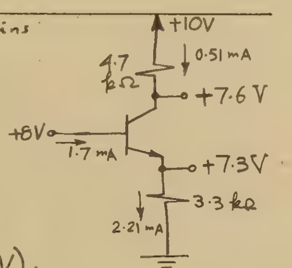
$$\beta_{Forced} = \frac{I_C}{I_B} = \frac{0.34}{0.3} = \underline{\underline{1.13}}$$

The transistor will remain saturated as long as its β is greater than 1.13.

9.34 $\beta = 100$.

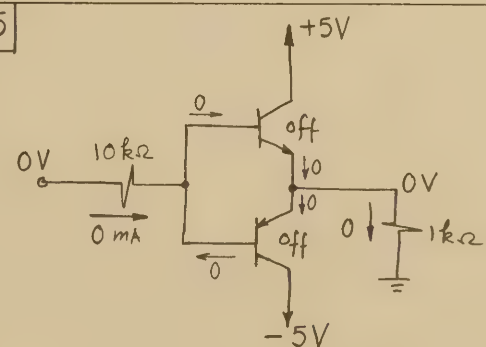


9.32 The transistor remains in saturation. v_E changes from $+5.3V$ to $+7.3V$ (thus, $\Delta v_E = +2V$). v_C changes from $+5.6V$ to $+7.6V$. (thus $\Delta v_C = +2V$).



Therefore the gain from base to emitter is +1 and from base to collector is +1. Hence the transistor operates as a three-terminal short-circuit, which is an appropriate model for a saturated transistor.

9.35

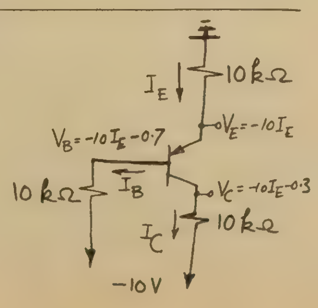


9.33 $I_B = \dfrac{-10\,I_E - 0.7 + 10}{10}$

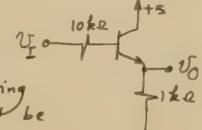
$$= -I_E + 0.93$$
$$I_C = \frac{-10\,I_E - 0.3 + 10}{10}$$
$$= -I_E + 0.97$$
$$I_E = I_B + I_C$$

Thus, $I_E = -2I_E + 1.90$

$$\Rightarrow I_E = \underline{0.63\ mA}$$



9.36 Refer to the results of Example 9.13 and Problems 9.35 and 9.35. Also note that because of the complementary symmetry of the circuit $v_O(+v_I) = -v_O(-v_I)$. Thus it is sufficient to investigate the transfer characteristics for $0 \le v_I \le +10V$. Over this range the pnp transistor will be off and the circuit simplifies to the one shown.



For $0 \le v_I \le +0.7$ (approx.) the transistor will be off or conducting a negligible current and v_O will be zero.

For $v_I > 0.7V$ (approx.) the transistor turns on and operates (initially) in the active mode. The emitter current will be

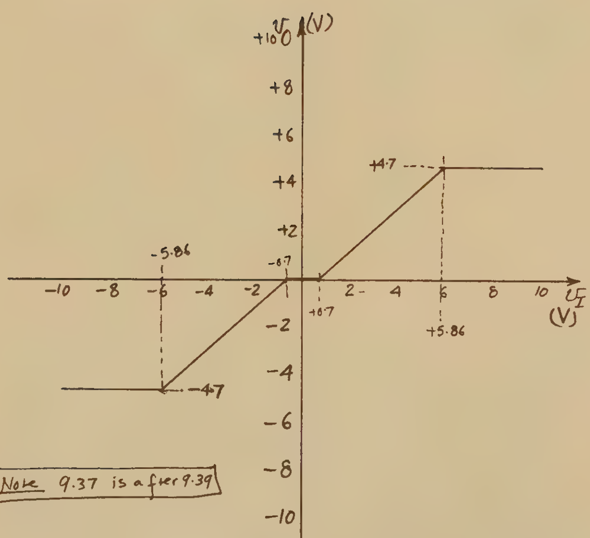
$$i_E = (v_I - 0.7) \Big/ \left(1\,k\Omega + \frac{10\,k\Omega}{\beta + 1}\right)$$

For $\beta = 100$,

$$i_E \simeq \frac{v_I - 0.7}{1.1} = 0.91\,v_I - 0.64, \quad mA$$

Thus, $v_O = i_E \times 1\,k\Omega = 0.91\,v_I - 0.64 = 0.91(v_I - 0.7)$

The transistor will remain in the active mode until the CBJ becomes forward-biased by about 0.4V or so, i.e. until v_O reaches +4.7V and correspondingly v_I reaches $\frac{4.7}{0.91} + 0.7 = +5.86$ V. From that point on, the transistor enters the saturation region and operate at a β (β_{forced}) that is lower that the nominal β value (100). v_O remains approximately 4.7V. In fact, however, v_O increases slightly as the transistor is driven deeper and deeper into saturation, i.e. as β_{forced} is decreased. Over the range $+5.86 \le v_I \le +10$, v_O will probably increase to 4.8 or 4.9 V. A piecewise linear approximation to the transfer characteristic is shown below.



Note 9.37 is after 9.39

$\boxed{9.38}$ V_2 is the value of v_I at which the BJT enters the saturation region. At this point the collector-base junction is forward biased by about 0.4 V (approx.), i.e. $v_O \simeq +0.3$V and the transistor is still operating at a β equal to its nominal value. Thus we can write

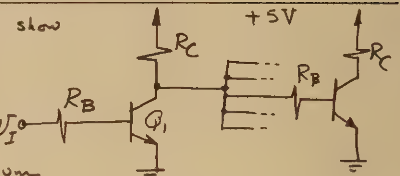
$$i_C = (V_{CC} - 0.3)/R_C \quad \text{and} \quad i_B = \frac{i_C}{\beta} = \frac{v_I - 0.7}{R_B}$$

$$\frac{v_I - 0.7}{R_B} = \frac{V_{CC} - 0.3}{\beta R_C}$$

Substituting $v_I = V_2$ obtains

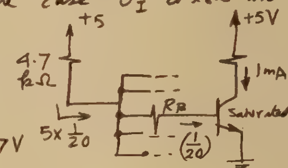
$$V_2 = 0.7 + \left(\frac{R_B}{\beta R_C}\right)(V_{CC} - 0.3)$$

$\boxed{9.39}$ The Figure show the RTL inverter to be designed, driving the maximum specified fan-out of 5 identical inverters.



First, note that when v_I is high, Q_1 will be saturated. In this state the maximum current throug R_C should be the specified 1mA. Thus $R_C = \frac{5 - 0.3}{1\text{ mA}} = \underline{\underline{4.7\,k\Omega}}$

Secondly, consider the case v_I is low and Q_1 off. In this case the circuit becomes as shown and we can write



$$5 = 4.7 \times 5 \times \tfrac{1}{20} + R_B \times \tfrac{1}{20} + 0.7\text{V}$$

$$\Rightarrow R_B = \underline{\underline{62.5\,k\Omega}}$$

$\boxed{9.37}$ Refer to Fig. 9.42. When $v_O = V_{CC}/2$, the transistor is operating in the active region with $I_C = \frac{V_{CC} - V_{CC}/2}{R_C} = \frac{V_{CC}}{2R_C}$. To find the slope of the transfer characteristic, i.e. the gain $\frac{\Delta v_O}{\Delta v_I}$ at this operating point, we consider an increment in v_I, Δv_I. Correspondingly we have

$$\Delta i_B = \frac{\Delta v_I}{R_B + r_\pi} = \frac{\Delta v_I}{R_B + (\beta+1) r_e}$$

where $r_e = \frac{V_T}{I_E} \simeq \frac{\alpha V_T}{I_C} = \left(\frac{\beta}{\beta+1} V_T\right) / \left(\frac{V_{CC}}{2R_C}\right)$

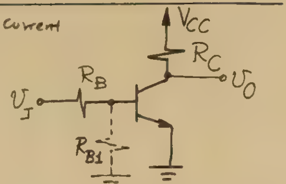
Thus, $\Delta i_B = \Delta v_I \Big/ \left\{ R_B + \frac{2\beta V_T}{V_{CC}} R_C \right\}$

and,
$$\Delta i_C = \beta\, \Delta i_B = \frac{\beta\, \Delta v_I}{R_B + 2\beta\left(\frac{V_T}{V_{CC}}\right) R_C}$$

$$\Delta v_O = -\Delta i_C R_C = \frac{-\beta\, \Delta v_I\, R_C}{R_B + 2\beta \frac{V_T}{V_{CC}} R_C}$$

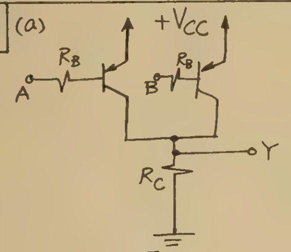
Thus,
$$\frac{\Delta v_O}{\Delta v_I} = -\frac{\beta R_C}{R_B + 2\beta R_C \frac{V_T}{V_{CC}}}$$

9.40 When the collector current is 1% of maximum, $v_{BE} = 0.5$ V. Thus the turn-on threshold (for high β) is $\underline{0.5\text{V}}$.



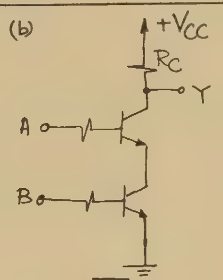
To raise the turn-on threshold connect a resister R_{B1} from base to ground as indicated. Now when $v_{BE} = 0.5$V and again assuming large β we find that V_I (which is the value of the turn-on threshold) $= 0.5\left(1+\frac{R_B}{R_{B1}}\right)$. For a turn-on threshold of 1.5V, $\frac{R_B}{R_{B1}} = 2$; thus $R_{B1} = R_B/2 = 10/2 = \underline{\underline{5\,k\Omega}}$.

9.41 (a)



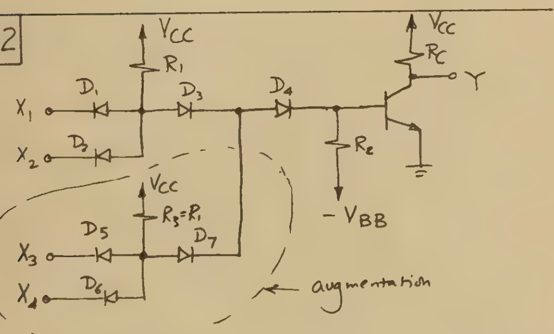
$Y = \overline{A B}$

(b)



$Y = \overline{A B}$

9.42



$Y = \overline{X_1X_2 + X_3X_4}$

9.43 This circuit is a bistable, i.e. it has two stable states. It can exist in either state indefinitely until triggered (by applying the appropriate value of v_I) to change state. In one stable state both transistors are cut-off and $v_O = +10$V. In the other stable state both transistors are on and saturated and $v_O = V_{BE}|_{Q_1} + V_{ECsat}|_{Q_2} \simeq 1$V.

To derive the input characteristic and the transfer characteristic, assume that the circuit is in the off state. Let v_I increase from 0V. When $v_I = 0$V, $i_I = 0$ and $v_O = +10$V. As v_I increases above 0V, $i_I = v_I/2.2\,k\Omega$ and v_{BE1} begins to increase above zero. This causes Q_1 to begin to conduct. Its collector current which will initially be very small will constitute the base current of Q_2. Thus Q_2 will conduct and supply a current I to the input network. (see Fig. 1). In this case both Q_1 and Q_2 will be in the active mode. Because Q_1 is conducting a small current, its base current will be negligibly small. However, i_I is no longer $v_I/2.2$, rather it will be smaller than that. Furthermore, as v_I is increased the transistors conduct more current and i_I decreases. As an example let us find the value of v_I and i_I for $I = 0.1$mA. In this case the collector current of Q_1 will be $0.1/101 \simeq 10^{-3}$mA and thus its V_{BE} will be $0.7 - 4\times 0.06 = 0.46$ V. Thus $I_1 = \frac{0.46}{2} = 0.23$mA and $i_I = 0.23 - 0.1 = \underline{0.13}$mA. The corresponding value of v_I is $0.46 + 0.13\times 0.2 = \underline{0.486}$V.

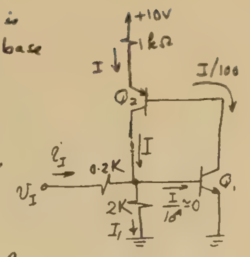


Fig. 1

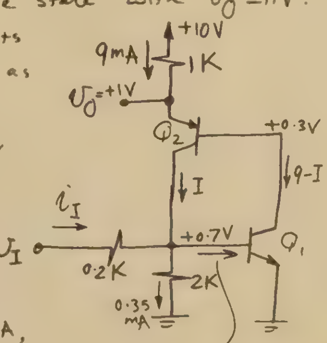
As v_I is increased, a value will be reached at which Q_2 supplies a current I that is sufficient to "keep the circuit going" with $i_I = 0$. This critical value of v_I can be calculated as follows. Let $i_I = 0$ then $v_I = V_{BE1} = 0.7 + 0.025\ln\left(\frac{I/100}{10\text{ mA}}\right) = I \times 2\,k\Omega$

Solution of this equation results in $I = 0.246$mA and $v_I = 0.492$ V $\simeq \underline{+0.5\text{V}}$.

Increasing v_I further causes a current increment into the base of Q_1. The collector current of Q_1 increases and the collector current of Q_2 increases even more. This in turn increases the base current of Q_1 even further and the regenerative action (positive feedback) continues until Q_1 and Q_2 both saturate. This all occurs for v_I close to the critical value of $\underline{0.5\text{V}}$ (approx.) and ends up with the circuit in the other stable state with $v_O \simeq +1$V. In this case the currents and voltages become as shown in Fig. 2. The input current is given by

$$i_I = \frac{v_I - 0.7}{0.2} \quad (1)$$

Thus for $v_I = 0.5$V, $i_I = -1$mA; for $v_I = 0.6$V, $i_I = -0.5$mA, and so on. The i-v curve now is a straight line of slope $= \frac{1}{0.2\,k\Omega}$



$(i_I + I - 0.35)$

Fig. 2

Now let us consider reducing v_I. It can be seen from Fig. 2 that both transistors

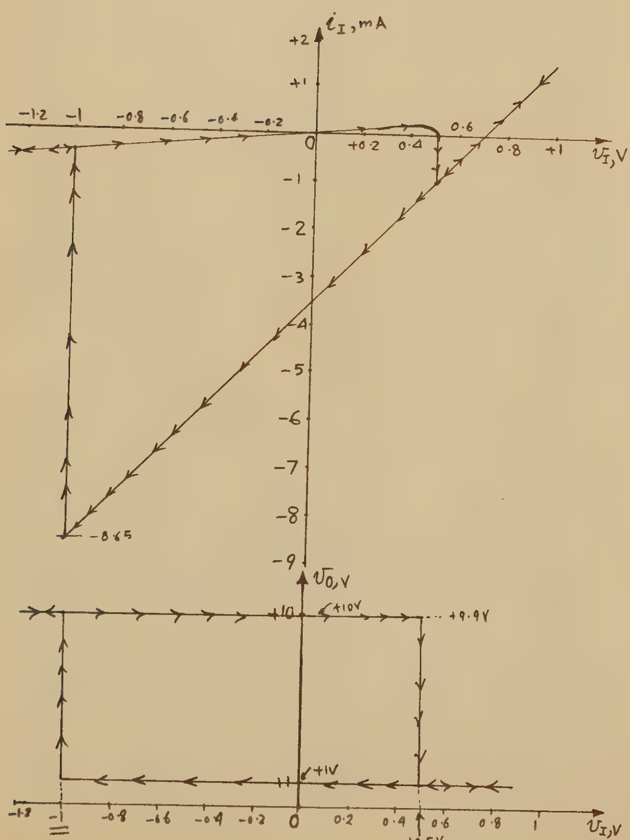
will remain saturated, v_O will remain at +1V, and i_I will be given by equation (1) until v_I is made sufficiently negative to cause the net current flowing into the base of Q_1 (i.e. $i_I + I - 0.35$) to become zero. Since the maximum value of I is 9 mA, this negative threshold value of v_I is found from

$$i_I + 9 - 0.35 = 0 \Rightarrow \frac{v_I - 0.7}{0.2} + 9 - 0.35 = 0$$

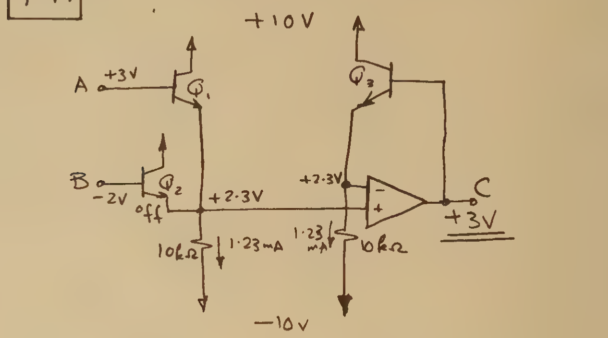
$$\Rightarrow v_I = -1.03\ V \simeq \underline{\underline{-1V}}.$$

When v_I is reduced to −1V Q_1 and Q_2 turn off (again we have a regenerative action) and the circuit reverts to the off state. v_O goes to +10V and i_I to $v_I / 2.2\ k\Omega$. Further reductions in v_I do not change the state of the circuit. In fact to change state v_I has to be raised to the positive threshold of $\simeq +0.5V$.

From the above we obtain the input and transfer characteristics depicted below.



9.44



Transistor Q_3 closes the negative feedback loop around the op amp. Since it is matched to Q_1 and Q_2 it provides a V_{BE} drop equal to that of Q_1 or Q_2, thus making V_C equal to the greater of V_A or V_B.

CHAPTER 10—EXERCISES

10.1 Refer to Fig. 10.2.

$$I_E = \frac{V_{CC}\,(R_2/R_1+R_2) - V_{BE}}{R_E + (R_1 \| R_2)/(\beta+1)}$$

Design 1: $R_1 = 80\ k\Omega$, $R_2 = 40\ k\Omega$

$$I_E = \frac{4 - 0.7}{3.3 + (26.7/(\beta+1))}$$

$\beta = 50$, $I_E = \underline{\underline{0.86\ mA}}$

$\beta = 150$, $I_E = \underline{\underline{0.95\ mA}}$

Design 2: $R_1 = 8\ k\Omega$, $R_2 = 4\ k\Omega$

$$I_E = \frac{4 - 0.7}{3.3 + (2.67/(\beta+1))}$$

$\beta = 50$, $I_E = \underline{0.98\ mA}$

$\beta = 150$, $I_E = \underline{0.995\ mA}$

10.2 Refer to Fig. 10.3.

Design 1 $R_1 = 80\ k\Omega$, $R_2 = 40\ k\Omega$, $I_E = \dfrac{3.3}{3.3 + \frac{26.7}{101}} = 0.93\ mA$

$$r_e = \frac{25\ mV}{0.93\ mA} = 27\ \Omega$$

(a) For $R_{E1} = 0$, $R_{in} = 80\,k\Omega \| 40\,k\Omega \| (101 \times 0.027)\,k\Omega$
$= 2.47\ k\Omega \simeq \underline{2.5\ k\Omega}$

(b) For $R_{E1} = 425\ \Omega$, $R_{in} = 80\,k\Omega \| 40\,k\Omega \| (101 \times 0.452)\,k\Omega$
$= \underline{16.4\ k\Omega}$ (Note: Answer in the book is obtained assuming $I_E \simeq 1$ mA).

Design 2 $I_E \simeq 1\,mA$, $r_e = 25\,\Omega$, $R_1 = 8\,k\Omega$, $R_2 = 4\,k\Omega$.

(a) $R_{E1} = 0$, $R_{in} = 8\,k\Omega // 4\,k\Omega // (101 \times 0.025)\,k\Omega$
$= \underline{1.3\,k\Omega}$

(b) $R_{E1} = 425\,\Omega$, $R_{in} = 8\,k\Omega // 4\,k\Omega // (101 \times 0.45)\,k\Omega$
$= \underline{\underline{2.5\,k\Omega}}$

10.3 Design 1 (a) $\frac{v_b}{v_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{2.5}{2.5+4} = 0.38$

(b) $\frac{v_b}{v_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{16.4}{16.4+4} = 0.80$

Design 2 (a) $\frac{v_b}{v_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{1.3}{1.3+4} = 0.25$

(b) $\frac{v_b}{v_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{2.5}{2.5+4} = 0.38$

10.4

Design 1 (a) $\frac{v_o}{v_b} \simeq -\frac{(R_C // R_L)}{r_e + R_{E1}} = \frac{-(4//4)}{0.027+0} = -74$

$\frac{v_o}{v_s} = -74 \times 0.38 = -28$ V/V

(b) $\frac{v_o}{v_b} \simeq -\frac{(R_C // R_L)}{r_e + R_{E1}} = \frac{-(4//4)}{0.027+0.425} = -4.42$

$\frac{v_o}{v_s} = -4.42 \times 0.8 = -3.5$ V/V

Design 2 (a) $\frac{v_o}{v_b} \simeq -\frac{(4//4)}{0.025+0} = -80$

$\frac{v_o}{v_s} = -80 \times 0.25 = -20$

(b) $\frac{v_o}{v_b} \simeq -\frac{(4//4)}{0.025+0.425} = -4.44$

$\frac{v_o}{v_s} = -4.44 \times 0.38 = -1.7$

10.5 (a) $v_{be\,max} = 10\,mV \Rightarrow v_{b\,max} = 10\,mV$

$v_{s\,max} = 10/0.38 = \underline{26.3\,mV}$

(b) $v_{be\,max} = 10\,mV \Rightarrow v_{b\,max} = 10\left(\frac{r_e + R_{E1}}{r_e}\right)$

$= 10\left(\frac{0.027+0.425}{0.027}\right)$

$= 167.4$ mV

$v_{s\,max} = 167.4/0.8 = \underline{209\,mV}$

10.6 (a) $V_o = 26.3 \times 28 = \underline{0.74\,V}$

(b) $V_o = 209 \times 3.5 = \underline{0.73\,V}$

Note: some numerical inaccuracies have occured. Theoretically the maximum output should be the same in both cases.

10.7 $V_B \simeq 15 \times \frac{10}{10+5} = +10\,V$

$V_E = 10 + V_{EB} \simeq +10.7\,V$

$I_E = \frac{15-10.7}{8.6} = \underline{0.5\,mA}$

$R_{in} = r_e // 8.6\,k\Omega \simeq r_e = \underline{50\,\Omega}$

$\frac{v_o}{v_s} = \frac{v_e}{v_s} \times \frac{v_o}{v_e} = \frac{R_{in}}{R_{in}+50\,\Omega} \times \frac{\alpha \times 16\,k\Omega}{r_e}$

$\simeq \frac{50}{50+50} \times \frac{16{,}000}{50} = \underline{\underline{160\,V/V}}$

Since the dc voltage at the collector is $+8\,V$ and that at the base is $+10\,V$, the collector signal swing is limited to 2V (otherwise the transistor saturates). Thus the maximum amplitude of $v_s = \frac{2\,V}{160} = \underline{\underline{12.5\,mV}}$.

10.8 $I_E = \left[V_{CC}\frac{R_{B2}}{R_{B1}+R_{B2}} - V_{BE}\right] \Big/ \left[R_E + \frac{R_{B1}//R_{B2}}{\beta+1}\right]$

$= (7.5-0.7)/\left[2+\frac{50}{101}\right] = 2.725\,mA$

$I_C = 0.99\,I_E \simeq 2.7\,mA$

$I_B = \frac{I_C}{\beta} = 0.027\,mA$

$V_E = I_E R_E = +5.45\,V$

$V_B = V_E + V_{BE} = +6.15\,V$

$I_{R_{B1}} = \frac{+15-6.15}{100} = 0.0885\,mA$

$I_{R_{B2}} = \frac{6.15}{100} = 0.0615\,mA$

10.9 $R_{in} = R_{B1} // R_{B2} // (\beta+1)[r_e + (R_E // R_L)]$

$R_{B1} = R_{B2} = 100\,k\Omega$, $\beta = 100$, $r_e = \frac{25\,mV}{2.73\,mA} = 9.16\,\Omega$,

$R_E = 2\,k\Omega$, and $R_L = 1\,k\Omega$.

$R_{in} = 100 // 100 // 101\left[0.00916 + \frac{1\times 2}{1+2}\right]$

$= \underline{28.9\,k\Omega}$

$\frac{v_b}{v_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{28.9}{28.9+5} = 0.852$

$\frac{v_e}{v_b} = \frac{(R_E // R_L)}{(R_E // R_L) + r_e} = \frac{(2/3)}{\frac{2}{3}+0.00916} = 0.986$

Thus, $\frac{v_o}{v_s} = \frac{v_e}{v_s} = \frac{v_b}{v_s}\frac{v_e}{v_b} = 0.852 \times 0.986 = \underline{\underline{0.84\,V/V}}$

10.10 $R_{in}\Big|_{R_L=\infty} = R_{B1} // R_{B2} // [(\beta+1)(r_e+R_E)]$

$= 100 // 100 // (101 \times 2.00916) = 40.1\,k\Omega$

$\frac{v_b}{v_s}\Big|_{R_L=\infty} = \frac{40.1}{40.1+5} = 0.889$

$\frac{v_e}{v_b}\Big|_{R_L=\infty} = \frac{R_E}{R_E+r_e} = \frac{2}{2+0.00916} = 0.995$

Thus, $\frac{v_o}{v_s}\Big|_{R_L=\infty} = 0.889 \times 0.995 = \underline{\underline{0.885\,V/V}}$

$R_{out} = R_E // \left[r_e + \frac{(R_{B1} // R_{B2} // R_s)}{\beta+1}\right] = \underline{\underline{52.7\,\Omega}}$

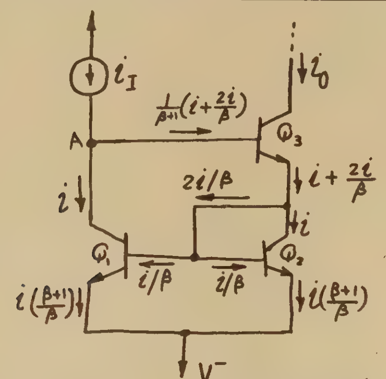
10.11 $\hat{V}_e = V_E / [1 + \frac{R_E}{R_L}]$

where $V_E = +5.45\text{ V}$, $R_E = 2\text{ k}\Omega$, and $R_L = 1\text{ k}\Omega$.

Thus, $\hat{V}_e = \dfrac{5.45}{1+\frac{2}{1}} = 1.817\text{ V}$

$$\hat{V}_s = \frac{\hat{V}_e}{\text{Gain}} = \frac{1.817}{0.84} = \underline{\underline{2.16\text{ V}}}$$

10.12

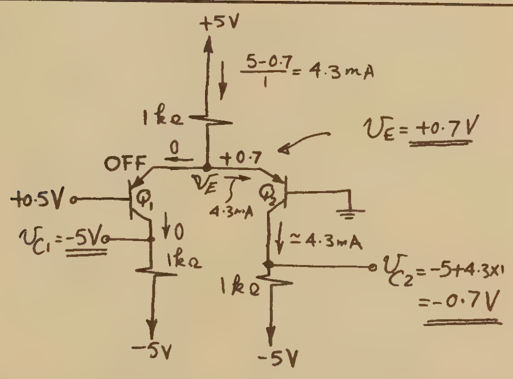


Node equation at A $\Rightarrow$ $i_I = i + \frac{1}{\beta+1}(i + \frac{2i}{\beta})$

For Q_3 we have: $i_0 = \dfrac{\beta}{\beta+1}\,(i + \frac{2i}{\beta})$

$$\text{Thus, } \frac{i_0}{i_I} = \frac{(\frac{\beta}{\beta+1})(1+\frac{2}{\beta})}{1+(\frac{1}{\beta+1})(1+\frac{2}{\beta})} = \underline{\underline{\frac{1}{\left[1+\frac{2}{\beta^2+2\beta}\right]}}}$$

10.13



10.14 From Equation (10.30)

$$i_{E1} = \frac{I}{1 + e^{(v_{B2}-v_{B1})/V_T}}$$

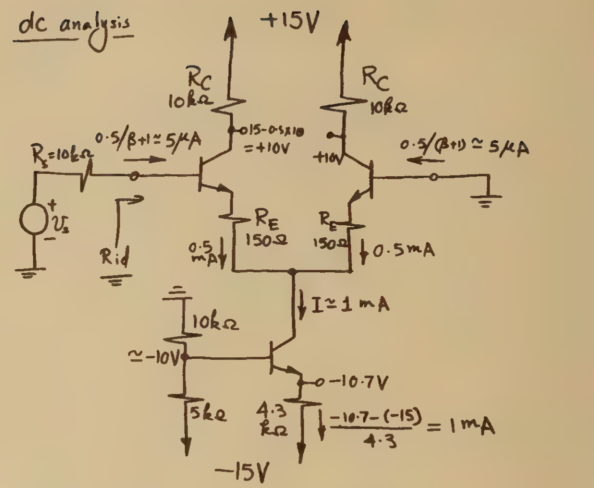
Thus, for $i_{E1} = 0.99I$ we must have a differential signal of

$$v_{B2} - v_{B1} = V_T \ln\left(\frac{1}{0.99} - 1\right) = -115\text{ mV}$$

i.e., $v_{B1} - v_{B2} = \underline{\underline{+115\text{ mV}}}$

10.15 See analysis on circuit diagram below

dc analysis



(a) $I = \underline{1\text{mA}}$ (b) $I_B = \underline{\underline{5\mu A}}$

(c) CM range $= -9.3\text{V}$ (neglecting the dc drop across R_E) to $+10\text{V}$.

(d) $R_{id} = 2(\beta+1)(r_e + R_E)$ where $r_e = \frac{25\text{mV}}{0.5\text{mA}} = 50\Omega$

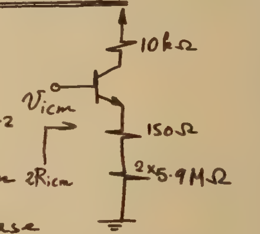
Thus, $R_{id} = 2\times101 \times 200 \simeq \underline{\underline{40\text{ k}\Omega}}$

(e) $\dfrac{v_{id}}{v_s} = \dfrac{R_{id}}{R_{id}+R_s} = \dfrac{40}{40+10} = 0.8$

$$\frac{v_o}{v_{id}} = -\frac{2R_C \times \alpha}{2r_e + 2R_E} \simeq \frac{-10}{0.2} = -50\text{ V/V}$$

Thus, $\dfrac{v_o}{v_{id}} = 0.8 \times -50 = \underline{\underline{-40\text{ V/V}}}$

(f) The equivalent common-mode half circuit is shown. It's gain is $\simeq \dfrac{10\times10^3}{2\times5.9\times10^6} = \dfrac{1}{2\times5.9}\times10^{-2}$



Since the output is taken differentially, the worst-case common-mode gain will be $\frac{1}{2\times5.9}\times10^{-2}\times\left.\frac{\Delta R_C}{R_C}\right|_{\text{worst case}}$

$= \frac{1}{2\times5.9}\times10^{-2}\times0.02$

$= \underline{\underline{1.7\times10^{-5}\text{ V/V}}}$

(g) $\text{CMRR} = 20\log\left|\dfrac{40}{1.7\times10^{-5}}\right| = \underline{\underline{127\text{ dB}}}$

(h) $R_{icm} = \{[(\beta+1)R] // (r_\mu/2)\}$

$= (101\times5.9 // 25)\text{ M}\Omega = \underline{\underline{24\text{ M}\Omega}}$

10.16 From Eqn. (10.50) the value of v_{id} that causes the current I to be carried entirely by Q_1 is $v_{id} = |V_P|\sqrt{\dfrac{I}{I_{DSS}}} = 2\sqrt{\dfrac{1}{2}} = \underline{1.4\text{ V}}$

Note that at this value of v_{id} $V_{GS1} \simeq 0.6\text{ V}$

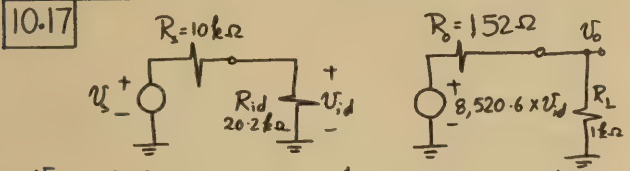
and $V_{GS2} \simeq -2V = V_P$. Thus Q_2 cuts off, which verifies the fact that all the bias current I flows through Q_1.

$$\frac{V_o\,(\text{differential})}{V_{id}} = -g_m R_d \qquad (\text{note: } V_o = V_{d_1} - V_{d_2})$$

where $g_m = \frac{2 I_{DSS}}{|V_P|}\sqrt{\frac{I_D}{I_{DSS}}} = \frac{2\times 2}{2}\sqrt{\frac{0.5}{2}} = 1\,\text{mA/V}$

Thus, $\frac{V_o}{V_{id}} = -1 \times 10 = \underline{\underline{-10\ \text{V/V}}}$

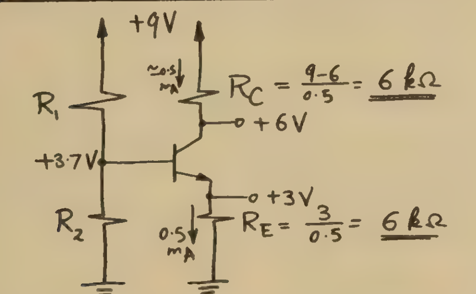
10.17



$$\frac{V_o}{V_s} = \frac{20.2}{20.2+10} \times 8{,}520.6 \times \frac{1}{1+0.152} = \underline{\underline{4{,}947.2\ \text{V/V}}}$$

CHAPTER 10—PROBLEMS

10.1



If β is very high, then we may neglect the base current and assume that the currents through R_1 and R_2 are equal at $\frac{0.5}{10} = 0.05$ mA. Thus $R_2 = \frac{3.7}{0.05} = \underline{74\,k\Omega}$ and $R_1 = \frac{9-3.7}{0.05} = \underline{106\,k\Omega}$.

If a transistor with $\beta = 100$ is substituted then I_E becomes

$$I_E = \frac{9 \times \frac{R_2}{R_1+R_2} - V_{BE}}{R_E + \frac{R_1 /\!/ R_2}{\beta+1}} = \frac{3}{6+0.43} = 0.47\,\text{mA}$$

i.e. I_E decreases by about 7%. This change is approximately equal to $\frac{(I_E / I_{Divider})}{\beta} \times 100\%$

10.2 $R_{in} = R_B /\!/ (\beta+1)(r_e + R_E)$

where $R_B = 10\,k\Omega$, $\beta = 100$, $r_e = \frac{25}{0.5} = 50\,\Omega$, and $R_E = 1\,k\Omega$. Thus,

$$R_{in} = 10 /\!/ 101 \times 1.05 = \underline{\underline{9.14\,k\Omega}}.$$

If R_E is bypassed, R_{in} becomes

$$R_{in} = R_B /\!/ (\beta+1) r_e = 10 /\!/ 101 \times 0.05 = \underline{\underline{3.36\,k\Omega}}$$

10.3 With the emitter resistor unbypassed:

$$\frac{V_o}{V_b} = -\frac{\alpha \times 5\,k\Omega}{(0.05+1)\,k\Omega} \simeq \underline{\underline{-4.76\ \text{V/V}}}$$

With R_E bypassed:

$$\frac{V_o}{V_b} = \frac{-\alpha \times 5\,k\Omega}{0.05\,k\Omega} \simeq \underline{\underline{-100\ \text{V/V}}}$$

With a source of $R_s = 1\,k\Omega$ connected and the emitter resistor unbypassed:

$$\frac{V_b}{V_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{9.14}{9.14+1} = 0.9$$

& $\frac{V_o}{V_s} = 0.9 \times -4.76 = \underline{\underline{-4.3\ \text{V/V}}}$.

With a source of $R_s = 1\,k\Omega$ connected and the emitter-resistor bypassed:

$$\frac{V_b}{V_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{3.36}{3.36+1} = 0.77$$

& $\frac{V_o}{V_s} = 0.77 \times -100 = \underline{\underline{-77\ \text{V/V}}}$.

10.4 $V_{be} = 10\,\text{mV} = \frac{r_e}{r_e+R_E} \times V_b = \frac{r_e}{r_e+R_e} \times 100$

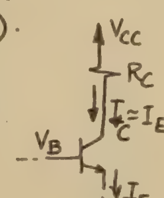
Thus, $R_e = 9\,r_e$

$$\underline{\underline{R_e = \frac{9 V_T}{I_E}}}$$

10.5 When $V_{be} = 10\,\text{mV}$, $i_e = \frac{10\,\text{mV}}{r_e} = \frac{10\,\text{mV}}{V_T/I_E}$

Thus $i_e = \frac{10\,\text{mV}}{25\,\text{mV}} I_E = 0.4\,I_E$

In other words, with $V_{be} = 10\,\text{mV}$, i_E increases by $\underline{40\%}$ (from I_E to $1.4 I_E$).



Originally with $R_C = R_{C1}$, we have $I_C R_{C1} = V_{CB}$

Thus $V_{CC} - V_B = 2 I_C R_{C1}$

Now we wish to increase R_C to R_{C2} so that with $V_{be} = 10\,\text{mV}$, $V_{CB} = 0$.

It follows that $1.4 I_C R_{C2} = V_{CC} - V_B$

Thus $1.4 I_C R_{C2} = 2 I_C R_{C1}$

$$\Rightarrow R_{C2} = \frac{1}{0.7} R_{C1} = 1.428 R_{C1}$$

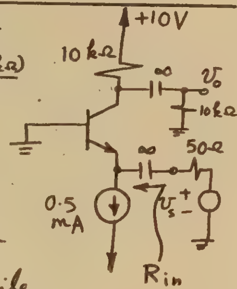
Since I_C is kept constant, the gain will be proportional to R_C. Thus the gain will be increased by $\underline{\underline{42.8\%}}$.

10.6 $R_{in} = r_e = \frac{V_T}{I_E} = \underline{\underline{50\,\Omega}}$.

$$\frac{v_o}{v_s} = \frac{R_{in}}{R_{in}+R_s} \cdot \frac{\alpha\,(10k\Omega // 10k\Omega)}{r_e}$$

$$\simeq \frac{1}{2} \times \frac{5{,}000}{50}$$

$$= \underline{\underline{50\ V/V}}.$$



Assuming that we can apply a maximum v_{be} of 10 mV while keeping the distortion acceptably small, then $v_{s\,max} = \underline{\underline{20\ mV}}$ and $v_{o\,max} = 20 \times 50 = \underline{\underline{1\ V}}$.

10.7 Refer to Fig. 10.5.

$$I_E = \left[V_{CC}\left(\frac{R_{B2}}{R_{B1}+R_{B2}}\right) - V_{BE}\right] \Big/ \left[R_E + \frac{R_{B1}//R_{B2}}{\beta+1}\right]$$

$$= (6-0.7) \Big/ \left[10 + \frac{500}{101}\right] = 0.35\ mA$$

$V_E = I_E R_E = \underline{\underline{3.5\ V}}$.

$V_B = V_E + V_{BE} = \underline{\underline{4.2\ V}}$.

$r_e = \frac{V_T}{I_E} = 71.4\ \Omega$

$$R_{in} = R_{B1}//R_{B2}//\{(\beta+1)[r_e + (R_E//R_L)]\}$$

$$= 1000//1000//\{101\,[0.0714 + (10//1)]\}$$

$$= \underline{\underline{82.7\ k\Omega}}$$

$$\frac{v_o}{v_s} = \frac{R_{in}}{R_{in}+R_s} \cdot \frac{(R_E//R_L)}{r_e + (R_E//R_L)}$$

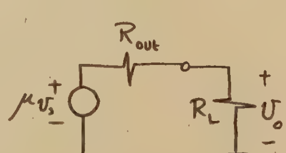
$$= \frac{82.7}{82.7+10} \times \frac{(10//1)}{0.0714 + (10//1)} = 0.892 \times 0.927$$

$$= \underline{\underline{0.827\ V/V}}$$

$$R_{out} = R_E // \left\{r_e + \frac{R_{B1}//R_{B2}//R_s}{\beta+1}\right\}$$

$$= \underline{\underline{168.5\ \Omega}}$$

The unloaded gain μ can be found from the circuit shown as follows



$$\frac{v_o}{v_s} = 0.827 = \mu \frac{R_L}{R_L + R_{out}}$$

$$\mu = 0.827\left[1 + \frac{168.5}{1000}\right] = \underline{\underline{0.966\ V/V}}$$

10.8 $V_A = -1.09 \times 1 = \underline{\underline{-1.09\ V}}$

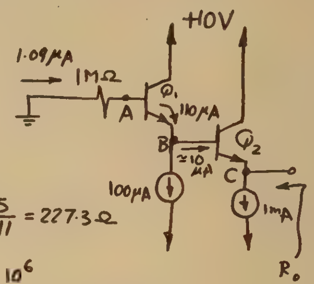
$V_B = -1.09 - 0.7 = \underline{\underline{-1.79\ V}}$

$V_C = -1.79 - 0.7 = \underline{\underline{-2.49\ V}}$

$$R_o = r_{e2} + \frac{r_{e1} + \frac{1M\Omega}{\beta+1}}{\beta+1}$$

where

$r_{e2} = 25\,\Omega$, $r_{e1} = \frac{25}{0.11} = 227.3\,\Omega$



Thus,

$$R_o = 25 + \frac{227.3 + \frac{10^6}{101}}{101} = \underline{\underline{125.3\ \Omega}}$$

10.9 At node B:

$$-(\beta+1)\frac{V_C + 1.4}{1000\,k\Omega} = \frac{V_C + 10.7}{100\,k\Omega} + \frac{I_{E2}}{\beta+1}$$

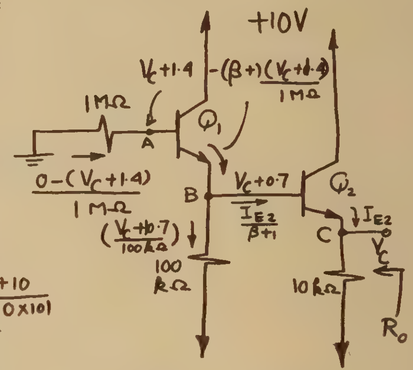
But $I_{E2} = \frac{V_C + 10}{10\,k\Omega}$

Thus,

$$-101 \times (V_C + 1.4)/1000 = \frac{V_C + 10.7}{100} + \frac{V_C + 10}{10 \times 101}$$

$$\Rightarrow V_C = \underline{\underline{-2.3\ V}}$$

$V_B = \underline{\underline{-1.6\ V}}$ $V_A = \underline{\underline{-0.9\ V}}$



$$I_{E2} = \frac{-2.3 + 10}{10} = 0.77\ mA$$

$$I_{E1} = \frac{0.77}{101} + \frac{10 - 1.6}{100} = 0.09\ mA$$

$r_{e1} = 277.8\ \Omega$ $r_{e2} = 32.5\,\Omega$

$$R_o = 10k\Omega // \left\{r_{e2} + \frac{\left[100k\Omega // \left(r_{e1} + \frac{1M\Omega}{101}\right)\right]}{\beta+1}\right\}$$

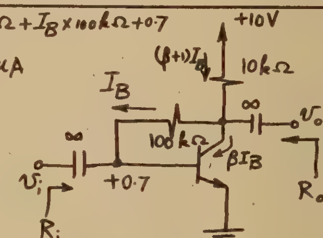
$$= \underline{\underline{122.5\ \Omega}}$$

10.10 $10 = (\beta+1) I_B \times 10k\Omega + I_B \times 100k\Omega + 0.7$

$$\Rightarrow I_B = \frac{9.3}{1010 + 100} = 8.4\ \mu A$$

$V_C = 0.7 + 8.4 \times 0.1$

$= \underline{\underline{1.54\ V}}$

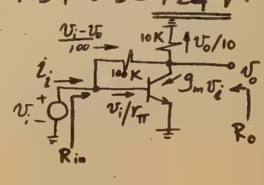


Since v_C can decrease to +0.3V then the maximum amplitude of the output unclipped sinusoid is $1.54 - 0.3 = \underline{\underline{1.24\ V}}$.

Small-signal Analysis:

Node equation at the collector $\Rightarrow$

$$g_m v_i + (v_o/10) = \frac{v_i - v_o}{100}$$



$$v_o\left(\frac{1}{10}+\frac{1}{100}\right)=-v_i\left(g_m-\frac{1}{100}\right)$$

where $g_m=\dfrac{I_C}{V_T}=\dfrac{\beta I_B}{V_T}=\dfrac{0.84\text{ mA}}{25\text{ mV}}=33.6\text{ mA/V}$

Thus

$$v_o=-v_i\,\frac{33.6-0.01}{0.11}\simeq-305\,v_i$$

$$\frac{v_o}{v_i}=\underline{\underline{-305\text{ V/V}}}$$

$$R_{in}\equiv\frac{v_i}{i_i}=\frac{v_i}{\dfrac{v_i}{r_\pi}+\dfrac{v_i-v_o}{100}}=\frac{v_i}{\dfrac{v_i}{r_\pi}+\dfrac{v_i+305v_i}{100}}$$

where $r_\pi=\dfrac{\beta}{g_m}=\dfrac{100}{33.6}\simeq3\text{ k}\Omega$

Thus, $R_{in}=\dfrac{1}{\frac{1}{3}+\frac{306}{100}}=\underline{\underline{295\,\Omega}}$

To find R_o, set $v_i=0$, it follows that

$R_o=10\text{ k}\Omega\,//\,100\text{ k}\Omega=\underline{\underline{9.1\text{ k}\Omega}}$.

10.11 (a) $I_E=\dfrac{10.7-0.7}{10+\frac{10}{51}}=0.98\text{ mA}$

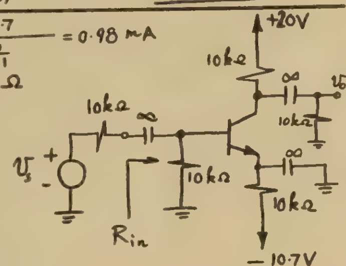
$r_e=\dfrac{25}{0.98}=25.5\,\Omega$

$r_\pi=(\beta+1)r_e=1.3\text{ k}\Omega$

$R_{in}=10//1.3=1.15\text{ k}\Omega$

$\dfrac{v_o}{v_s}=\dfrac{1.15}{10+1.15}\times\dfrac{-(10//10)\times\alpha}{0.0255}$

$=\underline{\underline{-19.8\text{ V/V}}}$



(b) $I_{E1}=\dfrac{10-0.7}{9.3+\frac{10}{51}}=0.98\text{ mA}$

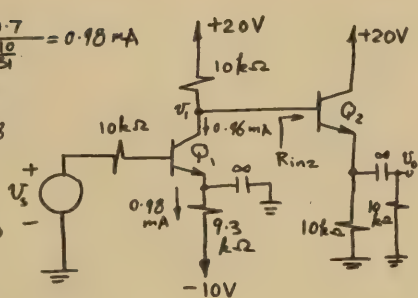
$I_{C1}=\alpha\times0.98$

$=0.98\times0.98$

$=0.96\text{ mA}$

$V_{C1}\simeq20-0.96\times10$

$=+10.4\text{V}$



$V_{E2}\simeq10.4-0.7=9.7\text{ V}$

$I_{E2}=\dfrac{9.7}{10}=0.97\text{ mA}\qquad I_{B2}=\dfrac{0.97}{51}=0.019\text{ mA}$

Thus a better approximation for V_{C1} is

$V_{C1}=-(0.96-0.019)\times10+20=+10.6\text{V}$

$V_{E2}=10.6-0.7=9.9\text{V}\qquad I_{E2}=0.99\text{ mA}$

$r_{e1}=\dfrac{25}{0.98}=25.5\,\Omega\qquad r_{\pi1}=51\times25.5=1.3\text{ k}\Omega$

$\dfrac{v_1}{v_s}=\dfrac{1.3}{1.3+10}\times\dfrac{\alpha\times(10\text{ k}\Omega//R_{in2})}{r_{e1}}$

where $R_{in2}=(\beta+1)\left[r_{e2}+(10\text{ k}\Omega//10\text{ k}\Omega)\right]$

$r_{e2}=\dfrac{25}{0.99}\simeq25\,\Omega$

Thus, $R_{in2}=51\left[0.025+5\right]=256.3\text{ k}\Omega$

$\dfrac{v_1}{v_s}=\dfrac{1.3}{11.3}\times\dfrac{-0.98\times9.62}{0.0255}=\underline{\underline{-29.5\text{ V/V}}}$

$\dfrac{v_o}{v_1}=\dfrac{5}{5+0.025}=\underline{\underline{0.995\text{ V/V}}}$

$\dfrac{v_o}{v_s}=-29.5\times0.995=\underline{\underline{-29.4\text{ V/V}}}$

(c) $I_{E1}=\dfrac{10-0.7}{9.3}=1\text{mA}$

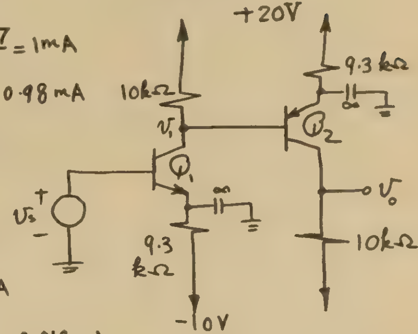
$I_{C1}=\alpha\times1=0.98\text{ mA}$

$V_{C1}\simeq+10.2\text{V}$

$V_{E2}\simeq10.9\text{V}$

$I_{E2}=\dfrac{20-10.9}{9.3}$

$=0.98\text{ mA}$



$I_{B2}=\dfrac{0.98}{51}=0.019\text{ mA}$

$V_{C1}=-(0.98-0.019)\times10+20=10.4\text{V}$

$V_{E2}=+11.1\text{V}\qquad I_{E2}=0.96\text{ mA}$

$r_{e1}=\dfrac{25}{0.98}=25.5\,\Omega\qquad r_{\pi1}=51\times25.5=1.3\text{ k}\Omega$

$r_{e2}=\dfrac{25}{0.96}=26\,\Omega\qquad r_{\pi2}=51\times26=1.33\text{ k}\Omega$

$\dfrac{v_1}{v_s}=-\dfrac{\alpha\times(10//r_{\pi2})}{0.0255}=\underline{\underline{-45.1\text{ V/V}}}$

$\dfrac{v_o}{v_1}=-\dfrac{\alpha\times10}{0.026}=\underline{\underline{-37.7\text{V/V}}}$

$\dfrac{v_o}{v_s}=-45.1\times-37.7=\underline{\underline{1{,}700\text{ V/V}}}$

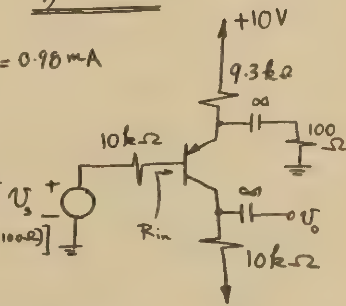
(d) $I_E=\dfrac{10-0.7}{9.3+\frac{10}{51}}=0.98\text{mA}$

$r_e=\dfrac{25}{0.98}=25.5\,\Omega$

$r_\pi=51\times25.5=1.3\text{k}\Omega$

$R_{in}=(\beta+1)\left[r_e+(9.3\text{k}\Omega//100\,\Omega)\right]$

$=6.4\text{ k}\Omega$



$\dfrac{v_o}{v_s}=\dfrac{6.4}{6.4+10}\times\dfrac{-\alpha\times10}{0.0255+(9.3//0.1)}$

$=\underline{\underline{-30.5\text{ V/V}}}$

10.12 $\beta=\infty$; base current $=0$

dc analysis is shown on diagram;

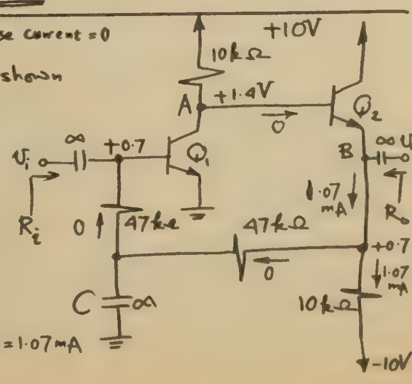
$V_A=\underline{\underline{+1.4\text{V}}}$

$V_B=\underline{\underline{+0.7\text{V}}}$

$I_{E1}=\dfrac{10-1.4}{10}$

$=0.86\text{ mA}$

$I_{E2}=\dfrac{0.7-(-10)}{10}=1.07\text{mA}$



(Please note error in circuit diagram in 1st printing; lower end of 10-kΩ resistor should be connected to −10 V.)

$r_{e1} = \frac{25}{0.86} = 29\,\Omega \qquad r_{e2} = \frac{25}{1.07} = 23.4\,\Omega$

(a) With $C = \infty$

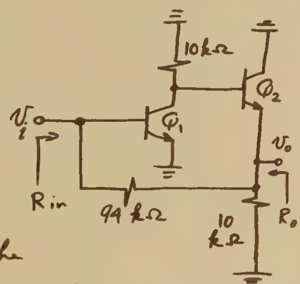
$$\frac{v_o}{v_s} = -\frac{10}{0.029} \times \frac{(10 /\!/ 47)}{0.0234 + (10/\!/47)} \simeq \underline{\underline{-344\ \text{V/V}}}$$

$$R_i = \underline{\underline{47\ \text{k}\Omega}}$$

$$R_o = 10\,\text{k}\Omega /\!/ 47\,\text{k}\Omega /\!/ 0.0234\,\text{k}\Omega \simeq \underline{\underline{23.3\,\Omega}}$$

(b) With C Removed

The ac equivalent circuit becomes as shown. The gain remains approximately the same as before (because the gain of the emitter-follower Q_2 remains approx. unity), i.e. $\frac{v_o}{v_i} \simeq \underline{\underline{-344\ \text{V/V}}}$.



To find R_i we use Miller's theorem:

$$R_i = \frac{94\,\text{k}\Omega}{1 - \frac{v_o}{v_i}} = \frac{94}{1+344} \simeq \underline{\underline{272\,\Omega}}$$

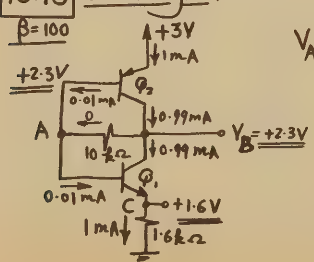
The output resistance is obtained by reducing v_i to zero, thus

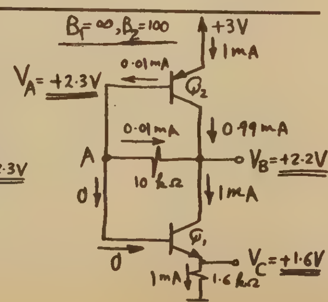
$$R_o = 10\,\text{k}\Omega /\!/ 94\,\text{k}\Omega /\!/ 0.0234\,\text{k}\Omega \simeq \underline{\underline{23.3\,\Omega}}.$$

Signal Swing

If $V_{CEsat} = 0.3$V then the peak signal allowed at the collector of Q_1 is 1.1V. Now since the gain of Q_2 is $\simeq 1$, it follows that the maximum unclipped signal amplitude at the output is $\underline{\underline{1.1\text{V}}}$.

10.13 dc analysis:

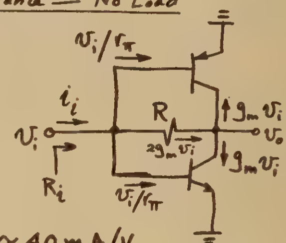




For the case with $\beta_1 = \beta_2 = 100$, the largest unclipped sine-wave output is $\underline{0.4\text{V}}$ in amplitude. (This takes v_B up to +2.7V and down to +1.9V). The corresponding value for the case with $\beta_1 = \infty$ and $\beta_2 = 100$ is $\underline{\underline{0.3\text{ V}}}$ peak.

(a) Gain and Input Resistance — No Load

The ac equivalent circuit is shown.



$$v_o = v_i - 2g_m v_i R$$

$$\frac{v_o}{v_i} = 1 - 2g_m R$$

where $g_m = g_{m1} = g_{m2} \simeq 40$ mA/V

& $R = 10$ kΩ

Thus, $\frac{v_o}{v_i} \simeq \underline{\underline{-800\ \text{V/V}}}$.

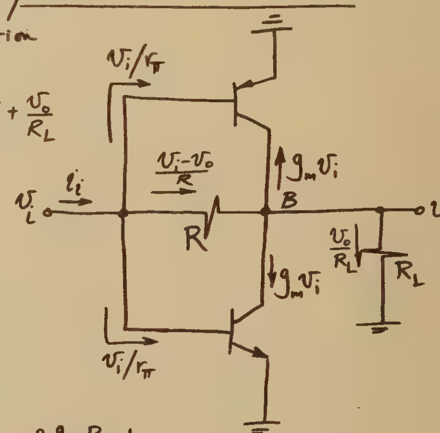
$$i_i = \frac{2v_i}{r_\pi} + 2g_m v_i = \frac{2v_i}{r_\pi}(1 + g_m r_\pi)$$

Thus, $R_i \equiv \frac{v_i}{i_i} = \frac{r_\pi}{2(1+g_m r_\pi)} = \frac{r_\pi}{2(1+\beta)} = \frac{r_e}{2} = \underline{\underline{12.5\,\Omega}}.$

(b) Gain and Input Resistance — 1-kΩ Load

Node Equation at B:

$$\frac{v_i - v_o}{R} = 2g_m v_i + \frac{v_o}{R_L}$$



Thus,

$$\frac{v_o}{v_i} = -\frac{2g_m R - 1}{1 + \frac{R}{R_L}}$$

$$= -\frac{2 \times 40 \times 10 - 1}{1 + \frac{10}{1}} = \underline{\underline{-72.6\ \text{V/V}}}$$

$$i_i = \frac{2v_i}{r_\pi} + \frac{v_i - v_o}{R} = \frac{2v_i}{r_\pi} + \frac{v_i}{R} \times 73.6$$

$$R_i \equiv \frac{v_i}{i_i} = 1 \Big/ \left(\frac{2}{r_\pi} + \frac{73.6}{R}\right) = 1 \Big/ \left(\frac{2}{2.5} + \frac{73.6}{10}\right)$$

$$\simeq \underline{\underline{123\,\Omega}}$$

10.14 From the complementary symmetry of the circuit we see that $V_C = 0V$. The rest of the dc analysis is shown on the circuit diagram.

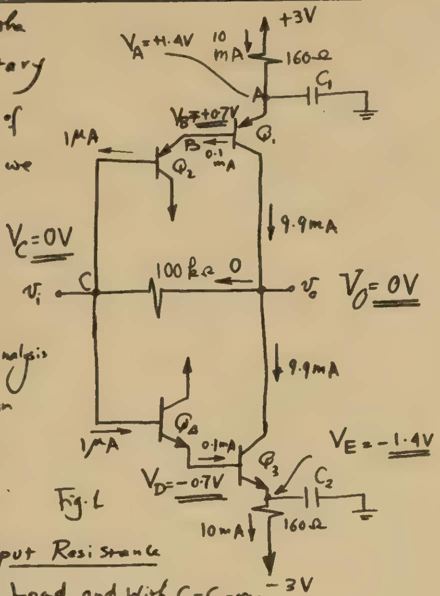


Fig.1

Gain and Input Resistance With No Load and With $C_1 = C_2 = \infty$:

The ac equivalent circuit is shown in Fig. 2, from which we can write:

$$i_{b2} = i_{b3} = \frac{v_i}{(\beta_3+1)[r_{e3} + (\beta_4+1)r_{e4}]}$$

where $r_{e4} = \frac{25}{10} = 2.5\Omega$ and $r_{e3} = \frac{25}{0.1} = 250\Omega$

Thus, $i_{b2} = i_{b3} \simeq \frac{v_i}{50\ k\Omega}$

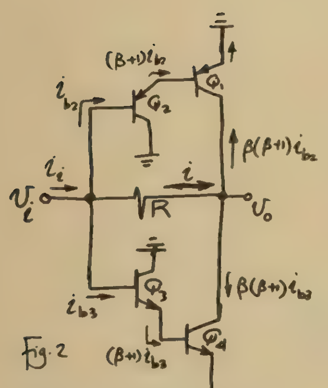


Fig.2

Thus, $i \simeq 2\beta^2 i_b = \frac{2\times10^4 v_i}{50\ k\Omega}$

$v_o = v_i - iR = v_i - 4\times10^4 v_i$

$\frac{v_o}{v_i} \simeq \underline{-4\times10^4\ V/V}$

$i_i = i + 2i_b \simeq 2\beta^2 i_b$

$= \frac{2\times10^4}{50k\Omega} v_i$

$R_i = \frac{v_i}{i_i} = \frac{50\ k\Omega}{2\times10^4} = \underline{2.5\Omega}$

With 200-Ω Load:

See Fig. 3. As before,

$i_{b2} = i_{b3} = \frac{v_i}{50k\Omega}$

A node equation at the output provides:

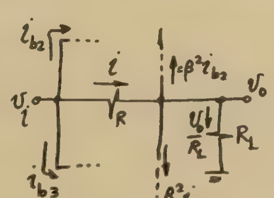


Fig.3

$$\frac{v_i - v_o}{R} = 2\beta^2 i_b + \frac{v_o}{R_L} \quad (1)$$

$$\Rightarrow \frac{v_o}{v_i} \simeq -4\times10^4/(1+\frac{R}{R_L}) = -\frac{4\times10^4}{1+\frac{100}{0.2}}$$

$$\simeq \underline{-80\ V/V}.$$

$i = \frac{v_i - v_o}{R} = \frac{81\ v_i}{R} \simeq \frac{v_i}{1.25\ k\Omega}$

$i_i \simeq i \Rightarrow R_i = \frac{v_i}{i_i} = \underline{1.25\ k\Omega}$

Gain and Input Resistance With C_1 & C_2 Removed

(a) No Load

The ac equivalent circuit of Fig. 2 applies if we add a 160Ω resistor in the emitter lead of Q_1 and Q_4. We then have

$$i_{b2} = i_{b3} = \frac{v_i}{(\beta_3+1)[r_{e3} + (\beta_4+1)(r_{e4}+0.16)]}$$

Substituting $\beta_3 = \beta_4 = 100$, $r_{e4} = 2.5\Omega = 0.0025\ k\Omega$ and $r_{e3} = 250\Omega$ results in

$$i_b = i_{b2} = i_{b3} \simeq \frac{v_i}{1650\ k\Omega}$$

$$i \simeq 2\beta^2 i_b = \frac{2\times10^4}{1{,}650\ k\Omega} v_i$$

$$v_o = v_i - iR = v_i - \frac{2\times10^4\times100}{1{,}650} v_i$$

$$\simeq \underline{-1212\ V/V}$$

$$R_i \equiv \frac{v_i}{i_i} \simeq \frac{v_i}{i} = \frac{v_i}{\frac{2\times10^4}{1{,}650k\Omega}v_i} = \underline{82.5\ \Omega}$$

(b) With a 200Ω Load

Fig. 3 applies with $i_{b2} = i_{b3} = i_b = \frac{v_i}{1{,}650k\Omega}$. Eqn. (1) applies and leads to

$$\frac{v_o}{v_i} = -\frac{1212}{1+\frac{100}{0.2}} = \underline{-2.4\ V/V}$$

$$i_i \simeq i = \frac{v_i - v_o}{R} = \frac{3.4}{R} \Rightarrow R_i \simeq \frac{R}{3.4} = \underline{29.4\ k\Omega}$$

10.15 (a) The output negative peak begins to clip when it causes a load current equal to the 1 mA bias current. This occurs when the negative peak at the output is 1 V, i.e. when the output is 2V peak-to-peak. Since the gain ≃ 1 it follows that the corresponding input amplitude is also $\underline{2V\ \text{peak-to-peak}}$.

(b) The output positive peak begins to clip when $v_E|_{max} = +1 - V_{CEsat} = 1 - 0.3 = +0.7V$. Since $V_E = -0.7V$, the positive peak is 1.4 V in amplitude. Thus a $\underline{2.8V}$ $\underline{\text{peak-to-peak}}$ input sine wave results in the output positive peak being clipped.

10.16 For β very high,

$$V = V_{BE} + \frac{V_{BE}}{R_1} R_2$$

$$= \left(1+\frac{R_2}{R_1}\right) V_{BE}$$

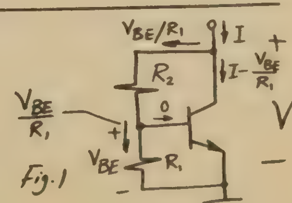


Fig.1

To find the incremental resistance refer to Fig. 2.

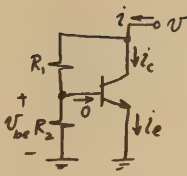


Fig. 2

$$v_{be} = v\,\frac{R_2}{R_1+R_2}$$

$$i = i_c + \frac{v}{R_1+R_2}$$

$$\simeq i_e + \frac{v}{R_1+R_2} = \frac{v_{be}}{r_e} + \frac{v}{R_1+R_2}$$

$$= v\,\frac{R_2}{R_1+R_2}\,\frac{1}{r_e} + \frac{v}{R_1+R_2}$$

Thus the incremental resistance $r \equiv \frac{v}{i}$ is given by $r = (R_1+R_2) \,//\, \left[r_e\left(1+\frac{R_1}{R_2}\right)\right]$.

For $\beta = 10$, $R_1 = R_2 = 1\,k\Omega$, $I = 10\,mA$ & $V_{BE} = 0.7V$:

For dc analysis refer to Fig. 3. Node equation at the base:

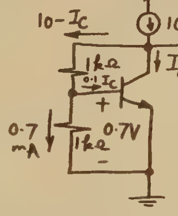


Fig. 3

$$10 - I_C = 0.1 I_C + 0.7$$

$$\Rightarrow I_C = 8.45\,mA$$

$$V = (10 - I_C)\times 1 + 0.7 = \underline{2.25\,V}$$

To find the incremental resistance $r \equiv \frac{v}{i}$ refer to Fig. 4.

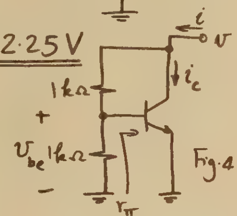


Fig. 4

$$v_{be} = v\,\frac{(1k\Omega \,//\, r_\pi)}{1k\Omega \,//\, r_\pi + 1k\Omega}$$

where $r_e = \frac{25}{8.45} \simeq 3\,\Omega$ $\quad r_\pi = (\beta+1)\,r_e = 33\,\Omega$

Thus $v_{be} \simeq v\,\frac{33}{1033} = 0.032\,v$

$$i_c = g_m v_{be} = \frac{\alpha}{r_e} v_{be} = \frac{10}{11}\times\frac{1}{3}\times 0.032\,v$$

$$= 9.7\times 10^{-3}\,v$$

$$i = \frac{v}{1033} + i_c$$

Thus $\frac{i}{v} = \frac{1}{1033\,\Omega} + 9.7\times10^{-3}\,v$

$$r \equiv \frac{v}{i} = \underline{93.7\,\Omega}$$

The value of I has to be greater than the value required to develop 0.7V across the base ~~an~~ emitter junction. For instance, from Fig. 1 we see that $I > \frac{V_{BE}}{R_1}$ otherwise the transistor would turn off.

10.17 The largest possible positive output voltage is determined by the transistor cut off: When v_i goes sufficiently negative to cause a current decrement (in the transistor emitter) of 1mA, the transistor cuts off. Let the corresponding value of v_o be denoted $\hat{V}_o$. We can write

$$\frac{\hat{V}_o}{1k\Omega} + \frac{\hat{V}_o}{7k\Omega} = 1\,mA \Rightarrow \hat{V}_o = \underline{\tfrac{7}{8}\,V}$$

In other words the maximum collector voltage possible is $3 + \frac{7}{8} = 3\frac{7}{8}$ V.

The largest possible negative output voltage is determined by transistor saturation: It is the value of v_o obtained when the total instantaneous collector voltage v_C reaches -0.4 V $(V_E + V_{CEsat})$. The corresponding value of v_o is $\underline{-3.4\,V}$ (because the quiescent collector voltage is $+3$V).

10.18 Refer to Fig. 10.9. To make the three V_{BE}'s equal we design for the current in the divider equal to the desired output current I_O, i.e. $\frac{V_1 - 2V_{BE}}{R_1+R_2} = I_O$ ----- (1)

Now if we assume $\beta \gg 1$, then I_O is given by

$$I_O = \left[\frac{V_1 - 2V_{BE}}{R_1+R_2}\times R_2 + V_{BE}\right] \Big/ R_E \quad \text{---(2)}$$

To make I_O independent of V_{BE} we must have

$$-\frac{2V_{BE}}{R_1+R_2}\times R_2 + V_{BE} = 0 \Longrightarrow \boxed{R_1 = R_2} \quad (3)$$

Under this condition (Eqn. (3)) the output current becomes (from (2))

$$I_O = \frac{V_1}{2R_E} \quad \text{..... (4)}$$

Substituting for I_O from (4) into (1) and further substituting $R_1 = R_2$ yields

$$\frac{V_1 - 2V_{BE}}{2R_1} = \frac{V_1}{2R_E}$$

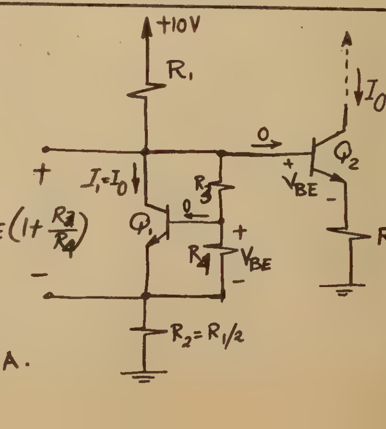
which gives the required value of R_1 as

$$R_1 = R_E\left(1 - \frac{2V_{BE}}{V_1}\right)$$

In summary the condition for I_O being independent of V_{BE} is: $\underline{R_1 = R_2 = R_E\left(1 - \frac{2V_{BE}}{V_1}\right)}$

10.19 $\beta = \infty$

In order to have $V_{BE1} = V_{BE2}$ we shall operate Q_1 and Q_2 at equal collector currents of 10mA.



$$I_0 = \frac{\dfrac{10 - V_{BE}\left(1+\frac{R_3}{R_4}\right)}{R_1 + R_2} \times R_2 + V_{BE}\left(1+\frac{R_3}{R_4}\right) - V_{BE}}{R} \quad (1)$$

Substituting $R_2 = \frac{1}{2}R_1$ we see that the terms containing V_{BE} can be eliminated by selecting $\frac{R_3}{R_4}$ such that

$$-\frac{1}{3}\left(1+\frac{R_3}{R_4}\right) + \left(1+\frac{R_3}{R_4}\right) - 1 = 0$$

$$\Rightarrow \underline{\underline{R_3 = \tfrac{1}{2}R_4}}$$

This makes I_0 insensitive to variations in V_{BE}; $I_0 = \frac{10}{3R}$. For $I_0 = 10\,mA$, $R = \underline{\underline{\frac{1}{3}\,k\Omega}}$. Assuming $V_{BE} = 0.7V$ and selecting the current through R_3 and R_4 to be $\frac{1}{10}I_0 = 1\,mA$, we find that $R_4 = \frac{0.7V}{1\,mA} = \underline{\underline{0.7\,k\Omega}}$ and $R_3 = \underline{\underline{0.35\,k\Omega}}$. Now the current through R_1 and R_2 is $11\,mA$ and the voltage drop across $R_1 + R_2$ is $10 - 0.7 \times \frac{3}{2} = 8.95\,V$. Thus $R_1 = \underline{\underline{542\,\Omega}}$ and $R_2 = \underline{\underline{271\,\Omega}}$.

10.20 Eqn (1) in the solution to Problem 10.19 says that to eliminate the dependence of I_0 on V_{BE} we must have

$$\frac{R_2}{R_1+R_2}\left(1+\frac{R_3}{R_4}\right) + 1 = 1 + \frac{R_3}{R_4}$$

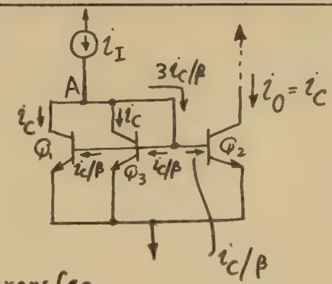
$$\Rightarrow \boxed{\frac{R_3}{R_4} = \frac{R_2}{R_1}}$$

Thus if R_1 is held constant while R_2 is reduced then R_3 must be reduced in order to maintain the insensitivity to V_{BE}. In this case I_0 is given by $I_0 = 10\,\frac{R_2}{R_1+R_2}\cdot\frac{1}{R}$. Thus,

$$R = \frac{10}{I_0}\,\frac{(R_2/R_1)}{1+(R_2/R_1)}.$$

For a given I_0, we see that as $\left(\frac{R_2}{R_1}\right)$ is reduced R should be reduced according to this relationship. In the limit as $R_2 = 0$, $R_3 = 0$, and $R = 0$. The circuit then becomes a variant of the current mirror in Fig. 10.12 with Q_1 acting as a diode-connected transistor. In this current mirror $I_0 = \frac{10 - V_{BE}}{R_1}$ which is obviously dependent on V_{BE}. Note that R_4 no longer serves a useful purpose.

10.21 Q_1, Q_2 and Q_3 will conduct equal collector currents, denoted i_C. For $\beta = \infty$, $i_0 = i_C$ and $i_I = 2i_C$; thus the current transfer ratio $\underline{\underline{\frac{i_0}{i_I} = \frac{1}{2}}}$. For finite β we write a node equation at A and obtain

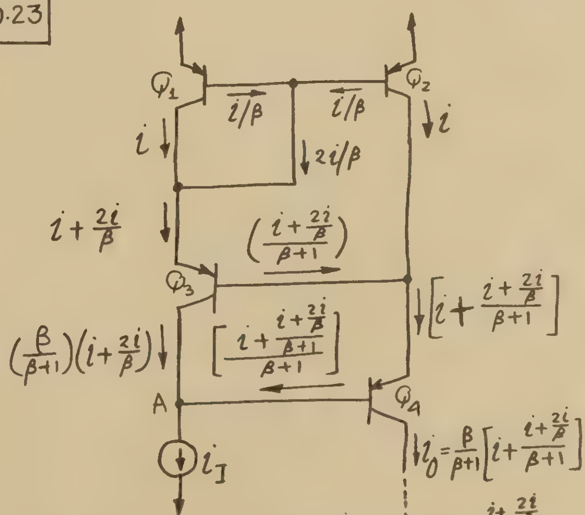


$$i_I = 2i_C + \frac{3i_C}{\beta}. \text{ Thus,}$$

$$\frac{i_0}{i_I} = \frac{i_C}{2i_C + \frac{3i_C}{\beta}} = \underline{\underline{\frac{1}{2+\frac{3}{\beta}}}}$$

10.22 If β is assumed very large then from Fig. 10.13 we see that the current transfer ratio from input I to output 1 is $\underline{\underline{\text{unity}}}$ and the current transfer ratio from input I to output 2 is $\underline{\underline{\text{two}}}$.

10.23



Node equation at A: $i_I = \left(\frac{\beta}{\beta+1}\right)\left(i+\frac{2i}{\beta}\right) + \frac{i + \frac{i+\frac{2i}{\beta}}{\beta+1}}{\beta+1}$

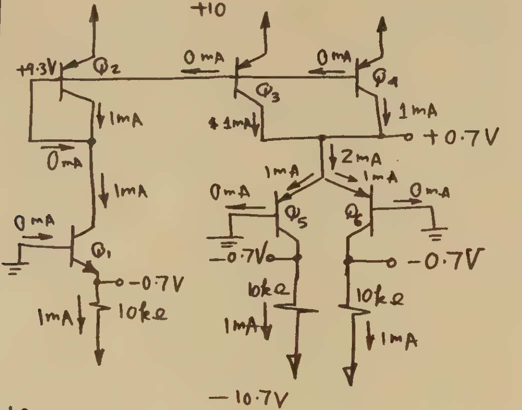
It can be easily shown that

$$\frac{i_0}{i_I} \simeq \frac{1}{1+\frac{2}{\beta}}$$

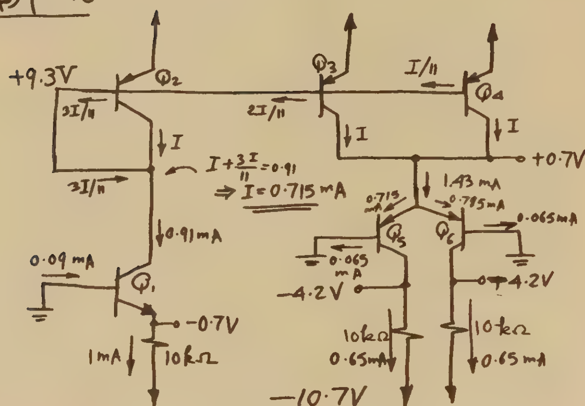
which is the same as that for the original mirror of Fig. 10.12 and is definitely worst than that of the Wilson Mirror of Exercise 10.12.

10.24

(a) $\beta = \infty$



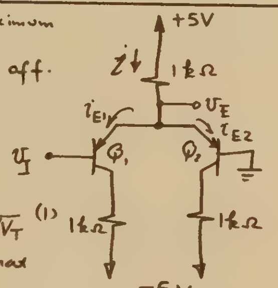
(b) $\beta = 10$



10.25 Q_2 conducts the maximum possible current when Q_1 is off. For Q_1 to conduct 1% of this maximum current then

$$\frac{i_{E2}}{i_{E1}} = 99 = \frac{I_S\, e^{v_E/V_T}}{I_S\, e^{(v_E - v_I)/V_T}} \quad (1)$$

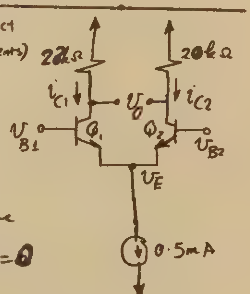
Here we have assumed that the total current i remains approximately constant, in otherwords v_E remains approx constant. Eqn. (1) yields $v_I = V_T \ln 99 = \underline{+0.115V}$



10.26 Neglecting the loading effect (i.e. the base currents) of the second stage, we see from the Figure that the input offset voltage $V_{off} = v_{B2} - v_{B1}$ must be of a value that causes $v_o = 0$

i.e. $i_{C2} \times 20 = i_{C1} \times 22$

In other words we desire $\frac{i_{C2}}{i_{C1}} = \frac{22}{20} = 1.1$



But $\frac{i_{C2}}{i_{C1}} = e^{(v_{B2} - v_{B1})/V_T} = e^{V_{off}/V_T}$.

Thus, $V_{off} = V_T \ln 1.1 = \underline{\underline{2.4\text{ mV}}}$.

10.27 $r_e = \frac{V_T}{I} = 25\,\Omega$

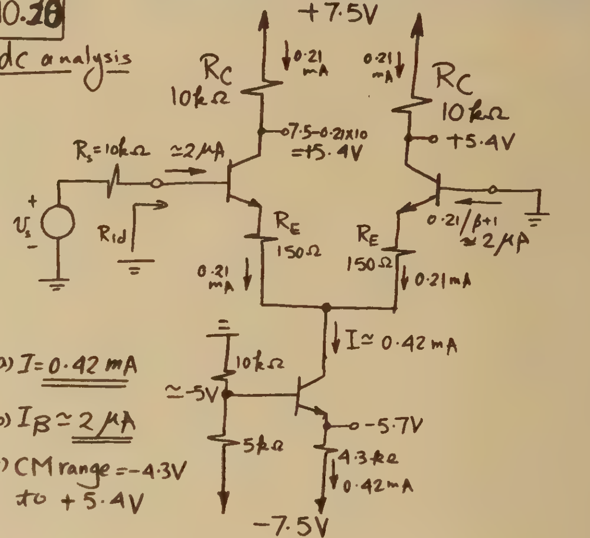
$R_{in} = (\beta+1)\,[r_e + 150\Omega + r_e] = \underline{20.2\text{ k}\Omega}$

$$\frac{v_o}{v_i} = \frac{\alpha \times 5\text{k}\Omega}{r_e + 150\Omega + r_e} \simeq \frac{5000}{200} = \underline{\underline{25\text{ V/V}}}$$

Note: Solutions to 10.28 and 10.29 follow 10.30

10.30

dc analysis



(a) $I = \underline{0.42\text{ mA}}$

(b) $I_B \simeq \underline{2\,\mu A}$

(c) CM range $= -4.3V$ to $+5.4V$

(d) $R_{id} = 2(\beta+1)(r_e + R_E)$ where $r_e = \frac{25}{0.21} = 119\,\Omega$

Thus, $R_{id} = 2 \times 101 \times 0.269 = \underline{\underline{54.3\text{ k}\Omega}}$

(e) $\frac{v_{id}}{v_s} = \frac{R_{id}}{R_{id} + R_s} = \frac{54.3}{54.3 + 10} = 0.84$

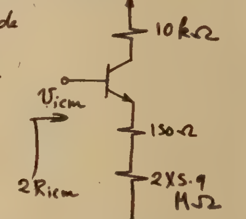
$\frac{v_o}{v_{id}} = -\frac{2R_C \times \alpha}{2r_e + 2R_E} \simeq -\frac{10}{0.269} = -37.17$ V/V

Thus, $\frac{v_o}{v_s} = -37.17 \times 0.84 = \underline{\underline{-31.2\text{ V/V}}}$

(f) The equivalent common-mode half circuit is shown. Its' gain is $\simeq \frac{10 \times 10^3}{2 \times 5.9 \times 10^6} = \frac{1}{2 \times 5.9} \times 10^{-2}$

Since the output is taken differentially, the worst-case common-mode gain will be $\frac{1}{2 \times 5.9} \times 10^{-2} \times \left.\frac{\Delta R_C}{R_C}\right|_{\text{worst case}}$

$= \frac{1}{2 \times 5.9} \times 10^{-2} \times 0.02 = \underline{\underline{1.7 \times 10^{-5}\text{ V/V}}}$



(g) CMRR $= 20 \log \left| \frac{31.2}{1.7 \times 10^{-5}} \right| = \underline{\underline{125\text{ dB}}}$

(h) $R_{icm} = \{[(\beta+1)R] \,//\, (r_\mu/2)\} = \underline{\underline{24\text{ M}\Omega}}$.

Comparing these results to those for the case of ±15V power supplies (Exercise 10.15) we see that the dc characteristics (specifically I_B) have been improved. Also the differential input

resistance has been increased. The price paid is a reduction in gain (from 40 V/V to 31.2 V/V) and a slight (2 dB) reduction in CMRR. Also, the CM range is decreased.

10.28 For differential output:

$$\text{Common-mode Gain} = \frac{(11-10)\times 10^{-3}}{1} = \underline{10^{-3}\ \text{V/V}}$$

$$\text{Differential Gain} = \frac{105\ \text{mV}}{1\ \text{mV}} = 105\ \text{V/V}$$

$$\text{Common-Mode Rejection Ratio} = \frac{105}{10^{-3}} = \underline{1.05\times 10^{5}}$$

$$\text{CMRR} = 20\log 1.05\times 10^{5} = \underline{100.4\ \text{dB}}$$

For single-ended output

$$\text{Worst-case Common-mode Gain} = \frac{11\times 10^{-3}}{1} = \underline{11\times 10^{-3}\ \text{V/V}}$$

To find the differential gain we note that there appears to be a 10% mismatch between the two sides, thus since $\frac{2R_C+\Delta R}{\text{total resistance in emitters}} = 105$ and $\frac{\Delta R}{R_C} = 0.1$

then, total resistance in emitters $= \frac{2R_C(1+0.05)}{105} = \frac{R_C}{50}$

$$\text{Thus, Differential gain} = \frac{R_C+\Delta R_C}{\text{total resistance in emitters}} = 50\times 1.1 = 55\ \text{V/V}$$

$$\text{Common-Mode Rejection Ratio} = \frac{55}{11\times 10^{-3}} = \underline{5{,}000} \text{ or } \underline{74\ \text{dB}}$$

10.29 (a) Single-ended Output

$$\text{Common-mode Gain} \simeq \frac{R_C}{2R} = \frac{10\ \text{k}\Omega}{2\times 1\ \text{M}\Omega} = \underline{5\times 10^{-3}\ \text{V/V}}$$

(b) Differential Output

$$\text{Common-mode Gain} \simeq \frac{R_C}{2R}\cdot\frac{\Delta R_C}{R_C} = 5\times 10^{-3}\times 0.1 = \underline{5\times 10^{-4}\ \text{V/V}}$$

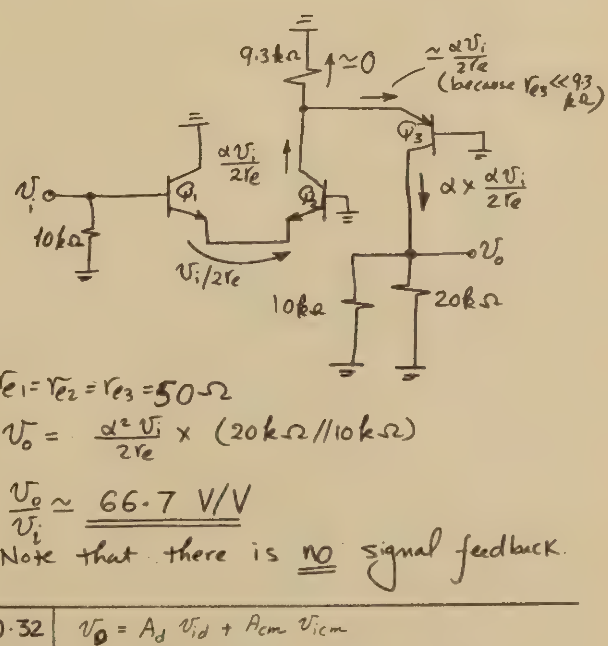
10.30 Solution for 10.30 is given following 10.27.

10.31 This circuit embodies dc negative feedback which will stabilize the dc operating point of the circuit. To see how this comes about assume that the 1-mA bias current divides equally between Q_1 and Q_2. It follows that $I_{E1} = I_{E2} = 0.5$ mA and $I_{B1} = I_{B2} \simeq 5\ \mu$A. Thus $V_{B1} \simeq -50$ mV. Also, $I_{C2} = 0.99\times 0.5 = 0.495$ mA. Now because of the pnp device (call it Q_3), $V_{C2} = +0.7$ V. Thus the current ~~from~~ through the 9.3-kΩ resistor will be $\frac{10-0.7}{9.3} = 1$ mA. The emitter current of Q_3 will be $1-0.495 = 0.505$ mA and its collector current will be $0.99\times 0.505 \simeq 0.5$ mA

Thus the current through the 20-kΩ resistor will be $0.5 - I_{B2} = 0.5 - 0.005 = 0.495$ mA which cause $V_O = -10 + 0.495\times 20 = -0.1$ V. This means that $V_{B2} = -0.15$ V which violates the ~~[illegible]~~ assumption that the 1-mA bias divides equally. What will happen then is that the feedback will force Q_2 to conduct a current very slightly less than 0.5 mA (Q_1 will of course have to conduct a current very slightly greater than 0.5 mA). The feedback loop will provide ~~this~~ and stabilize this very slight imbalance. Exact evaluation of the dc quantities is not warranted as V_{B2} will be approximately equal to $V_{B1} \simeq \underline{-50\ \text{mV}}$,

$I_{E1} \simeq I_{E2} \simeq I_{E3} \simeq 0.5$ mA, and $V_O \simeq \underline{0\ \text{V}}$.

For small-signal analysis the circuit simplifies to that shown in the Figure below:



$r_{e1} = r_{e2} = r_{e3} = 50\ \Omega$

$$v_o = \frac{\alpha^2 v_i}{2r_e}\times(20\ \text{k}\Omega \,//\, 10\ \text{k}\Omega)$$

$$\frac{v_o}{v_i} \simeq \underline{66.7\ \text{V/V}}$$

Note that there is $\underline{\text{NO}}$ signal feedback.

10.32 $v_O = A_d v_{id} + A_{cm} v_{icm}$

where $A_d = \frac{\alpha R_C}{2r_e} \simeq \frac{R_C}{2r_e} = \frac{R_C}{2\frac{V_T}{I/2}} = \frac{IR_C}{4V_T}$,

$A_{cm} \simeq \frac{\alpha R_C}{2R} \simeq \frac{R_C}{2R}$,

and $v_{id} = v_1 - v_2$, and $v_{icm} = \frac{1}{2}(v_1+v_2)$

Thus $v_O = \left(\frac{IR_C}{4V_T}\right)(v_1-v_2) + \left(\frac{R_C}{2R}\right)\left(\frac{v_1+v_2}{2}\right)$

10.33 $|A_{cm}| = \frac{R_C}{2R}$ $\quad A_d = \left|\frac{IR_C}{4V_T}\right|$

$$CMRR = \frac{IR}{2V_T}$$

For $CMRR = 10^4$ and $I = 1\,mA$,

$$10^4 = \frac{10^{-3} \times R}{2 \times 25 \times 10^{-3}} \Rightarrow \underline{\underline{R = 0.5\,M\Omega}}$$

10.34 From Eqn. 10.50,

$$|v_{id}| = |V_P|\sqrt{\frac{I}{I_{DSS}}} = 3V.$$

For Q_2 to carry all the current, $\underline{\underline{v_{id} \equiv v_{G1} - v_{G2} = -3V.}}$

Note that this should be obvious by inspection: Refer to Fig. 10.26 and observe that for Q_2 to carry all of I which is equal to I_{DSS} then $v_{GS2} = 0$, i.e. $v_S = v_{G2}$.

Now since Q_1 has just cut off then $v_{GS1} = V_P = -3V$.

It follows that $v_{id} \equiv v_{G1} - v_{G2} = v_{G1} - v_S = v_{GS1} = -3V$.

$$\frac{v_o}{v_{id}} = g_m R_d = \frac{2I_{DSS}}{|V_P|}\sqrt{\frac{I_D}{I_{DSS}}}\,R_d$$

$$= \frac{2 \times 2}{3}\sqrt{\frac{1}{2}} \times 12 = \underline{11.3\ V/V}$$

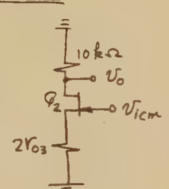
10.35 Each device in the differential pair is biased at $I_D = \frac{1}{2}I_{DSS} = 2\,mA$. Thus, $g_m = \frac{2 \times 4}{2}\sqrt{\frac{2}{4}} = 2.83\,mA/V$ and $r_o = \frac{100}{2.83} = 35.3\,k\Omega$.

(a) Diff. Gain $\equiv \frac{v_o}{v_{id}} = g_m(R_d // r_o)$

$$= 2.83 \times (10 // 35.3) = \underline{\underline{22\ V/V}}$$

(b) The equivalent common-mode half circuit is shown. r_{o3} is the output resistance of the biasing device, $r_{o3} = \frac{100}{g_{m3}} = \frac{100}{2I_{DSS}/|V_P|}$

$$= \frac{100}{2 \times 4/2} = 25\,k\Omega$$



If for simplicity we neglect the effect of r_{o2} then

$$\text{Common-mode Gain} \simeq \frac{10\,k\Omega}{2r_{o3} + \frac{1}{g_{m2}}}$$

$$= \frac{10}{50 + \frac{1}{2.83}} \cong 0.2\ V/V$$

Thus, $CMRR = 20\log\left(\frac{22}{0.2}\right) = \underline{\underline{40.8\ dB}}$

(c) The upper limit on common-mode range is determined by the amplifying FETs leaving the pinch-off region. Since $V_D = 30 - 2 \times 10 = +10V$ and $|V_P| = 2V$, the upper limit of common-mode range is $+8V$. The lower limit is determined by the biasing FET leaving the pinch-off region. Since $V_{GS1} = V_{GS2} = V_P\left(1 - \sqrt{\frac{2}{4}}\right) = -0.6V$, the lower limit is $-8 - (-0.6) = -7.4V$. Thus, the CM range is $\underline{\underline{-7.4 \to +8V}}$.

10.36 (a) There is $\underline{\text{no effect}}$ on dc bias.

(b) Refer to the solution to Example 10.4. R_{i2} changes to

$$R_{i2} = (\beta+1)\,[r_{e4} + r_{e5} + 200\Omega] = 25.25\,k\Omega.$$

The gain of the first stage changes to

$$A_1 = \frac{(25.25\,k\Omega // 40\,k\Omega)}{200\Omega} = 77.4\ V/V$$

The gain of the second stage changes to

$$A_2 = -\frac{R_3 // R_{i3}}{r_{e4} + r_{e5} + 200\Omega} = -\frac{(3k\Omega // 234.8\,k\Omega)}{250\Omega} = -11.85\ V/V$$

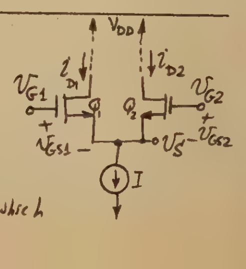
The overall voltage gain changes to

$$\frac{v_o}{v_{id}} = 77.4 \times -11.85 \times -6.42 \times 1 = \underline{\underline{5887.7\ V/V}}$$

10.37 (a) When the positive supply changes to +16V. Refer to Fig. 10.17. I_{E1} and I_{E2} remain unchanged. V_{C1} and V_{C2} change to $+11V$. I_{E4} and I_{E5} remain unchanged. V_{C5} becomes $+13V$ and V_{E7} becomes $13.7V$. Nevertheless, I_{E7} remains unchanged. V_O remains unchanged at $0V$. We conclude that the power-supply rejection is infinite (or equivalently the supply-voltage sensitivity is zero) for positive-supply changes. Note, however, that our calculations are approximate and are based on first-order device models.

(b) Refer to Fig. 10.17 and let the negative supply voltage change to $-14V$. V_{B3} changes to $-9.3V$ thus causing I_{E3} to become $0.46\,mA$ and I_{E6} to become $1.86\,mA$. I_{E1} and I_{E2} change to $0.23\,mA$, V_{C1} and V_{C2} become $+10.4V$, I_{E4} and I_{E5} change to $0.93\,mA$. V_{C5} becomes $+12.21V$ and V_E becomes $+12.91V$. I_{E7} becomes $0.91\,mA$ and V_{B8} becomes $-14 + 0.91 \times 15.7 = +0.287V$. Thus V_O becomes $-0.4V$. Referred to the input this change is $\frac{0.4V}{8{,}500} = \underline{\underline{47\,\mu V}}$. In other words a $1V$ change in the negative-supply voltage gives rise to a $47\,\mu V$ input offset voltage. Thus the supply-voltage sensitivity is $47\,\mu V/V$.

10.38 (a) Q_2 just cut off when v_{GS2} is reduced to V_T; i.e. $v_{GS2} = V_T$ (1). At this point Q_1 conducts all the bias current I; thus $I = \frac{1}{2}\beta(v_{GS1} - V_T)^2$ which



leads to $v_{GS1} = \sqrt{2I/\beta} + V_T$ ---(2)

Subtracting Eqn. (1) from Eqn. (2) yields

$$v_{GS1} - v_{GS2} \equiv v_{id} = \underline{\underline{\sqrt{2I/\beta}}}.$$

For the numerical values given,

$$v_{id} = \sqrt{2\times 2/0.5} = \underline{\underline{2.83\,V}}$$

(b) $g_m = \beta(V_{GS} - V_T)$

where V_{GS} is obtained from

$$\frac{I}{2} = \frac{1}{2}\beta(V_{GS}-V_T)^2 \Rightarrow V_{GS}-V_T = \sqrt{\frac{I}{\beta}}$$

Thus, $\underline{\underline{g_m = \sqrt{\beta I}}}$

For the numerical values given:

$$g_m = \sqrt{0.5\times 2} = 1 \text{ mA/V}$$

(c) $i_{D1} = \frac{1}{2}\beta(v_{GS1} - V_T)^2$

$$\Rightarrow v_{GS1} = \sqrt{\frac{2}{\beta}}\sqrt{i_{D1}} + V_T \quad (3)$$

$$i_{D2} = \frac{1}{2}\beta(v_{GS2} - V_T)^2$$

$$\Rightarrow v_{GS2} = \sqrt{\frac{2}{\beta}}\sqrt{i_{D2}} + V_T \quad (4)$$

Subtracting (4) from (3) leads to

$$v_{id} \equiv v_{GS1} - v_{GS2} = \sqrt{\frac{2}{\beta}}\left(\sqrt{i_{D1}} - \sqrt{i_{D2}}\right)$$

and,

$$i_{D2} \simeq \frac{I}{2} - \sqrt{\beta I}\left(\frac{v_{id}}{2}\right) \quad \text{---} (8)$$

But $g_m = \sqrt{\beta I}$, thus

$$i_{D1} = \frac{I}{2} + g_m\left(\frac{v_{id}}{2}\right) \quad \text{----} (9)$$

$$i_{D2} = \frac{I}{2} - g_m\left(\frac{v_{id}}{2}\right) \quad \text{---} (10)$$

which are the results we could have written from the small-signal analysis.

The small-signal condition is

$$v_{id} \ll 2\sqrt{\frac{I}{\beta}}$$

which for the numerical values given yields

$$v_{id} \ll 2\sqrt{\frac{2}{0.5}} = 4\,V$$

Thus, $\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{\beta/2}\; v_{id}$

But $i_{D1} + i_{D2} = I$. Thus,

$$\sqrt{i_{D1}} - \sqrt{I - i_{D1}} = \sqrt{\beta/2}\; v_{id}$$

$$i_{D1} + (I - i_{D1}) - 2\sqrt{i_{D1}(I - i_{D1})} = \frac{\beta}{2} v_{id}^2$$

$$\sqrt{i_{D1}(I - i_{D1})} = \frac{1}{2}I - \frac{1}{4}\beta v_{id}^2$$

$$i_{D1}(I - i_{D1}) = \frac{1}{4}\left(I - \frac{1}{2}\beta v_{id}^2\right)^2$$

Solving this quadratic equation in i_{D1} yields

$$i_{D1} = \frac{I}{2} \pm \sqrt{\beta I}\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{\beta v_{id}^2}{4I}}$$

From physical considerations we see that the positive sign applies. Thus,

$$i_{D1} = \frac{I}{2} + \sqrt{\beta I}\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{\beta v_{id}^2}{4I}} \quad (5)$$

Since $i_{D2} = I - i_{D2}$, we have

$$i_{D2} = \frac{I}{2} - \sqrt{\beta I}\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{\beta v_{id}^2}{4I}} \quad (6)$$

The small-signal approximation is

$$(\beta v_{id}^2/4I) \ll 1, \text{ i.e. } v_{id} \ll 2\sqrt{\frac{I}{\beta}}$$

and results in

$$i_{D1} \simeq \frac{I}{2} + \sqrt{\beta I}\left(\frac{v_{id}}{2}\right) \quad (7)$$

CHAPTER 11—EXERCISES

$\boxed{11.1}$ Using the voltage-divider rule,

$$T(s) \equiv \frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1 + Z_2}, \text{ where } Z_2 = \frac{1}{sC} /\!/ R_2 \;\&\; Z_1 = R_1$$

$$= \frac{Y_1}{Y_1 + Y_2} = \frac{1/R_1}{\frac{1}{R_1} + \frac{1}{R_2} + sC}$$

$$= \underline{\underline{\frac{1/CR_1}{s + 1/C(R_1/\!/R_2)}}}$$

$\boxed{11.2}$

$$F_L(s) = \frac{s(s+0.25)}{(s+0.5)(s+1)}$$

$$|F_L(j\omega)| = \frac{\omega\sqrt{\omega^2 + 0.25^2}}{\sqrt{\omega^2 + 0.5^2}\sqrt{\omega^2 + 1}}$$

$$|F_L(j\omega_L)| = \frac{\omega_L\sqrt{\omega_L^2 + 0.25^2}}{\sqrt{\omega_L^2 + 0.5^2}\sqrt{\omega_L^2 + 1}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \omega_L = \underline{1.15 \text{ rad/s}} \quad (\text{Exact value}).$$

An approximate estimate of ω_L can be obtained using the superposition method from Eq. 11.23

$$\omega_L \simeq \rho_1 = \underline{1.5 \text{ rad/s}}$$

The large error is due to: (1) The poles are closely spaced and (2) A close-by zero exists. Nevertheless note that the approximate value is conservative.

11.3 Refer to Fig. 11.9b. Setting $C_{gd} = C_{ds} = 0$ results in $\underline{\underline{R_{gs} = R'}}$. Then setting $C_{gs} = 0$ and $C_{ds} = 0$ results in the circuit for determining C_{gd}, shown in Fig. E11.3. From this latter Figure we see that $V_{gs} = I_x R'$. A node equation at d yields:

$$\frac{V_d}{R_L'} + g_m V_{gs} + I_x = 0$$

Thus $V_d = -R_L' I_x (1 + g_m R')$

Now we find V_{gd} as, $V_{gd} \equiv V_g - V_d = V_{gs} - V_d = I_x R' + R_L' I_x (1 + g_m R')$.

Thus, $R_{gd} \equiv \dfrac{V_{gd}}{I_x} = \underline{\underline{R' + (1 + g_m R')R_L'}}$

Finally to find R_{ds} we set $C_{gs} = C_{ds} = 0$ in the circuit of Fig. 11.9b. We then find that

$$R_{ds} = R_L'$$

11.4 $\omega_H \simeq \dfrac{1}{b_1} = 1/\sum C_i R_i$

$$= 1/\left[C_{gs} R_{gs} + C_{gd} R_{gd} + C_{ds} R_{ds}\right]$$

$$= 1/\left[1 \times 10^{-12} \times 80.8 \times 10^{3} + 1 \times 10^{-12} \times 1.16 \times 10^{6} + 1 \times 10^{-12} \times 3.33 \times 10^{3}\right]$$

$$= 1/\left[80.8\text{ ns} + 1.16\ \mu\text{s} + 3.33\text{ ns}\right]$$

$$\omega_H = 1/(1244.1\text{ ns}) = 803.8\text{ krad/s}$$

$$f_H = \frac{\omega_H}{2\pi} = \underline{127.9\text{ kHz}}$$

Percentage Contribution of $C_{gd} = \dfrac{1160}{1244.1} \times 100 = \underline{\underline{93.2\%}}$

11.5 Refer to Fig. E11.5.

$$v_\pi = v_{ce} \frac{r_\pi}{r_\pi + r_\mu}$$

$$i_c = \frac{v_{ce}}{r_o} + g_m v_\pi + \frac{v_{ce}}{r_\pi + r_\mu}$$

$$= \frac{v_{ce}}{r_o} + v_{ce} \frac{g_m r_\pi}{r_\pi + r_\mu} + \frac{v_{ce}}{r_\pi + r_\mu}$$

Thus $\dfrac{i_c}{v_{ce}} = \dfrac{1}{r_o} + \dfrac{\beta_0 + 1}{r_\pi + r_\mu}$

$$\simeq \underline{\underline{\frac{1}{r_o} + \frac{\beta_0}{r_\mu}}}$$

11.6 $g_m = I_C/V_T = 1\text{ mA}/25\text{ mV} = \underline{\underline{40\text{ mA/V}}}$

$$r_\pi = \frac{h_{fe}}{g_m} = \frac{100}{40} = \underline{\underline{2.5\text{ k}\Omega}}$$

$$r_x = h_{ie} - r_\pi = 2.6 - 2.5 = \underline{\underline{100\ \Omega}}.$$

$$r_\mu = \frac{r_\pi}{h_{re}} = \frac{2.5\text{ k}\Omega}{0.5 \times 10^{-4}} = \underline{\underline{50\text{ M}\Omega}}.$$

$$r_o = \left(h_{oe} - \frac{h_{fe}}{r_\mu}\right)^{-1}$$

$$= \left(1.2 \times 10^{-5} - \frac{100}{50 \times 10^{6}}\right)^{-1} = \left(10^{-5}\right)^{-1}$$

$$= 10^{5}\ \Omega = \underline{\underline{100\text{ k}\Omega}}$$

11.7 $|h_{fe}| = 10$ at 50 MHz. Thus, $|h_{fe}| = 1$ at 500 MHz, i.e.

$$\omega_t = 2\pi \times 500 \times 10^{6} = \frac{g_m}{C_\pi + C_\mu}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi \times 5 \times 10^{8}} = \frac{40 \times 10^{-3}}{\pi \times 10^{9}} = 12.7\text{ pF}$$

Thus, $C_\pi = 12.7 - C_\mu = 12.7 - 2$

$$= \underline{\underline{10.7\text{ pF}}}$$

11.8 Refer to Figs. 11.21 and 11.22.

$g_m = 40\text{ mA/V}$, $r_\pi = \dfrac{100}{40} = 2.5\text{ k}\Omega$, $R_B = R_1 // R_2 \simeq 2.7\text{ k}\Omega$,

$R_L' = R_L // R_C // r_o \simeq 2.3\text{ k}\Omega$.

$$A_M = \frac{R_B}{R_B + R_s} \cdot \frac{r_\pi}{r_\pi + r_x + (R_B // R_s)} \cdot (-g_m R_L')$$

$$= -\frac{2.7}{2.7 + 4} \times \frac{2.5}{2.5 + 0.05 + 1.6} \times 40 \times 2.3$$

$$= \underline{\underline{-22.3\text{ V/V}}}$$

<u>High-Frequency Analysis:</u>

$$C_T = C_\pi + C_\mu(1 + g_m R_L')$$

$$= 13.9 + 2(1 + 40 \times 2.3) \simeq 200\text{ pF}$$

This capacitance interacts with a resistance given by $r_\pi // [r_x + (R_B // R_s)]$

$$= 2.5 // [0.05 + 1.6] \simeq 1\text{ k}\Omega$$

Thus, $f_H = \dfrac{\omega_H}{2\pi} = \dfrac{1}{2\pi \times 200 \times 10^{-12} \times 1 \times 10^{3}} = \underline{\underline{796\text{ kHz}}}$.

<u>Low-Frequency Analysis</u>

Refer to the analysis on page 495.

$$R_{C1} = R_s + [R_B // (r_x + r_\pi)] = 4 + [2.7 // 2.55] = 5.31\text{ k}\Omega$$

$$R_E' = R_E // \frac{r_\pi + r_x + (R_B // R_s)}{\beta_0 + 1} = 3.3 // \left(\frac{2.5 + 0.05 + 1.6}{101}\right) = 40.6\ \Omega$$

$$R_{C2} = R_L + (R_C // r_o) = 4 + (6//100) = 9.66\ k\Omega$$

$$\omega_L \simeq \frac{1}{1\times10^{-6}\times 5.31\times10^3} + \frac{1}{10\times10^{-6}\times 40.6} + \frac{1}{1\times10^{-6}\times 9.66\times10^3}$$

$$= 188.3 + 2463 + 103.5 = 2754.8$$

$$f_L = \frac{\omega_L}{2\pi} = \underline{\underline{438\ \text{Hz}}}$$

The zero due to C_E has a frequency

$$f_Z = \frac{1}{2\pi C_E R_E} = \frac{1}{2\pi\times10\times10^{-6}\times3.3\times10^3}$$

$$\simeq \underline{\underline{5\ \text{Hz}}}$$

11.9 Rise time $= 2.2\,\tau_H = \dfrac{2.2}{\omega_H}$

$$= \frac{2.2}{2\pi f_H} = \frac{2.2}{2\pi\times796\times10^3}\ s$$

$$= \underline{0.44\ \mu s}$$

Pulse height $= 44.8\times22.3 = \underline{1\ V}$

$$\text{Sag}, \Delta V = \frac{t_P}{\tau_L}\times V_P = \frac{10\ \mu s}{1/\omega_L}\times 1\,V$$

$$= \frac{10\times10^{-6}}{1/(2\pi\times438)}\times 1 \simeq \underline{\underline{0.028\,V}}$$

11.10 Refer to Fig. 11.25. $V_{B1} \simeq V_{CC}\dfrac{R_3}{R_1+R_2+R_3} = 15\times\dfrac{8}{18+4+8}$

$$= +4\,V$$

Thus, $I_{E1} = \dfrac{4-0.7}{3.3} = 1\,mA$

$$I_{E2} \simeq I_{E1} = 1\,mA$$

The midband gain A_M can be determined from Eqn. (11.64) as

$$A_M = -40\,(6//4)\,\frac{(4//8)}{(4//8)+4}\cdot\frac{2.5}{2.5+0.05+(4//8//4)}$$

$$\simeq \underline{-23.1\ V/V}$$

Note: This value is slightly higher than that for the common-emitter amplifier in Exercise 11.8 because in the cascode circuit the output resistance of Q_2 is greater than r_o and, at any rate, the r_o has been eliminated from the transistor model for simplicity.

Using Eqn. (11.62) we obtain

$$f_L = \frac{1}{2\pi\,R_s'\,(C_{\pi1}+2C_{\mu1})}$$

where

$$R_s' = r_{\pi1} // [r_{x1} + (R_3//R_2//R_s)] = 1\ k\Omega$$

Thus, $f_1 = \dfrac{1}{2\pi\times10^3\times(13.9+4)\times10^{-12}}$

$$= \underline{\underline{8.9\ \text{MHz}}}$$

Eqn (11.61) $\Rightarrow f_2 = \dfrac{1}{2\pi C_{\pi2} r_{e2}} = \dfrac{1}{2\pi\times13.9\times10^{-12}\times25} = \underline{\underline{456\ \text{MHz}}}$

Eqn (11.63) $\Rightarrow f_3 = \dfrac{1}{2\pi C_{\mu2} R_L'} = \dfrac{1}{2\pi\times2\times10^{-12}\times(4//6)\times10^3} = \underline{\underline{33\ \text{MHz}}}$

$$f_H \simeq f_1 = \underline{8.9\ \text{MHz}}$$

11.11

$$A_M = \frac{R_E}{R_E + r_e + \frac{R_s + r_x}{\beta_0+1}} = \frac{1}{1 + 0.025 + \frac{1+0.1}{101}}$$

$$= 0.97\ V/V.$$

$$\omega_P = 1\Big/\left\{\left(C_\mu + \frac{C_\pi}{1+g_m R_E}\right)[R_s' // (1+g_m R_E) r_\pi]\right\}$$

where

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{40\times10^{-3}}{2\pi\times400\times10^6} = 15.9\ pF.$$

$$C_\pi = 15.9 - C_\mu = 15.9 - 2 = 13.9\ pF$$

Thus, $\omega_P = 1\Big/\left[\left(2 + \dfrac{13.9}{1+40\times1}\right)\times10^{-12}\times[1.1 // 41\times2.5]\times10^3\right]$

$$f_P = \frac{\omega_P}{2\pi} = \underline{\underline{62.5\ \text{MHz}}}$$

11.12 Refer to Fig. 11.29. Capacitor C_{C1} sees a resistance R_{C1},

$$R_{C1} = R_s + R_{in} = 4 + 38 = 42\ k\Omega.$$

Capacitor C_E sees a resistance R_{C_E} given by

$$R_{C_E} = R_{E2} // \left\{ r_{e2} + \frac{\left[R_{E1} // \left(r_{e1} + \frac{R_1//R_2//R_s}{\beta_0+1}\right)\right]}{\beta_0+1} \right\}$$

$$= 3.6 // \left\{0.025 + \frac{\left[4.3 // \left(0.025 + \frac{100//100//4}{101}\right)\right]}{101}\right\}$$

$$\simeq 25\,\Omega$$

Capacitor C_{C2} sees a resistance R_{C2} given by

$$R_{C2} = R_C + R_L = 8\ k\Omega$$

We can now use the superposition method to determine f_L as follows

$$f_L = \frac{\omega_L}{2\pi} \simeq \frac{1}{2\pi}\left[\frac{1}{C_{C1}R_{C1}} + \frac{1}{C_E R_{CE}} + \frac{1}{C_{C2}R_{C2}}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{1\times10^{-6}\times42\times10^3} + \frac{1}{47\times10^{-6}\times25} + \frac{1}{1\times10^{-6}\times8\times10^3}\right]$$

$$\simeq \underline{159\ \text{Hz}}$$

The zero introduced by C_E has a frequency $= \dfrac{1}{2\pi C_E R_{E2}} = \underline{\underline{0.94\ \text{Hz}}}$

11.13 Refer to Fig. E11.13(b). $I_{E2} = 5\text{ mA}$; then

$I_{E1} = \frac{5}{101} \simeq 0.05\text{ mA}$. $r_{e1} = 500\,\Omega$, $r_{e2} = 5\,\Omega$.

$$R_{in} = (\beta_0+1)\left[r_{e1} + (\beta_0+1)(r_{e2}+R_E)\right]$$

$$= 101\left[0.5 + 101(0.005+1)\right]\text{ k}\Omega$$

$$= \underline{10.3\text{ M}\Omega}$$

$$\frac{V_o}{V_s} = \frac{R_{in}}{R_{in}+R_s}\;\frac{R_E}{R_E + r_{e2} + \frac{r_{e1}}{\beta_0+1}}$$

$$= \frac{10.3}{10.3+0.1}\;\frac{1}{1+0.005+\frac{0.5}{101}} \simeq \underline{0.98\text{ V/V}}$$

$$R_{out} = R_E \,//\, \left\{ r_{e2} + \frac{r_{e1} + \frac{R_s}{\beta_0+1}}{\beta_0+1} \right\}$$

$$= 1000 \,//\, \left\{ 5 + \frac{500 + \frac{100{,}000}{101}}{101} \right\}$$

$$\simeq \underline{20\,\Omega}$$

11.14 From Eqn.(11.76),

$$A_0 = -g_m R_C \,\frac{2r_\pi}{2r_\pi + R_s + 2r_x}$$

$$= -20 \times 10\,\frac{2\times 5}{2\times 5 + 10 + 0.1} \simeq \underline{-100\text{ V/V}}$$

or, $\underline{40\text{ dB}}$.

Eqn.(11.75) $\Rightarrow f_H = \frac{\omega_P}{2\pi} = \frac{1}{2\pi\,[10//(10.1)]\,[3+1\times 200]\times 10^{-9}} \simeq 156\text{ kHz}$

Gain-bandwidth product = 100 X 156 = 15.6 MHz

11.15 Using Eqn. (11.77)

$$A_0 = \frac{-(\beta+1)(r_e+R_E)}{R_s/2 + (\beta+1)(r_e+R_E)}\;\frac{\alpha R_C}{R_E + r_e}$$

$$= \frac{-101\times 0.2}{5 + 101\times 0.2}\;\frac{0.99\times 10}{0.2} = \underline{40\text{ V/V}} \text{ or } \underline{32\text{ dB}}$$

Eqn.(11.78) $\Rightarrow R_\pi = r_\pi \,//\, \frac{R_s' + R_E}{1 + g_m R_E}$

$$= 5 \,//\, \frac{5.05+0.15}{1+20\times 0.15} \simeq \underline{1\text{ k}\Omega}$$

Eqn.(11.79) $\Rightarrow R_\mu = R_C + \frac{1 + R_E/r_e + g_m R_C}{1/r_\pi + (1/R_s')(1+R_E/r_e)}$

$$= 10 + \frac{1 + 150/50 + 20\times 10}{\frac{1}{5} + \frac{1}{5.05}\times\left(1+\frac{150}{50}\right)}$$

$$\simeq 214\text{ k}\Omega$$

Eqn.(11.80) $\Rightarrow \tau = C_\pi R_\pi + C_\mu R_\mu = 6\times 1 + 2\times 214 = 434\text{ ns}$

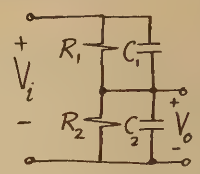
$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi\times 434\times 10^{-9}} = \underline{367\text{ kHz}}$$

Gain-bandwidth product = $40 \times 367 = \underline{14.7\text{ MHz}}$

CHAPTER 11—PROBLEMS

11.1

$$T = \frac{V_o}{V_i} = \frac{Z_2}{Z_1+Z_2} = \frac{Y_1}{Y_1+Y_2}$$



$$= \frac{\frac{1}{R_1} + sC_1}{\frac{1}{R_1} + sC_1 + \frac{1}{R_2} + sC_2}$$

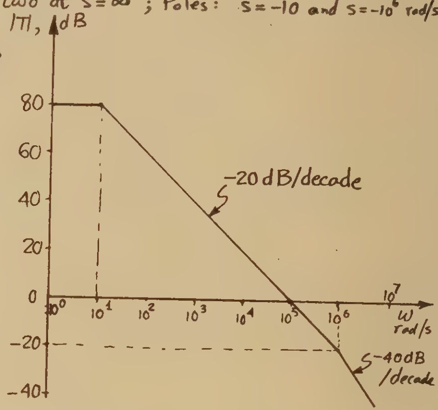
$$= \frac{C_1}{C_1+C_2}\;\frac{s + 1/C_1R_1}{s + 1/(C_1+C_2)(R_1//R_2)}$$

$T(s)$ can be made independent of frequency (i.e. s) by arranging that $C_1R_1 = (C_1+C_2)(R_1//R_2)$

i.e. $\frac{C_1}{C_1+C_2} = \frac{R_2}{R_1+R_2} \Rightarrow \underline{C_1R_1 = C_2R_2}$

11.2 Zeros: two at $s=\infty$; Poles: $s=-10$ and $s=-10^6$ rad/s

ω, rad/s	Gain, dB
0	80
10	77
100	60
10^5	0
10^6	−23

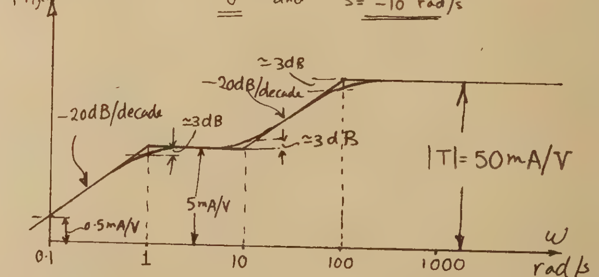


Unity-gain is obtained at $\omega = 10^5$ rad/s, at this frequency the phase shift is

$$\phi = -\tan^{-1}\left(\frac{10^5}{10}\right) - \tan^{-1}\left(\frac{10^5}{10^6}\right) = \underline{-95.7^\circ}$$

11.3 Poles: $s = \underline{-1\text{ rad/s}}$ and $s = \underline{-100\text{ rad/s}}$

Zeros: $s = \underline{0}$ and $s = \underline{-10\text{ rad/s}}$



ω, rad/s	0.1	1	10	100	1,000
|T|, mA/V	0.5	$5/\sqrt{2} = 3.54$	$5\sqrt{2} = 7.07$	$50/\sqrt{2} = 35.4$	50.

At $\omega = 100$ rad/s the various poles and zeros contribute the following approximate phase components: Pole at 100 rad/s: -45°; zero at 10 rad/s: $+90^\circ - 5.7^\circ$; pole at 1 rad/s: -90°; zero at 0: $+90^\circ$. Summing up these values leads to the phase at $\omega = 100$ rad/s being $\underline{+39.3^\circ}$. An exact

evaluation of the phase of the given function yields $+39.9^\circ$.

11.4 The high-frequency gain is given by

$$A_H(s) = \frac{A_M}{\left(1+\frac{s}{10^5}\right)\left(1+\frac{s}{10^6}\right)^2}$$

$$|A_H(j\omega)| = \frac{A_M}{\sqrt{1+\frac{\omega^2}{10^{10}}}\left(1+\frac{\omega^2}{10^{12}}\right)}$$

At $\omega = \omega_H$, $|A_H(j\omega_H)| = A_M/\sqrt{2}$. Thus

$$2 = \left(1+\frac{\omega_H^2}{10^{10}}\right)\left(1+\frac{\omega_H^2}{10^{12}}\right)^2$$

$$\Rightarrow \omega_H = \underline{0.98 \times 10^5} \text{ rad/s}$$

An approximate value for ω_H can be obtained using the superposition method as follows: The value of the coefficient of s in the denominator polynomial of $A_H(s)$ is,

$$b_1 = \frac{1}{10^5} + \frac{2}{10^6} = 1.2 \times 10^{-5}$$

Thus, $\omega_H \simeq \frac{1}{b_1} = \frac{10^5}{1.2} = \underline{\underline{0.83 \times 10^5 \text{ rad/s}}}$

11.5 $A_L(s) = A_M \dfrac{s(s+0.5)}{(s+1)(s+2)}$

$$|A_L(j\omega)| = A_M \frac{\omega\sqrt{\omega^2+0.25}}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$

Since $|A_L(j\omega_L)| = A_M/\sqrt{2}$, then

$$2 = \frac{(\omega_L^2+1)(\omega_L^2+4)}{\omega_L^2(\omega_L^2+0.25)}$$

$$\Rightarrow \omega_L = \underline{\underline{2.3 \text{ rad/s}}} \quad \text{(Exact vale).}$$

An approximate value for ω_L can be obtained using the superposition method as follows. The coefficient of s in the denominator of $A_L(s)$ is $e_1 = 3$.

Thus, $\omega_L \simeq e_1 = \underline{3 \text{ rad/s}}$

Although the error is large the estimate is on the conservative side.

11.6 (a) Using the Miller's approximation we find that the total input capacitance

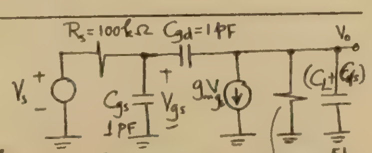


Fig. 1 $g_m = 5$ mA/V . $R_L = 10\,k\Omega$ 51 pF

is given by: $C_T = 1 + 1(1 + g_m R_L) = 52$ pF

Thus the dominant pole created by the input circuit has a frequency ω_{P1},

$$\omega_{P1} = \frac{1}{C_T R_s} = \frac{1}{52\times10^{-12} \times 100\times10^3} = \underline{\underline{192.3 \text{ k rad/s}}}$$

The nondominant pole at the output has a frequency ω_{P2},

$$\omega_{P2} = \frac{1}{(51+1)\times10^{-12} \times 10\times10^3} = \underline{\underline{1923 \text{ k rad/s}}}$$

Based on these two values we conclude that

$$\omega_H \simeq \omega_{P1} = \underline{192.3} \text{ k rad/s}$$

(b) Setting $C_{gd} = C_L + C_{ds} = 0$ and $V_s = 0$ we see that the resistance seen by C_{gs} is

$$R_{ds} = R_s = 100\ k\Omega$$

Setting $C_{gs} = C_L + C_{ds} = 0$ and $V_s = 0$ we obtain the circuit shown for determining the resistance R_{gd} seen by C_{gd}.

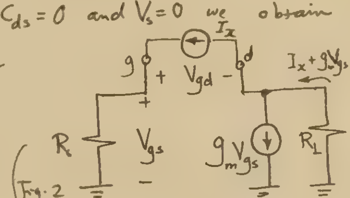


Fig. 2

For this circuit we can write

$$V_{gs} = I_x R_s$$

$$\& \quad V_d = -R_L(I_x + g_m V_{gs})$$

Thus $V_d = -R_L V_{gs}\left(g_m + \frac{1}{R_s}\right)$

and, $V_{gd} = V_{gs}\left[1 + R_L\left(g_m + \frac{1}{R_s}\right)\right] = I_x\left[R_s + R_L(1+g_m R_s)\right]$

$$\text{Thus } R_{gd} \equiv \frac{V_{gd}}{I_x} = R_s + R_L(1 + g_m R_s)$$

$$= 100 + 10(1 + 5\times100) = 5.11\ M\Omega$$

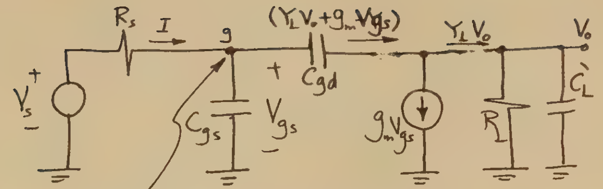
Finally setting $C_{gs} = C_{gd} = 0$ and $V_s = 0$ the resistance seen by $C_L + C_{ds}$ is found to be equal to R_L, i.e. $10\ k\Omega$. We can now write

$$\omega_H \simeq 1\Big/\sum_i C_i R_i$$

$$= \frac{1}{1\times10^{-12}\times100\times10^3 + 1\times10^{-12}\times5.11\times10^6 + 51\times10^{-12}\times10\times10^3}$$

$$\simeq \underline{\underline{175 \text{ k rad/s}}}$$

(c) The exact value of ω_H can be determined from an analysis of the circuit in Fig. 1 to find the transfer function $T(s) \equiv \frac{V_o}{V_s}$.



$$V_{gs} = V_o + \frac{1}{sC_{gd}}\left[Y_L V_o + g_m V_{gs}\right]$$

$$I = sC_{gs} V_{gs} + Y_L V_o + g_m V_{gs}$$

$$= \frac{1}{R_s}\left[V_s - V_{gs}\right]$$

Thus, $V_s = V_{gs} + (g_m + sC_{gs})R_s V_{gs} + Y_L R_s V_o$ (1)

From Fig. 3 we have

$$V_{gs}\left[1 - \frac{g_m}{sC_{gd}}\right] = V_o\left[1 + \frac{Y_L}{sC_{gd}}\right] \quad \text{---(2)}$$

Substituting for V_{gs} from (2) into (1) yields

$$\frac{V_s}{V_o} = Y_L R_s + \left(\frac{1 + \frac{Y_L}{sC_{gd}}}{1 - \frac{g_m}{sC_{gd}}}\right)\left(1 + g_m R_s + sC_{gs}R_s\right)$$

Substituting $Y_L = \frac{1}{R_L} + sC_L'$ and after some manipulations we obtain

$$\frac{V_o}{V_s} = \frac{-(g_m - sC_{gd})R_L}{s^2\left[C_L' R_L (C_{gs}+C_{gd})R_s + C_{gd}R_L C_{gs} R_s\right] + s\left[C_L' R_L + (C_{gs}+C_{gd})R_s + C_{gd}R_L(1+g_m R_s)\right] + 1}$$

Thus, $\frac{V_o}{V_s} = \frac{-50\ (1 - s/5\times10^9)}{s^2 \times 0.103\times10^{-12} + s\times5.72\times10^{-6} + 1}$

The transfer function has a zero with a frequency of 5×10^9 rad/s, and two poles whose frequencies are 1.942×10^5 rad/s and 5.53×10^7 rad/s. It follows that the response is dominated by the pole at 1.942×10^5 rad/s; thus $\omega_H \simeq$ 194.2 krad/s. This value is very close to that obtained using the Miller approximation.

11.7 $A_M = \frac{R_{in}}{R_{in}+R} \times -g_m\ (R_D // R_L)$

where $R_{in} = R_{G1} // R_{G2} = 22\,M\Omega // 10\,M\Omega = 6.9\,M\Omega$

Thus, $A_M = -\frac{6.9}{6.9+1} \times 10 \times \frac{5\times10}{5+10} = -29$ V/V.

<u>High-Frequency Response</u>

$$C_T = C_{gs} + C_{gd}\left(1 + g_m(R_D//R_L)\right)$$

$$= 1 + 1\times\left(1 + 10\times\frac{50}{15}\right) = 35.3\ pF$$

$$f_H = \frac{1}{2\pi C_T (R//R_{in})} = \frac{1}{2\pi\times35.3\times10^{-12}(1//6.9)\times10^6}$$

$$= 5.2\ kHz$$

<u>Low-Frequency Response</u>

Pole due to C_{C1}: $f_{P1} = \frac{1}{2\pi C_{C1}(R_{in}+R)}$

$$= \frac{1}{2\pi\times0.01\times10^{-6}\times7.9\times10^6}$$

$$= 2\ Hz$$

Pole due to C_E: $f_{P2} = \frac{1}{2\pi C_S (R_S // 1/g_m)}$

$$= \frac{1}{2\pi\times10\times10^{-6}(5//0.1)\times10^3}$$

$$= 159\ Hz$$

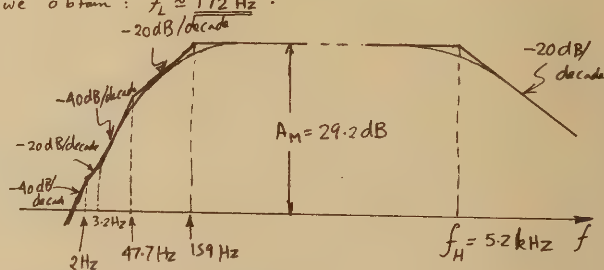
Zero due to C_E: $f_Z = \frac{1}{2\pi C_S R_S} = \frac{1}{2\pi\times10\times10^{-6}\times5\times10^3}$

$$= 3.2\ Hz$$

Pole due to C_{C2}: $f_{P3} = \frac{1}{2\pi C_{C2}(R_D//R_L)} = \frac{1}{2\pi\times10^{-6}\times3.3\times10^3}$

$$= 47.7\ Hz$$

The low-frequency response will be dominated by the two poles: at 159 Hz and 47.7 Hz. An approximate value for f_L can be obtained by neglecting the effect of the pole at 47.7 Hz, thus $f_L \simeq 159$ Hz. Taking the second pole into account we obtain: $f_L \simeq$ 172 Hz.



11.8 The bandwidth of this amplifier is too narrow (i.e. ω_H is not much larger than ω_L) to allow a faithful reproduction of the input pulse. Unfortunately this also means that the approximate methods used in the Text would not work. Therefore, to determine the shape of the output waveform we shall provide an exact solution.

Let the input pulse be of width T (0.1 ms) and of height P (0.1 V). Since the amplifier transfer function has two dominant poles, it can be expressed as

$$A(s) = A_M \cdot \frac{s}{s+\omega_L} \cdot \frac{1}{1+\frac{s}{\omega_H}}$$

Thus, $V_o(s)$ will be

$$V_o(s) = A(s)\, V_i(s)$$

where $V_i(s) = \frac{P}{s} - \frac{P}{s} e^{-sT}$

Thus $$V_o(s) = \frac{A_M P}{(s+\omega_L)(1+\frac{s}{\omega_H})} - \frac{A_M P}{(s+\omega_L)(1+\frac{s}{\omega_H})} e^{-sT}$$

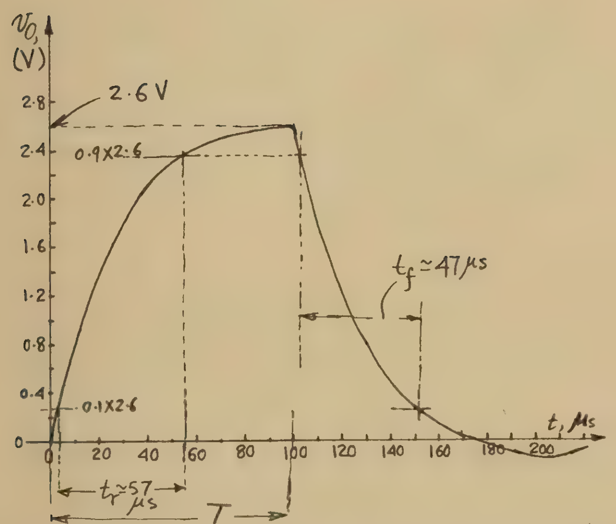
$$= \left(\frac{A_M P}{1-\frac{\omega_L}{\omega_H}}\right)\frac{1}{s+\omega_L} - \left(\frac{A_M P}{1-\frac{\omega_L}{\omega_H}}\right)\frac{1}{s+\omega_H}$$

$$-\left(\frac{A_M P}{1-\frac{\omega_L}{\omega_H}}\right)\frac{1}{s+\omega_L} e^{-sT} + \left(\frac{A_M P}{1-\frac{\omega_L}{\omega_H}}\right)\frac{1}{s+\omega_H} e^{-sT}$$

$$v_o(t) = \left(\frac{A_M P}{1-\frac{\omega_L}{\omega_H}}\right)\left(e^{-\omega_L t} - e^{-\omega_H t}\right)$$

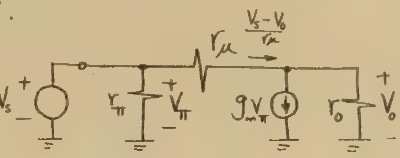
$$-\left(\frac{A_M P}{1-\frac{\omega_L}{\omega_H}}\right)\left(e^{-\omega_L (t-T)} - e^{-\omega_H (t-T)}\right) u(t-T)$$

Substituting $A_M = 29$, $P = 0.1\,V$, $\omega_L = 2\pi \times 159$ rad/s, $\omega_H = 2\pi \times 5200$ rad/s, and $T = 0.1$ ms, we obtain the plot shown below. From this plot we observe that because of the narrow bandwidth no sag is visibly obvious. We can nevertheless determine the rise and fall times, as indicated



11.9 Please note that in the first printing of the text the following sentence was missed. "Neglect r_x".

If $R_L \gg r_o$, the amplifier equivalent circuit becomes



as shown. $V_\pi = V_s$ and,

$$\frac{V_o}{r_o} = \frac{V_s - V_o}{r_\mu} - g_m V_\pi$$

$$= \frac{V_s - V_o}{k \beta r_o} - g_m V_s$$

Thus,

$$V_o = \frac{V_s}{k\beta} - \frac{V_o}{k\beta} - \mu V_s$$

$$V_o\left(1 + \frac{1}{k\beta}\right) = -\left(\mu - \frac{1}{k\beta}\right) V_s$$

$$\underline{\underline{\frac{V_o}{V_s} = -\frac{\mu - (1/k\beta)}{1 + (1/k\beta)}}} \simeq -\mu, \text{ for } k\beta \gg 1$$

$$R_{in} = r_\pi \,//\, \frac{V_s}{(V_s - V_o)/r_\mu} = r_\pi \,//\, \frac{r_\mu}{1 - \frac{V_o}{V_s}}$$

$$\simeq r_\pi \,//\, \frac{r_\mu}{1+\mu} \simeq r_\pi \,//\, \frac{r_\mu}{\mu}$$

$$R_{in} = r_\pi \,//\, \frac{k \beta r_o}{g_m r_o}$$

For $k = 1$,

$$R_{in} = r_\pi \,//\, \frac{\beta}{g_m} = r_\pi \,//\, r_\pi = \underline{\underline{\frac{r_\pi}{2}}}$$

11.10 Using Eqn. (11.39),

$$R_o \simeq \left(r_o \, \frac{1 + (R_E/r_e)}{1 + (R_E/r_\pi)} \right) // \, r_\mu \qquad (11.39)$$

with $R_E = 10\,k\Omega$, $g_m = I_C/V_T = 1/0.025 = 40$ mA/V, $r_e \simeq 25\,\Omega$, $r_\pi = \beta_0/g_m = 100/40 = 2.5\,k\Omega$, $r_o = \frac{\mu}{g_m} = \frac{1000}{40} = 25\,k\Omega$, and $r_\mu = \beta_0 r_o = 2.5\,M\Omega$, results in

$$R_o = \left(25 \times \frac{1 + \frac{10}{0.025}}{1 + \frac{10}{2.5}}\right) // \, 2500 \;\; k\Omega$$

$$= 2005 \,//\, 2500 \; k\Omega = \underline{1.11\; M\Omega}$$

If R_E is replaced by r_o then

$$R_o = \left(25 \times \frac{1 + \frac{25}{0.025}}{1 + \frac{25}{2.5}}\right) k\Omega \; // \, 2.5\, M\Omega$$

$$= \underline{\underline{1.19\; M\Omega}}$$

11.11 $g_m = I_C/V_T = 10/0.025 = 400\,mA/V$

$= \underline{0.4\,A/V}$.

$\beta_0 = h_{fe} = 100 \qquad r_\pi = \beta_0/g_m = \frac{100}{0.4} = \underline{250\,\Omega}$

$r_x = h_{ie} - r_\pi = 350 - 250 = \underline{100\,\Omega}$

$r_\mu = \frac{r_\pi}{h_{re}} = \frac{250\,\Omega}{0.5\times10^{-4}} = \underline{5\,M\Omega}$

$r_o = \left(h_{oe} - \frac{\beta_0}{r_\mu}\right)^{-1} = \left(1.2\times10^{-4} - \frac{100}{5\times10^6}\right)^{-1}$

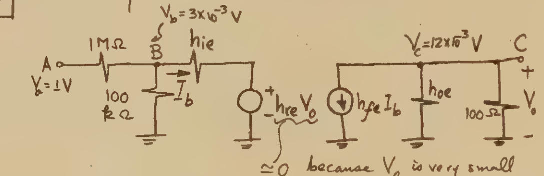
$= 10^4\,\Omega = \underline{10\,k\Omega}$.

11.12 At $f = 50\,MHz$, $|h_{fe}| = 10$. Thus $f_T = \underline{500\,MHz}$.

$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{0.4}{2\pi\times500\times10^6} = 127.3\,pF$

$C_\pi = 127.3 - 2 = \underline{125.3\,pF}$.

11.13 The ac equivalent circuit is



$\frac{V_b}{V_a} = 3\times10^{-3} = \frac{(h_{ie}//100\,k\Omega)}{(h_{ie}//100\,k\Omega) + 1M\Omega}$

$\Rightarrow h_{ie} \simeq \underline{3.0\,k\Omega}$.

$V_o \equiv V_c = 12\times10^{-3}\,V \simeq h_{fe} I_b \times 100\,\Omega$ but $I_b \simeq \frac{V_b}{h_{ie}} \simeq 1\,\mu A$

Thus, $h_{fe} = \frac{12\times10^{-3}}{10^{-6}\times100} = \underline{120}$

Since $r_x \simeq 0$ then $r_\pi \simeq h_{ie} = \underline{3\,k\Omega}$

Now, $g_m = \frac{h_{fe}}{r_\pi} = \frac{120}{3} = \underline{40\,mA/V}$

$r_e \simeq 1/g_m = \underline{25\,\Omega}$

$\beta = h_{fe} = \underline{120}$

Since $g_m = I_C/V_T \simeq I/V_T$ then

$I = 40\times0.025 = \underline{1\,mA}$.

11.14 $A_M = \frac{R_{in}}{R_{in}+R_s}\cdot\frac{r_\pi}{r_\pi + r_x}\cdot(-g_m(R_C//R_L))$

where $R_{in} = R_1//R_2//(r_\pi + r_x) = 10//(r_\pi + r_x)$

and $r_\pi = \frac{\beta_0}{g_m} = \frac{100}{40} = 2.5\,k\Omega$

$R_{in} \simeq 10//2.5 = 2\,k\Omega$

Thus, $A_M = \frac{2}{10+2}\cdot\frac{2.5}{2.5+0.05}\cdot(-40\times1.5)$

$\simeq \underline{-10\,V/V}$.

$C_T = C_\pi + (1 + g_m(R_L//R_C))\,C_\mu$

$= 13.9 + (1 + 40\times1.5)\times2 \simeq 136\,pF$

$R_{eq} = [(R_s//R_1//R_2) + r_x]//r_\pi$

$\simeq 5//2.5 = 1.67\,k\Omega$

$\omega_H = \frac{1}{C_T R_{eq}}$

$f_H = \frac{1}{2\pi\times136\times10^{-12}\times1.67\times10^3} \simeq \underline{700\,kHz}$

Low-Frequency Response:

$R_{C1} = R_s + [R_1//R_2//(r_\pi + r_x)]$

$\simeq 10 + (10//2.5) = 12\,k\Omega$

$R_E' = R_E // \frac{r_\pi + r_x + (R_1//R_2//R_s)}{\beta_0 + 1}$

$= 3k// \frac{2.5 + 0.05 + 5}{101} \simeq 73.2\,\Omega$

$R_{C2} = R_L + R_C = 6\,k\Omega$

Thus, $\omega_L \simeq \frac{1}{C_{C1}R_{C1}} + \frac{1}{C_E R_E'} + \frac{1}{C_{C2}R_{C2}}$

$= \frac{1}{10^{-6}\times12\times10^3} + \frac{1}{10^{-5}\times73.2} + \frac{1}{10^{-6}\times6\times10^3}$

$= 83.3 + 1366.1 + 166.7 = 1616$ rad/s

$f_L = \frac{1616}{2\pi} = \underline{257\,Hz}$

$f_Z = \frac{1}{2\pi C_E R_E} = \frac{1}{2\pi\times10^{-5}\times3\times10^3} = \underline{5.3\,Hz}$

Note: In the above calculations the effects of r_x and r_o were neglected.

11.15 $A_M = \frac{R_{in}}{R_{in}+R_s}\cdot\frac{r_\pi}{r_\pi + r_x + (\beta_0+1)R_{E1}}\cdot(-g_m(R_L//R_C))$

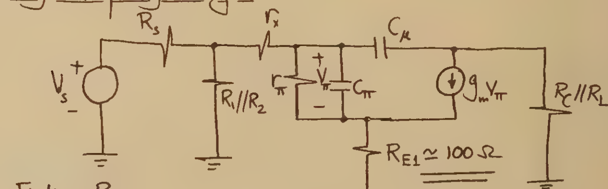
where $R_{in} = R_1//R_2//[r_x + r_\pi + (\beta_0+1)R_{E1}]$

$= 10//[0.05 + 2.5 + 101\times0.1] \simeq 5.56\,k\Omega$

Thus, $A_M = \frac{5.56}{5.56+10}\cdot\frac{2.5}{2.5+0.05+101\times0.1}\times -40\times1.5$

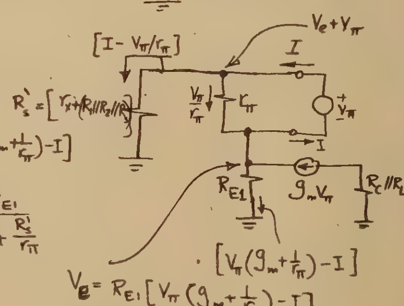
$= -4.3\,V/V$

High-Frequency Analysis



Finding R_π

See analysis on Figure.



$(I - \frac{V_\pi}{r_\pi})R_s' = R_{E1}[V_\pi(g_m + \frac{1}{r_\pi}) - I] + V_\pi$

$\Rightarrow R_\pi = \frac{V_\pi}{I} = \frac{R_s' + R_{E1}}{1 + \frac{R_{E1}}{r_e} + \frac{R_s'}{r_\pi}}$

$V_e = R_{E1}[V_\pi(g_m + \frac{1}{r_\pi}) - I]$

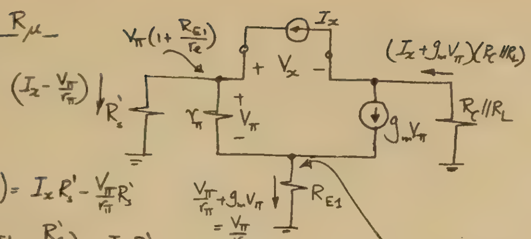
where

$R_s' = 0.05 + 10//10$

$\simeq 5\,k\Omega$

$R_\pi = \frac{5.1}{1 + \frac{100}{25} + \frac{5}{2.5}} = \frac{5.1}{7} = 0.73\,k\Omega$

Finding R_μ



$$V_\pi\left(1+\frac{R_{E1}}{r_e}\right) = I_xR_s' - \frac{V_\pi}{r_\pi}R_s'$$

$$V_e = \frac{V_\pi}{r_e}\times R_{E1}$$

$$V_\pi\left(1+\frac{R_{E1}}{r_e}+\frac{R_s'}{r_\pi}\right) = I_xR_s'$$

$$V_\pi = \frac{I_xR_s'}{1+\frac{R_{E1}}{r_e}+\frac{R_s'}{r_\pi}} \qquad (1)$$

But, $V_x = V_\pi\left(1+\frac{R_{E1}}{r_e}\right) + (I_x+g_mV_\pi)(R_C//R_L)$ (2)

Substituting for V_π from (1) into (2) yields

$$R_\mu \equiv \frac{V_x}{I_x} = (R_C//R_L) + \frac{R_s'\left[1+\frac{R_{E1}}{r_e}+g_m(R_C//R_L)\right]}{1+\frac{R_{E1}}{r_e}+\frac{R_s'}{r_\pi}}$$

$$= 1.5 + \frac{5\left[1+\frac{100}{25}+40\times1.5\right]}{1+\frac{100}{25}+\frac{5}{2.5}} \simeq 48\ k\Omega$$

$$f_H \simeq \frac{1}{2\pi\tau} = \frac{1}{2\pi(C_\pi R_\pi + C_\mu R_\mu)}$$

$$= \frac{1}{2\pi(13.9\times10^{-12}\times0.73\times10^3+2\times10^{-12}\times48\times10^3)}$$

$$\simeq \underline{1.5\ \text{MHz}}$$

it follows that $s_Z = -1/C_E(R_E+R_{E1}) \simeq -\frac{1}{C_ER_E}$

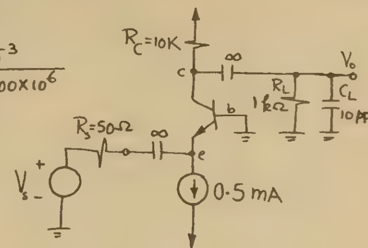
Thus $f_Z \simeq 5.3$ Hz, as before.

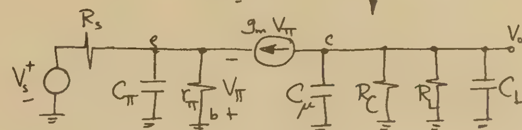
11.16 $C_\pi + C_\mu = \frac{g_m}{\omega_T}$

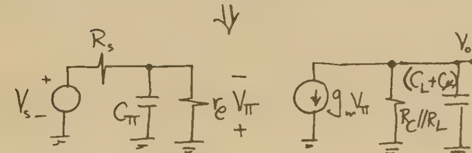
$$= \frac{20\times10^{-3}}{2\pi\times400\times10^6}$$

$$= 8\ \text{pF}$$

$$C_\pi = 8-2 = 6\ \text{pF}$$







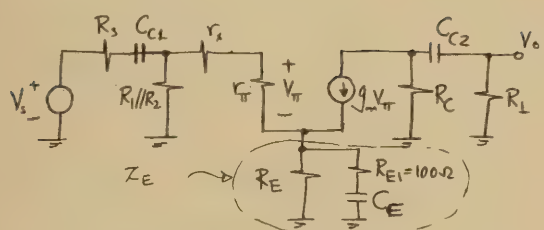
$$f_{P1} = \frac{1}{2\pi C_\pi(R_s//r_e)} = \frac{1}{2\pi\times6\times10^{-12}\times25}$$

$$= \underline{1061\ \text{MHz}}$$

$$f_{P2} = \frac{1}{2\pi(C_L+C_\mu)(R_C//R_L)} \simeq \frac{1}{2\pi\times12\times10^{-12}\times0.91\times10^3}$$

$$= \underline{14.6\ \text{MHz}}$$

Low-Frequency Response



$$R_{C1} \simeq R_s + \left[R_1//R_2 // (r_x + r_\pi + (\beta_0+1)R_{E1}\right]$$

$$= 10 + [10 // (0.05+2.5+101\times0.1)]$$

$$\simeq 10 + 10//12.5 = 15.56\ k\Omega$$

$$R_E' = R_{E1} + \left\{R_E // \frac{r_\pi + r_x + (R_1//R_2//R_s)}{\beta_0+1}\right\}$$

$$= 0.1 + \left\{3 // \frac{2.5+0.05+5}{101}\right\} \simeq 173.2\ \Omega$$

$$R_{C2} = R_L + R_C = 6\ k\Omega$$

$$f_L = \frac{1}{2\pi}\left[\frac{1}{10^{-6}\times15.56\times10^3} + \frac{1}{10^{-5}\times173.2} + \frac{1}{10^{-6}\times6\times10^3}\right]$$

$$= \underline{128.6\ \text{Hz}}$$

C_E introduces a zero at $s=s_Z$ where $Z_E(s_Z)=\infty$ or $Y_E(s_Z)=0$. Since $Y_E(s) = \frac{1}{R_E} + \frac{1}{R_{E1}+\frac{1}{sC_{E1}}}$

$$A_M = \frac{r_e}{r_e+R_s}\times g_m(R_C//R_L)$$

$$= \frac{50}{50+50}\times 20\times0.91 = \underline{9.1\ \text{V/V}}$$

$$f_H \simeq f_{P2} = \underline{14.6\ \text{MHz}}$$

11.17 The three high-frequency poles are given by Equations (11.61), (11.62) and (11.63):

$$f_2 = \frac{1}{2\pi C_{\pi2}r_{e2}} \simeq f_T = 400\ \text{MHz}$$

$$f_1 = \frac{1}{2\pi(C_{\pi1}+2C_{\mu1})R_s'}$$

where $C_{\pi1}+C_{\mu1} = \frac{g_m}{\omega_T} = \frac{20\times10^{-3}}{2\pi\times400\times10^6} \simeq 8\ \text{pF}$

$$C_{\pi1} = 8-1 = 7\ \text{pF}$$

$$r_{\pi1} = \frac{\beta_0}{g_m} = \frac{100}{20} = 5\ k\Omega$$

Using Eq. (11.60): $R_s' = \{5 // [0.1+1]\} = 0.9\ k\Omega$

Thus, $f_1 = \frac{1}{2\pi\times9\times10^{-12}\times0.9\times10^3} = \underline{19.6\ \text{MHz}}$

$$f_3 = \frac{1}{2\pi(C_{\mu2}+C_L)R_L'} = \frac{1}{2\pi\times6\times10^{-12}\times1\times10^3}$$

$$= \underline{26.5\ \text{MHz}}$$

An estimate of f_H can be obtained using the superposition method as follows:

$$\frac{1}{f_H} \simeq \frac{1}{19.6} + \frac{1}{26.5} + \frac{1}{400} = 0.09126$$

$f_H \simeq \underline{11 \text{ MHz}}$

The midband gain can be obtained by substituting in Eqn. (11.64),

$$A_M = -20 \times 1 \times \frac{5}{5+0.1+1} = \underline{-16.4 \text{ V/V}}$$

If R_s is reduced to 100 Ω: $R_s' = 5 // 0.2 = 0.19$ kΩ

$f_2 \simeq \underline{400 \text{ MHz}}$ $\quad f_1 = \frac{1}{2\pi \times 9 \times 10^{-12} \times 0.19 \times 10^3} = \underline{93.1 \text{ MHz}}$

$f_3 = \underline{26.5 \text{ MHz}}$

$$f_H \simeq \left\{ \frac{1}{26.5} + \frac{1}{93.1} + \frac{1}{400} \right\}^{-1} = \underline{19.6 \text{ MHz}}$$

$$A_M = -20 \times 1 \frac{5}{5+0.1+0.1} = \underline{-19.2 \text{ V/V}}.$$

$\boxed{11.18}$ $A_M = \frac{R_E}{R_E + r_e + \frac{R_s + r_x}{\beta_0 + 1}} = \frac{1}{1 + 0.05 + \frac{100.1}{101}} = \underline{0.49 \text{ V/V}}$

$C_\pi + C_\mu = \frac{20 \times 10^{-3}}{2\pi \times 400 \times 10^6}$

$= 8$ pF $\Rightarrow C_\pi = 6$ pF

$r_\pi = \beta_0 / g_m = 100/20 = 5$ kΩ

Using the equation at the bottom of page 504 we obtain:

$$f_P = \frac{1}{2\pi \left[2 + \frac{6}{1+20\times1}\right] \times 10^{-12} \times \left[100.1 // (1+20\times1)\times5\right] \times 10^3}$$

$= \underline{1.36 \text{ MHz}}$

$\boxed{11.19}$ Refer to Fig. 11.29 and note that the new values of components are:
$R_s = 4$ kΩ; $C_{C1} = 0.5\,\mu F$; $R_1 = R_2 = 200$ kΩ; $R_{E1} = 8.6$ kΩ; $R_{E2} = 7.2$ kΩ, $C_E = 23.5\,\mu F$; $R_C = 8$ kΩ; $C_{C2} = 0.5\,\mu F$; $R_L = 4$ kΩ.

dc Bias Calculations: $V_{B1} \simeq 5$ V; $V_{E1} \simeq 4.3$ V; $I_{E1} = \frac{4.3}{8.6} = 0.5$ mA; $V_{E2} \simeq 3.6$ V; $I_{E2} = \frac{3.6}{7.2} = 0.5$ mA.

Small-Signal Parameters: $g_m \simeq 20$ mA/V; $r_e \simeq 50\,\Omega$; $r_\pi \simeq 5$ kΩ; $C_\pi + C_\mu = 8$; $C_\mu = 2$ pF; $C_\pi = 6$ pF.

Midband Gain: $R_{in} = R_1 // R_2 // (\beta+1)\left[r_{e1} + (R_{E1} // r_{\pi2})\right]$

$= 200 // 200 // 101 \left[0.05 + (8.6 // 5)\right]$

$\simeq 76$ kΩ

$$\frac{V_{b1}}{V_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{76}{80} = 0.95$$

$$\frac{V_{e1}}{V_{b1}} = \frac{(R_{E1}//r_{\pi2})}{(R_{E1}//r_{\pi2}) + r_{e1}} = 0.98$$

$$\frac{V_o}{V_{e1}} = -g_{m2}(R_C // R_L) = -20 \times \frac{8\times4}{12} = -53.3$$

Thus, $A_M \equiv \frac{V_o}{V_s} = 0.95 \times 0.98 \times -53.3 \simeq \underline{-50 \text{ V/V}}$

High-Frequency Response: Refer to the equivalent circuit in Fig. 11.30a and to the manipulations performed on it to obtain the circuit in Fig. 11.30b, as explained on page 507 of the Text Book.

$R_s' = 3.85$ kΩ; $R_L' = 2.67$ kΩ

$R_{\mu1} = R_s // R_{in} = 4 // 76 = 3.8$ kΩ

$R_{\pi1} = r_{\pi1} // \frac{R_s' + R_{E1}'}{1 + g_m R_{E1}'}$ where $R_{E1}' = R_{E1} // r_{\pi2} = 3.16$ kΩ

Thus, $R_{\pi1} = 5 // \frac{3.85 + 3.16}{1 + 20\times3.16} \simeq 107\,\Omega$

$C_T = C_{\pi2} + C_{\mu2}(1 + g_{m2} R_L') = 6 + 2(1 + 20\times2.67)$

$= 114.7$ pF

$R_T = R_{E1}' // \left(\frac{r_{\pi1} + R_s'}{\beta_1 + 1}\right) = 3.16 // \frac{5+3.85}{101} = 85\,\Omega$

$R_L' = 2.67$ kΩ

Thus, $\tau = C_{\mu1} R_{\mu1} + C_{\pi1} R_{\pi1} + C_T R_T + C_{\mu2} R_L'$

$= 2\times10^{-12} \times 3.8\times10^3 + 6\times10^{-12} \times 0.107\times10^3 + 114.7\times10^{-12} \times 85$

$+ 2\times10^{-12} \times 2.67\times10^3 = 23.3$ ns

$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi \times 23.3 \times 10^{-9}} = \underline{6.83 \text{ MHz}}$$

$\boxed{11.20}$ Refer to Fig. 11.29 with the component values modified as in Problem 11.19. Capacitor C_{C1} sees a resistance R_{C1},

$R_{C1} = R_s + R_{in} = 4 + 76 = 80$ kΩ.

Capacitor C_E sees a resistance R_{CE} given by

$$R_{CE} = R_{E2} // \left\{ r_{e2} + \frac{\left[R_{E1} // (r_{e1} + \frac{R_1//R_2//R_s}{\beta_0+1})\right]}{\beta_0+1} \right\}$$

$$= 7.2 // \left\{ 0.05 + \frac{\left[8.6 // (0.05 + \frac{200//200//4}{101})\right]}{101} \right\}$$

$\simeq 50\,\Omega$

Capacitor C_{C2} sees a resistance R_{C2} given by

$R_{C2} = R_C + R_L = 12$ kΩ

Thus, $f_L = \frac{\omega_L}{2\pi} \simeq \frac{1}{2\pi}\left[\frac{1}{C_{C1}R_{C1}} + \frac{1}{C_E R_{CE}} + \frac{1}{C_{C2}R_{C2}}\right]$

$$= \frac{1}{2\pi}\left[\frac{1}{0.5\times10^{-6}\times80\times10^3} + \frac{1}{23.5\times10^{-6}\times50} + \frac{1}{0.5\times10^{-6}\times12\times10^3}\right]$$

$\simeq \underline{166 \text{ Hz}}$

The zero introduced by C_E has a frequency $= \frac{1}{2\pi C_E R_{E2}}$

$= \underline{0.94 \text{ Hz}}$

$\boxed{11.21}$

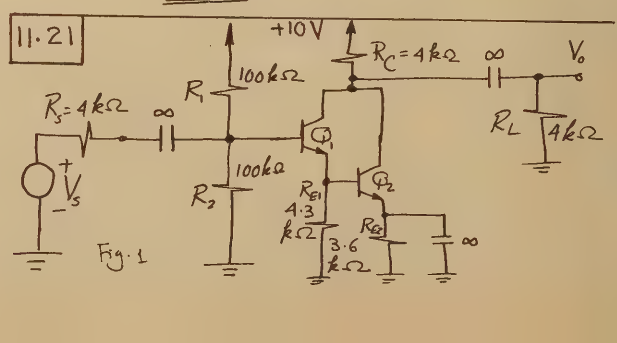


Fig. 1

dc Bias: Same as in Example 11.7; $I_{E1} = I_{E2} = 1\text{mA}$.

Small-Signal Parameters: $r_e = 25\Omega$; $g_m = 40$ mA/V; $r_\pi = 2.5$ kΩ; $C_\pi = 13.9$ pF; $C_\mu = 2$ pF.

Mid and High-Frequency Equivalent Circuit:

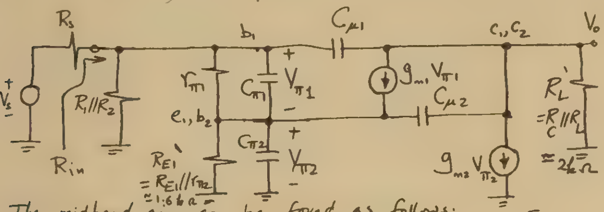


Fig. 2

The midband gain can be found as follows:

$$\frac{V_{b1}}{V_s} = \frac{R_{in}}{R_s + R_{in}}$$

where $R_{in} = R_1 // R_2 // [r_{\pi1} + (\beta_1 + 1) R_{E1}']$, $R_{E1}' \simeq 1.6$ kΩ

$$= 100 // 100 // [2.5 + 101 \times 1.6] \simeq 38 \text{ k}\Omega$$

Then, $\dfrac{V_{b1}}{V_s} = \dfrac{38}{4+38} = 0.9$

$V_{b1} = V_{\pi1} + V_{\pi2}$ and $V_o = -g_m (V_{\pi1} + V_{\pi2}) R_L'$

Thus, $\dfrac{V_o}{V_s} = 0.9 \times g_m R_L'$

$$= -0.9 \times 40 \times 2 = \underline{-72 \text{ V/V}}$$

Examination of the circuit in Fig. (1) reveals that the high-frequency response should be dominated by the pole created by the Miller-multiplied capacitance $C_{\mu1}$. The value of the resulting capacitance is: $C_{\mu1}\left(1 + \left|\frac{V_o}{V_{b1}}\right|\right) = 2 \times (1 + 80) = 162$ pF

This capacitance sees a total resistance of

$$R_{in} // R_s = 38 // 4 = 3.62 \text{ k}\Omega.$$

Thus, $f_H \simeq \dfrac{1}{2\pi \times 162 \times 10^{-12} \times 3.62 \times 10^3} = \underline{271 \text{ kHz}}$

To verify that this is a good estimate of the upper 3-dB cutoff frequency, we shall ~~re~~ use the superposition method to analyze the equivalent circuit of Fig. 2. The analysis is rather tedious but some of the results can be written by inspection of the circuit in Fig (1).

$$R_{\pi1} = r_{\pi1} // \frac{R_s' + R_{E1}'}{1 + g_m R_{E1}'}$$

where $R_s' = R_s // R_1 // R_2$. Note that $R_{\pi1}$ is identical to the expression derived in the Text.

$$R_{\pi1} \simeq 80\,\Omega$$

$$R_{\pi2} = R_{E1}' // \left(\frac{R_s' + r_{\pi1}}{\beta_1 + 1}\right)$$

This result could have been written by inspection of the circuit in Fig. 1.

$$R_{\pi2} = 59.1\,\Omega$$

$$R_{\mu1} = R_L' + (1 + g_m R_L')\left[R_s' // (r_{\pi1} + (\beta+1) R_{E1}')\right]$$

$$= 295 \text{ k}\Omega$$

$$R_{\mu2} = \left\{R_L' + \left[R_{E1}' // \frac{r_{\pi1} + R_s'}{\beta_1 + 1}\right]\left[1 + g_m R_L'\right]\right\}$$

$$= 6.8 \text{ k}\Omega$$

Thus, the effective time constant τ is given by $\tau = R_{\pi1} C_{\pi1} + R_{\mu1} C_{\mu1} + R_{\pi2} C_{\pi2} + R_{\mu2} C_{\mu2}$

$$= 80 \times 13.9 \times 10^{-12} + 295 \times 10^3 \times 2 \times 10^{-12} + 59.1 \times 13.9 \times 10^{-12}$$

$$+ 6.8 \times 10^3 \times 2 \times 10^{-12} = 605.5 \text{ ns}$$

& $f_H \simeq \dfrac{1}{2\pi\tau} = \dfrac{1}{2\pi \times 605.5 \times 10^{-9}} = \underline{\underline{263 \text{ kHz}}}$

which is close to the value obtained before.

Note in conclusion that by connecting the collector of Q_1 to that of Q_2 the high-frequency response of the amplifier is reduced by more than one order of magnitude!

$\boxed{11.22}$ $I_{E1} = I_{E2} = 0.5$mA

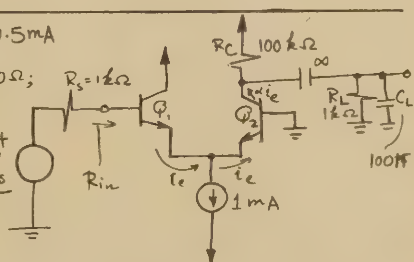
$g_m = 20$ mA/V; $r_e = 50\Omega$;

$r_\pi = 5$ kΩ;

$C_\pi + C_\mu = \dfrac{g_m}{\omega_T}$

$= \dfrac{20 \times 10^{-3}}{2\pi \times 400 \times 10^6}$

$= 8$ pF;



$C_\mu = 2$ pF ; $C_\pi = 6$ pF ; $\beta = 100$; $\alpha = 0.99$.

$$A_M = \frac{\alpha \times (100\text{ k}\Omega // 1\text{ k}\Omega)}{\frac{R_s}{\beta+1} + 2 r_e} = \underline{\underline{8.9 \text{ V/V}}}.$$

There are two poles at high frequency: one created at the input (f_{P1}) and the other at the output (f_{P2}).

$$f_{P1} = \frac{1}{2\pi (R_s // R_{in})\left(\frac{C_\pi}{2} + C_\mu\right)} = \frac{1}{2\pi (1 // 10) 10^3 \times (3 + 2) 10^{-12}}$$

$$= 35 \text{ MHz}$$

$$f_{P2} = \frac{1}{2\pi \times (R_C // R_L)(C_L + C_\mu)} = \frac{1}{2\pi \times (100 // 1) \times 10^3 \times 102 \times 10^{-12}}$$

$$= 1.6 \text{ MHz}$$

Thus,

$$f_H \simeq \underline{\underline{1.6 \text{ MHz}}}$$

11.23 Refer to Fig. E11.13(b). $I_{E2} = 5$ mA ; then

$I_{E1} = \frac{5}{11} \simeq 0.5$ mA. $r_{e1} = 50\,\Omega$, $r_{e2} = 5\,\Omega$

$$R_{in} = (\beta_1+1)\left[r_{e1} + (\beta_2+1)(r_{e2}+R_E)\right]$$

$$= 101\,[0.05 + 11\times(0.005+1)]$$

$$= \underline{1.12\ \text{M}\Omega}$$

$$\frac{V_o}{V_s} = \frac{R_{in}}{R_{in}+R_s}\ \frac{R_E}{R_E + r_{e2} + \frac{r_{e1}}{\beta_2+1}}$$

$$= \frac{1.12}{1.12+0.1}\ \frac{1}{1+0.005+\frac{0.05}{11}} = \underline{0.91\ \text{V/V}}$$

$$R_{out} = R_E // \left\{ r_{e2} + \frac{r_{e1} + \frac{R_s}{\beta_1+1}}{\beta_2+1} \right\}$$

$$= 1000 // \left\{ 5 + \frac{50 + \frac{1000}{101}}{11} \right\}$$

$$\simeq \underline{10\ \Omega}$$

11.24 $I = 2$ mA ; $g_m = 40$ mA/V ; $r_e = 25\,\Omega$; $r_\pi = 2.5$ kΩ.

From Eqn. (11.76),

$$A_0 = -g_m R_C \frac{2r_\pi}{2r_\pi + R_s + 2r_x} = -40\times 10\ \frac{5}{5+10+0.1}$$

$$= \underline{-132.5\ \text{V/V}} \quad \text{or} \quad \underline{42.4\ \text{dB}}.$$

Using Eqn. (11.75) we obtain

$$f_H = \frac{\omega_P}{2\pi} = \frac{1}{2\pi\,[5//10.1]\,[3+1\times 400]\times 10^{-9}}$$

$$= \underline{118\ \text{kHz}}$$

Gain-bandwidth product = 132.5 × 118 = <u>15.6 MHz</u>

11.25 Using Eqn. (11.77),

$$A_0 = -\frac{(\beta+1)(r_e+R_E)}{\frac{R_s}{2} + (\beta+1)(r_e+R_E)} \cdot \frac{\alpha R_C}{R_E + r_e}$$

$$= -\frac{101\times 0.5}{5+101\times 0.5} \cdot \frac{0.99\times 10}{0.5} = \underline{18\ \text{V/V}}$$

or <u>25 dB</u>.

Eqn. (11.78) $\Rightarrow R_\pi = r_\pi // \frac{R_s' + R_E}{1+g_m R_E}$

$$= 5 // \frac{5.05+0.45}{1+20\times 0.45} = \underline{0.5\ \text{k}\Omega}$$

Eqn. (11.79) $\Rightarrow R_\mu = R_C + \frac{1 + R_E/r_e + g_m R_C}{\frac{1}{r_\pi} + \left(\frac{1}{R_s'}\right)(1+R_E/r_e)}$

$$= 10 + \frac{1+(450/50)+20\times 10}{\frac{1}{5} + \frac{1}{5.05}\times\left(1+\frac{450}{50}\right)} = \underline{106\ \text{k}\Omega}$$

Eqn. (11.80) $\Rightarrow \tau = C_\pi R_\pi + C_\mu R_\mu = 6\times 0.5 + 2\times 106 = 215$ ns

$$f_H = \frac{1}{2\pi\tau} = \frac{10^9}{2\pi\times 215} = \underline{740\ \text{kHz}}$$

Gain-bandwidth product = 18 × 740 = <u>13.3 MHz</u>

11.26 Refer to Figs. 11.37 and 11.38. $I = 0.5$ mA ; $r_e = 100\,\Omega$

$g_m = 10$ mA/V ; $r_\pi = \frac{\beta_0}{g_m} = 10$ kΩ ; $C_\pi + C_\mu = \frac{10\times 10^{-3}}{2\pi\times 400\times 10^6}$

$= 4$ pF ; $C_\mu = 2$ pF ; $C_\pi = 2$ pF.

$R_{in} = 2(\beta+1)\,r_e \simeq 20$ kΩ.

$$\text{Low-frequency Gain} = \frac{R_{in}}{R_{in}+R_s} \times \frac{\alpha R_C}{2 r_e}$$

$$\simeq \frac{20}{20+20} \times \frac{10}{0.2} = \underline{25\ \text{V/V}}$$

$$f_{P1} = \frac{1}{2\pi(R_s//2r_\pi)(C_\pi/2 + C_\mu)}$$

$$= \frac{1}{2\pi\,(20//20)\times 10^3 \times (1+2)\times 10^{-12}} = 5.3\ \text{MHz}$$

$$f_{P2} = \frac{1}{2\pi R_C C_\mu} = \frac{1}{2\pi\times 10\times 10^3 \times 2\times 10^{-12}}$$

$= 8$ MHz

At $\omega = \omega_H$, $|A(j\omega_H)| = A_0/\sqrt{2}$ and

$$2 = \left(1 + \frac{\omega_H^2}{\omega_{P1}^2}\right)\left(1 + \frac{\omega_H^2}{\omega_{P2}^2}\right)$$

$\Rightarrow f_H \simeq 4$ MHz

11.27 (a) $g_m = 20$ mA/V ; $r_e = 50\,\Omega$; $r_\pi = 5$ kΩ ;

$C_\pi + C_\mu = \frac{20\times 10^{-3}}{2\pi\times 400\times 10^6} = 8$ pF ; $C_\mu = 2$ pF ; $C_\pi = 6$ pF.

$$A_M = \frac{R_{in}}{R_{in}+R_s} \times -g_m R_C = \frac{5}{10+5} \times -20\times 10$$

$= \underline{-66.7\ \text{V/V}}$.

$$f_H = \frac{1}{2\pi\,(R_s//r_\pi)\,[C_\pi + C_\mu(1+g_m R_C)]}$$

$$= \frac{1}{2\pi\,(10//5)\times 10^3\,[6+2(1+200)]\times 10^{-12}} = \underline{117\ \text{kHz}}$$

(b) Small-signal parameters are as in (a) above.

$$A_M = \frac{R_{in}}{R_{in}+R_s} \times -g_m R_C \simeq \underline{-66.7\ \text{V/V}}.$$

Using Eqn. (11.61): $f_2 = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi\times 6\times 10^{-12}\times 50}$

$= 530$ MHz

Using Eqn. (11.62): $f_1 = \frac{1}{2\pi R_s' (C_{\pi 1} + 2C_{\mu 1})}$

$$= \frac{1}{2\pi\times(10//5)\times 10^3\times 10\times 10^{-12}} = 4.8\ \text{MHz}$$

Using Eqn. (11.63) $f_3 = 1/2\pi C_{\mu 2} R_L' = \frac{1}{2\pi\times 2\times 10^{-12}\times 10\times 10^3} = 8$ MHz

To determine the upper 3-dB frequency f_H we

may neglect the pole at f_2 and use f_1 and f_3 as follows:

$$2 = \left(1+\frac{f_H^2}{f_1^2}\right)\left(1+\frac{f_H^2}{f_3^2}\right)$$

$$\Rightarrow f_H = \underline{\underline{3.8\ \text{MHz}}}$$

(c) Small-signal parameters as in (a) above.

$$A_M = \frac{R_{in}}{R_{in}+R_s}\cdot\frac{\alpha R_C}{2r_e}$$

where $R_{in} = 2r_\pi = 10\ k\Omega$.

Thus $A_M \simeq \frac{10}{10+10}\ \frac{10}{0.1} = \underline{\underline{50\ V/V}}$

Pole at input: $f_{P1} = \dfrac{1}{2\pi\,(10k\Omega \,//\, 2r_\pi)\left(\frac{C_\pi}{2}+C_{\mu1}\right)}$

$$= \frac{1}{2\pi \times 5\times10^3 \times 5\times10^{-12}} = 6.4\ \text{MHz}$$

Pole at output: $f_{P2} = \dfrac{1}{2\pi \times 10\times10^3 \times 2\times10^{-12}} \simeq 8\ \text{MHz}$

The upper 3dB frequency f_H is found from:

$$2 = \left(1+\frac{f_H^2}{6.4^2}\right)\left(1+\frac{f_H^2}{8^2}\right)$$

$$\Rightarrow f_H = \underline{\underline{4.6\ \text{MHz}}}$$

(d) Small-signal parameters as in (a) above.

$$R_{in} = (\beta_1+1)\left\{r_{e1} + (\beta_2+1)\,r_{e2}\right\}$$

$$\simeq 100 \times 5.05 = 505\ k\Omega$$

$$A_M = \frac{R_{in}}{R_{in}+R_s}\cdot\frac{r_{\pi2}}{r_{\pi2}+r_{e1}}\cdot(-g_{m2}R_C)$$

$$= \frac{505}{515}\cdot\frac{5}{5.05}\cdot(-20\times10) = \underline{\underline{-194\ V/V}}$$

To determine f_H we use similar analysis as in Example 11.7: $R_s' = R_s = 10\ k\Omega$; $C_T = C_{\pi2} + C_{\mu2}(1+g_{m2}R_L')$ $= 6 + 2(1+20\times10) = 408\ pF$; $R_{\mu1} = R_s // R_{in} = 10//505$ $= 9.8\ k\Omega$; $R_{\pi1} = \left(r_{\pi1} \,//\, \frac{R_s'+R_{E1}'}{1+g_m R_{E1}'}\right) = \left(5\,//\,\frac{10+5}{1+20\times5}\right)$ $= 144\ \Omega$; $R_T = \left(R_{E1}' \,//\, \frac{r_{\pi1}+R_s'}{\beta_1+1}\right) = \left(5 \,//\, \frac{5+10}{101}\right) = 144\ \Omega$; $R_{\mu2} = R_L = 10\ k\Omega$. Thus,

$$\tau = C_{\mu1}R_{\mu1} + C_{\pi1}R_{\pi1} + C_T R_T + C_{\mu2}R_{\mu2}$$

$$= 2\times9.8 + 6\times0.144 + 408\times0.144 + 2\times10 \quad ns$$

$$= 99.2\ ns$$

$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi\times99.2\times10^{-9}} = \underline{\underline{1.6\ \text{MHz}}}$$

(e) The circuit is a cascode. Small-signal parameters as in (a) above. $R_{in} = r_{\pi1} = 5\ k\Omega$.

$$A_M = -\frac{R_{in}}{R_{in}+R_s}\cdot g_{m1}\cdot\alpha_2\cdot R_C$$

$$\simeq -\frac{5}{10+5}\times20\times10 = \underline{\underline{-66.7\ V/V}}$$

Using Eqn. (11.61): $f_2 = \dfrac{1}{2\pi C_{\pi2}r_{e2}} = \dfrac{1}{2\pi\times6\times10^{-12}\times50}$
$= 530\ \text{MHz}$.

Using Eqn. (11.62): $f_1 = \dfrac{1}{2\pi R_s'(C_{\pi1}+2C_{\mu1})} = \dfrac{1}{2\pi(10//5)\times10^3\times10\times10^{-12}}$
$= 4.8\ \text{MHz}$

Using Eqn. (11.63): $f_3 = \dfrac{1}{2\pi C_{\mu2}R_L'} = \dfrac{1}{2\pi\times2\times10^{-12}\times10\times10^3} = 8\ \text{MHz}$

These results are identical to (b) above (as should have been expected!) and $f_H = \underline{\underline{3.8\ \text{MHz}}}$.

(f) From an ac point of view the circuit is identical to that in (c) above. Thus,

$A_M = \underline{\underline{50\ V/V}}$ and $f_H = \underline{\underline{4.6\ \text{MHz}}}$

$\boxed{11.28}$ $I_D = 1\,mA$; $g_m = \frac{2\times2}{2}\sqrt{\frac{1}{2}} = 1.414\ mA/V$

$$A_M = -g_m(R_d // R_L)$$

$$= -1.41\times5 = \underline{-7.1\ V/V}$$

$$C_T = C_{gs} + C_{gd}(1+7.1) = 2 + 2\times8.1 = 18.2\ pF$$

$$f_H \simeq \frac{1}{2\pi C_T R_s} = \frac{1}{2\pi\times18.2\times10^{-12}\times1\times10^6}$$

$$= \underline{8.7\ kHz}$$

The $1\mu F$ bypass capacitor introduces a $\underline{\underline{\text{zero at dc}}}$ and a pole at

$$f_{L1} = \frac{1}{2\pi\times10^{-6}\times\left(\frac{1}{g_m}\right)} = \frac{1}{2\pi\times10^{-6}\times\frac{1}{1.414}\times10^3}$$

$$= \underline{225\ Hz}$$

The $1\text{-}\mu F$ coupling capacitor introduces a $\underline{\underline{\text{zero at dc}}}$ and a pole at

$$f_{L2} = \frac{1}{2\pi\times10^{-6}\times20\times10^3} \simeq 8\ Hz.$$

Thus, $f_L \simeq f_{L1} = \underline{\underline{225\ Hz}}$

11.29 This is a cascode configuration. Each JFET is biased at $I_D = 1\,mA$ in the pinch-off region. $g_m = 1.414$ mA/V. $A_M = -g_m(R_d / R_L) = -1.414 \times 5 = \underline{-7.1\ V/V}$.

Equation (11.61) adapted to the FET circuit becomes

$$f_2 = \frac{1}{2\pi (C_{gs2} + C_{ds1})(1/g_{m2})}$$

$$= \frac{1}{2\pi \times 4 \times 10^{-12} (1/1.414) \times 10^3} = \underline{56.3\ MHz}$$

Equation (11.62) adapted to the FET circuit becomes

$$f_1 = \frac{1}{2\pi R_s (C_{gs1} + 2C_{gd1})} = \frac{1}{2\pi \times 10^6 \times 6 \times 10^{-12}}$$

$$= \underline{26.5\ kHz}$$

Equation (11.63) adapted to the FET circuit becomes

$$f_3 = \frac{1}{2\pi\ C_{gd2} R_L'} = \frac{1}{2\pi \times 2 \times 10^{-12} \times 5 \times 10^3} = \underline{16\ MHz}$$

Thus, $f_H \simeq f_1 = \underline{26.5\ kHz}$

The 1-μF bypass capacitor introduces a zero at dc and a pole at

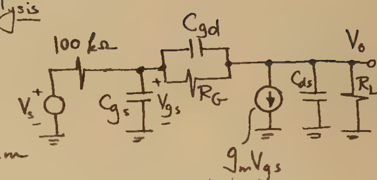
$$f_{L1} = \frac{1}{2\pi \times 10^{-6} \times (1/g_{m1})} = \underline{225\ Hz}$$

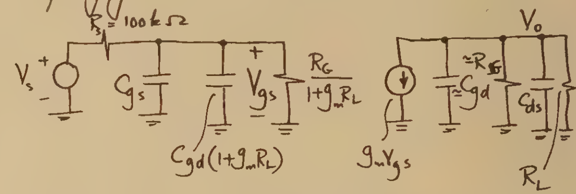
The 1μF coupling capacitor introduces a zero at dc and a pole at $f_2 = \frac{1}{2\pi \times 10^{-6} \times 20 \times 10^3} = \underline{8\ Hz}$. Thus $f_L \simeq \underline{225\ Hz}$

High-Frequency Analysis

$V_o \simeq -g_m V_{gs} R_L$

Thus we can use Miller's theorem to simplify the circuit to the one below





$$f_{P1} = \frac{1}{2\pi [C_{gs} + C_{gd}(1+g_m R_L)] [R_s // \frac{R_G}{1+g_m R_L}]}$$

$$= \frac{1}{2\pi \times 10 \times 10^{-12} \times (0.1 // 1.11) \times 10^6}$$

$$= \underline{173.5\ kHz}$$

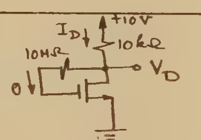
$$f_{P2} = \frac{1}{2\pi (C_{gd} + C_{ds})(R_L // R_G)} \simeq \frac{1}{2\pi \times 2 \times 10^{-12} \times 10 \times 10^3}$$

$$\simeq \underline{8\ MHz}$$

Thus, $f_H \simeq \underline{173.5\ kHz}$

11.30 dc analysis $I_D = \frac{1}{2}\beta (V_D - V_T)^2$

i.e. $I_D = 0.25\,(V_D - 2)^2$ ... (1)

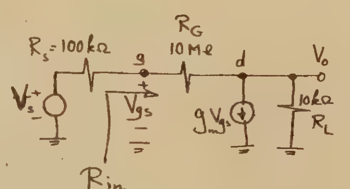


and $I_D = \frac{10 - V_D}{10k\Omega} = 1 - 0.1V_D$ (2)

Solving (1) together with (2) yields $I_D = 0.64\ mA$ and $V_D = 3.6\ V$. $g_m = \beta(V_{GS} - V_T) = 0.5(3.6 - 2)$

$= 0.8\ mA/V$.

Midband Gain:



$V_o \simeq -g_m R_L V_{gs}$

$= -0.8 \times 10\ V_{gs}$

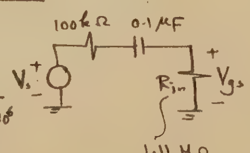
$= -8\ V_{gs}$

$R_{in} = \frac{R_G}{1+8} = \frac{10}{9} = 1.11\ M\Omega$

Thus, $V_{gs} = \frac{1.11}{0.1 + 1.11} V_s = 0.92\ V_s$

$\frac{V_o}{V_s} \simeq -0.92 \times 8 = \underline{-7.3\ V/V}$

Low-Frequency Analysis

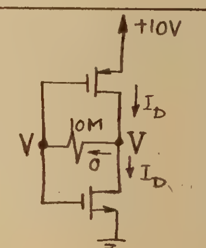


$$f_L = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times (1.11 + 0.1) \times 10^6}$$

$= \underline{1.3\ Hz}$

11.31 dc Analysis

$I_D = 0.25\,(10 - V - 2)^2 = 0.25(V-2)^2$



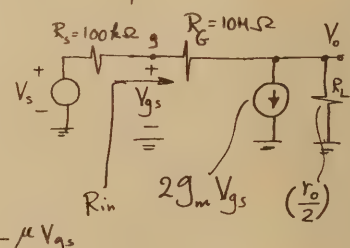
$\Rightarrow 8 - V = V - 2$

$\Rightarrow V = +5V$ (which should have been expected because of symmetry).

$I_D = 0.25\,(5-2)^2 = 2.25\ mA$.

$g_m = 0.5\ (5-2) = 1.5\ mA/V$.

Midband Gain



$r_o = \frac{\mu}{g_m} = \frac{100}{1.5}$

$= 66.7\ k\Omega$

$V_o \simeq -2g_m V_{gs} R_L$

$= -2\,g_m (\frac{r_o}{2}) V_{gs} = -\mu V_{gs}$

$= -100\ V_{gs}$

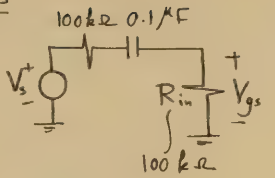
$\frac{V_g}{V_s} = \frac{R_{in}}{R_{in} + R_s}$ where $R_{in} = \frac{R_G}{1+\mu} \simeq 100\,k\Omega$

Thus $\frac{V_g}{V_s} = \frac{100}{100 + 100} = 0.5$

$A_M \equiv \frac{V_o}{V_s} = 0.5 \times -\mu = \underline{-50\ V/V}$

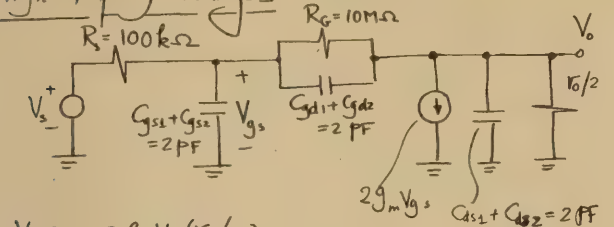
Low-Frequency Analysis

$$f_L = \frac{1}{2\pi \times 0.1\times10^{-6}\times 200\times10^{3}}$$



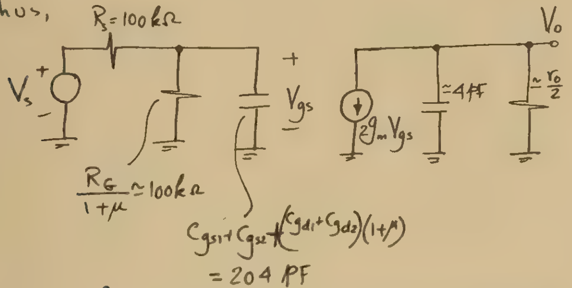
$= 8$ Hz

High-Frequency Analysis



$V_o \simeq -2g_mV_{gs}(r_o/2) = -\mu V_{gs}$

Thus,



Thus, $f_H = \dfrac{1}{2\pi\times 204\times10^{-12}\times 50\times10^{3}} = \underline{\underline{15.6\,kHz}}$

$\boxed{11.32}$ Each device is biased at $I_D = 0.5\,mA$. Thus

$0.5 = \frac{1}{2}\times 0.5(V_{GS}-V_T)^2$

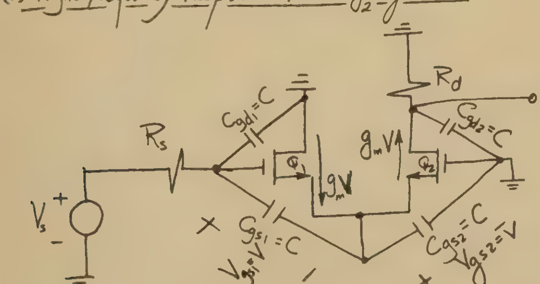
$\Rightarrow (V_{GS}-V_T) = \sqrt{2}$

$g_m = \beta(V_{GS}-V_T) = 0.5\times\sqrt{2} = \frac{1}{\sqrt{2}} = 0.707\,mA/V$

$A_M = \frac{1}{2}g_mR_d = \frac{1}{2}\times 0.707\times 10$

$= \underline{3.5\ V/V}$

(a) High-Frequency Response With g_2-grounded:

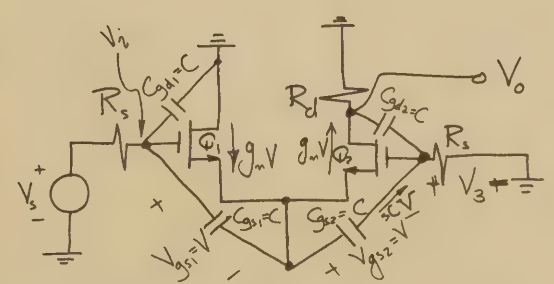


It can be shown by a more detailed analysis that the effect of C_{ds1} and C_{ds2} is negligible. Furthermore we can see that at the input we will have an input capacitance at least equal to $C_{gd1} = C = 1PF$. Since $R_s = 1M\Omega$ we see that the time constant in effect will be greater than $CR_s = 10^{-6}$ seconds. We may therefore neglect time constants much smaller ~~greater~~ than 10^{-6}s. For instance at the output the time constant in effect will be $CR_d = 10^{-12}\times10^{4} = 10^{-8}$s and we may therefore write $V_o \simeq g_mVR_d = g_mR_dV$. A node equation at the common sources shows that $V_{gs1} = V_{gs2} = V$. We therefore conclude that the input capacitance is $C + \frac{C}{2} = \frac{3}{2}C$ and that the dominant time constant is $\frac{3}{2}CR_s = \frac{3}{2}\times10^{-6}$ s leading to a dominant high-frequency pole at

$$f_P = \frac{1}{2\pi\times\frac{3}{2}\times10^{-6}} = \underline{\underline{106\ kHz}}$$

(b) High-Frequency Response With a Resistance Equal to R_s Connected Between g_2 and Ground:

Here again we shall neglect the effects of C_{ds1} and C_{ds2}.



We can also show that $V_{gs1} = V_{gs2} = V$, as before, and that

$V_o \simeq g_mVR_d = (g_mR_d)V$, also as before.

Denoting the voltage at G_2 by V_3 we can write a node equation at G_2 as,

$$sCV = \frac{V_3}{R_s} + sC(V_3 - g_mR_dV)$$

$$\Rightarrow V_3 = V\,\frac{sC(1+g_mR_d)}{sC+\frac{1}{R_s}}$$

Now we can write

$$V_i = 2V + V_3 = 2\frac{V_o}{g_mR_d} + V\,\frac{sC(1+g_mR_d)}{sC+\frac{1}{R_s}}$$

$$= V_o\frac{1}{(g_mR_d/2)}\left[1+\frac{1}{2}\frac{sCR_s}{1+sCR_s}(1+g_mR_d)\right]$$

A node equation at the input gives:

$$\frac{V_s - V_i}{R_s} = sCV_i + sCV$$

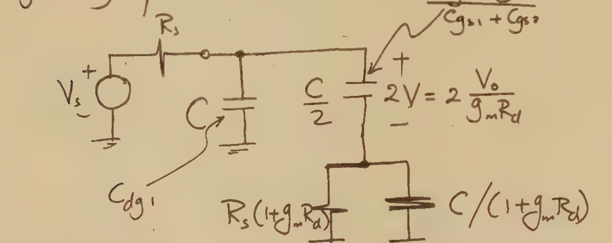
$$\Rightarrow \frac{V_o}{V_s} = (g_m R_d/2)\frac{1}{1 + sCR_s\left[\frac{3}{2} + \frac{1}{2}(1 + g_m R_d)\right]}$$

Thus $f_P = \dfrac{1}{2\pi C R_s\left[\frac{3}{2} + \frac{1}{2}(1+g_m R_d)\right]}$

$$= \frac{1}{2\pi \times 10^{-12} \times 10^{6}\left[\frac{3}{2} + \frac{1}{2}(1+7.07)\right]}$$

$$= \underline{\underline{28.8\ kHz}}$$

Finally we can easily show ~~the~~ using the above analysis that the input impedance has the following equivalent circuit:



11.33 $\beta_0 = 100$; $r_x = 100\,\Omega$; $I_E = 1\,mA$; $r_e = 25\,\Omega$; $g_m = 40\,mA/V$.

$r_\pi = 2.5\,k\Omega$; $C_\pi + C_\mu = \dfrac{40 \times 10^{-3}}{2\pi \times 400 \times 10^{6}} = 15.9\,pF$; $C_\mu = 2\,pF$;

$C_\pi = 13.9\,pF$.

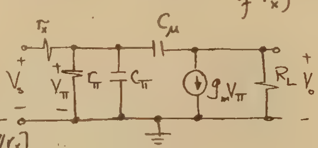
With $R_s = 0$

$A_M \simeq -g_m R_L = \underline{-40\ V/V}$ (Neglecting the effect of r_x)

$f_L \simeq \dfrac{1}{2\pi \times 10 \times 10^{-6} \times 25} = \underline{636.6\ Hz}$ (Neglecting the effect of r_x)

High-frequency response:

Pole at input,



$$f_{P1} = \frac{1}{2\pi[C_\pi + C_\mu(1+g_m R_L)][r_\pi // r_x]}$$

$$= \frac{1}{2\pi[13.9 + 2\times 41][2.5//0.1]\times 10^{-9}} = 17.30\,MHz$$

Pole at output:

$$f_{P2} \simeq \frac{1}{2\pi C_\mu R_L} = \frac{1}{2\pi \times 2 \times 1 \times 10^{-9}} \simeq 80\ MHz.$$

Thus,

$$f_H \simeq \left(\frac{1}{f_{P1}} + \frac{1}{f_{P2}}\right)^{-1} = \underline{\underline{14.2\ MHz}}$$

With $R_s = 1\,k\Omega$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_s} \cdot g_m R_L = -\frac{2.5}{3.6} \times 40 = -27.8\ V/V.$$

$$f_L = \frac{1}{2\pi \times 10 \times 10^{-6}\left(r_e + \frac{R_s + r_x}{\beta_0 + 1}\right)} = \underline{\underline{443.4\ Hz}}$$

The high-frequency response will be dominated by the pole at the input,

$$f_H \simeq f_{P1} = \frac{1}{2\pi[C_\pi + C_\mu(1+g_m R_L)][r_\pi // (r_x + R_s)]}$$

$$= \underline{2.17\ MHz}$$

11.34 Same parameters as in Problem 11.33.

With $R_s = 0$:

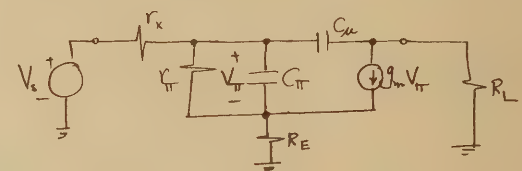
$$A_M \simeq \frac{-1\,k\Omega}{r_e + R_E} = -\frac{1\,k\Omega}{50\,\Omega} \quad (\text{neglecting } r_x)$$

$$= \underline{\underline{-20\ V/V}}$$

$$f_L \simeq \frac{1}{2\pi \times 10 \times 10^{-6}[R_E + r_e]} \quad (\text{neglecting } r_x)$$

$$= \frac{1}{2\pi \times 10^{-5} \times 50} = \underline{\underline{318.3\ Hz}}$$

High-frequency response:



R_π is given by Eqn. (11.78):

$$R_\pi = r_\pi // \frac{r_x + R_E}{1 + g_m R_E} = 2.5 // \frac{0.125}{1 + 40 \times 0.025} = 60.9\,\Omega$$

R_μ is given by Eqn. (11.79):

$$R_\mu = R_C + \frac{1 + \frac{R_E}{r_e} + g_m R_C}{\frac{1}{r_\pi} + \left(\frac{1}{r_x}\right)\left(1 + \frac{R_E}{r_e}\right)}$$

$$= 1 + \frac{1 + \frac{25}{25} + 40 \times 1}{\frac{1}{2.5} + \frac{1}{0.1}\left(1 + \frac{25}{25}\right)} = 3.06\,k\Omega$$

$$\tau = C_\pi R_\pi + C_\mu R_\mu = 13.9 \times 10^{-12} \times 60.9 + 2 \times 10^{-12} \times 3.06 \times 10^{3}$$

$$= 6.97\ ns$$

$$f_H \simeq \frac{1}{2\pi\tau} = \frac{1}{2\pi \times 6.97 \times 10^{-9}} = \underline{\underline{22.8\ MHz}}$$

With $R_s = 1\,k\Omega$

$$A_M \simeq \frac{(\beta_0+1)(r_e + R_E)}{(\beta_0+1)(r_e + R_E) + r_x + R_s}\left(-\frac{1000}{r_e + R_E}\right)$$

$$= \underline{-16.4\ V/V}$$

$$f_L = \frac{1}{2\pi \times 10 \times 10^{-6}\left[R_E + r_e + \frac{R_s + r_x}{\beta_0 + 1}\right]}$$

$$= \underline{261.4\ Hz}$$

$$R_\pi = r_\pi // \frac{R_s' + R_E}{1 + g_m R_E} = 2.5 // \frac{1.1 + 0.025}{1 + 40 \times 0.025} = 0.46\,k\Omega$$

$$R_\mu = R_C + \frac{1+\frac{R_E}{r_e}+g_mR_C}{\frac{1}{r_\pi}+\left(\frac{1}{R_s+r_x}\right)\left(1+\frac{R_E}{r_e}\right)}$$

$$= 1 + \frac{1+\frac{25}{25}+40\times 1}{\frac{1}{2.5}+\frac{1}{1.1}\left(1+\frac{25}{25}\right)} = 19.93\ k\Omega$$

$$\tau = C_\pi R_\pi + C_\mu R_\mu = 13.9\times 0.46+2\times 19.93 \text{ ns}$$

$$= 46.25 \text{ ns}$$

$$f_H \simeq \frac{1}{2\pi\tau} = \frac{1}{2\pi\times 46.25\times 10^{-9}} = \underline{\underline{3.44 \text{ MHz}}}$$

$\boxed{11.35}$ Same parameters as in Problems 11.33 and 11.34.

With $R_s = 0$

A_M is the same as in Problem 11.34; thus $A_M = \underline{\underline{-20V/V}}$

f_L is the same as in Problem 11.34; thus $f_L = \underline{\underline{318.3 \text{ Hz}}}$

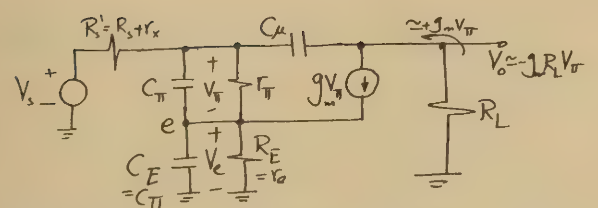
With $R_s = 1\ k\Omega$

A_M is the same as in Problem 11.34; thus $A_M = -16.4$ V/V

f_L is the same as in Problem 11.34; thus $f_L = 261.4$ Hz.

High-Frequency Response

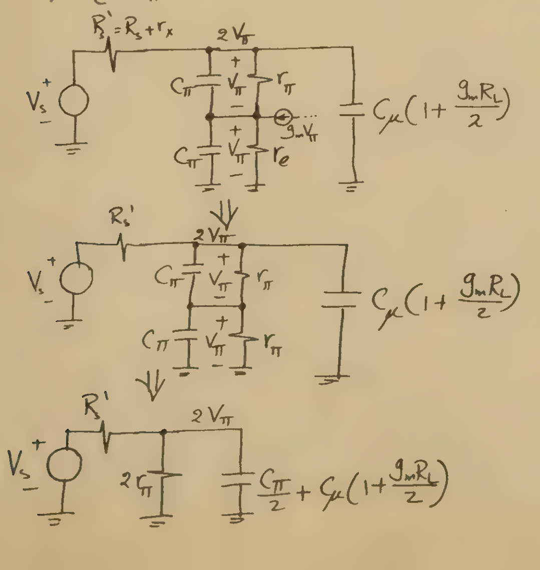
Note that $R_E = r_e$ and $C_E = C_\pi$. This important observations enables the simplifications depicted below.



Node equation at "e": $\left(\frac{1}{r_\pi}+g_m\right)V_\pi + sC_\pi V_\pi = \left(sC_E+\frac{1}{R_E}\right)V_e$

But $\frac{1}{r_\pi}+g_m = \frac{1}{r_e}$ and $R_E = r_e$ and $C_E = C_\pi$.

Thus, $V_e = V_\pi$ and we have



Since $V_o \simeq -g_mR_LV_\pi$, we conclude that

$$f_H \simeq \frac{1}{2\pi\left[\frac{C_\pi}{2}+C_\mu\left(1+\frac{g_mR_L}{2}\right)\right]\left[2r_\pi // R_s'\right]}$$

For the case $R_s = 0$, we have

$$f_H = \frac{1}{2\pi\left[\frac{13.9}{2}+2\left(1+\frac{40}{2}\right)\right]\left[5//0.1\right]} = \underline{\underline{33.2 \text{ MHz}}}$$

Note the improvement obtained over the case without the capacitor C_E (in Problem 11.34) where $f_H = 22.8$ MHz.

For the case $R_s = 1\ k\Omega$ we have:

$$f_H = \frac{1}{2\pi\left[\frac{13.9}{2}+2\left(1+\frac{40}{2}\right)\right]\left[5//1.1\right]} = \underline{\underline{3.61 \text{ MHz}}}$$

Comparison with the corresponding case without C_E (in Problem 11.34) indicates that although some improvement is obtained, it is not as dramatic as in the case when $R_s = 0$.

$\boxed{11.36}$ Parameters of Q_1 are as in Problems 11.33–11.35.

Parameters of Q_2 are: $\beta_0 = 100$; $g_{m2} = 200$ mA/V; $r_{e2} = 5\ \Omega$; $r_{\pi 2} = 500\ \Omega$; $r_{x2} = 100\ \Omega$; $C_{\mu 2} = 2$ pF

$$C_{\pi 2} + C_{\mu 2} = \frac{200\times 10^{-3}}{2\pi\times 400\times 10^{6}} = 80 \text{ pF}; \quad C_{\pi 2} = 78 \text{ pF}.$$

Midband Gain (Neglecting r_x)

With $R_s = 0$: $A_M = -g_{m1}\left(1k\Omega // r_{\pi 2}\right)\times g_{m2}\times 1\,k\Omega$

$$= 40\times\frac{1\times 0.5}{1.5}\times 200\times 1$$

$$= \underline{\underline{2666.7 \text{ V/V}}}.$$

With $R_s = 1\ k\Omega$: $A_M = \frac{r_{\pi 1}}{r_{\pi 1}+R_s}\times 2666.7$

$$= \underline{\underline{1905 \text{ V/V}}}$$

Low-Frequency Response (Neglecting r_x)

The bypass capacitor of Q_1 introduces a pole at,

$$f_{P1} = \frac{1}{2\pi\times 10\times 10^{-6}\left(r_{e1}+\frac{R_s}{\beta_0+1}\right)}$$

The bypass capacitor of Q_2 introduces a pole at

$$f_{P2} = \frac{1}{2\pi\times 10\times 10^{-6}\left(r_{e2}+\frac{1k\Omega}{\beta_0+1}\right)}$$

With $R_s = 0$: $f_{P1} = \frac{1}{2\pi \times 10^{-5} \times 25} = 636.6\ \text{Hz}$

$$f_{P2} = \frac{1}{2\pi \times 10^{-5}\left(5 + \frac{1000}{101}\right)} = 1{,}068\ \text{Hz}$$

The lower 3-dB frequency f_L can be found from

$$2 = \left(1 + \frac{f_{P1}^2}{f_L^2}\right)\left(1 + \frac{f_{P2}^2}{f_L^2}\right)$$

$$\Rightarrow f_L = \underline{\underline{1.34\ \text{kHz}}}.$$

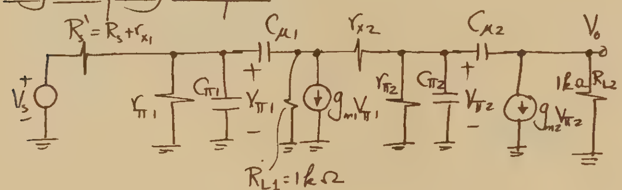
With $R_s = 1\,\text{k}\Omega$: $f_{P1} = \frac{1}{2\pi \times 10^{-5} \times \left(25 + \frac{1000}{101}\right)} = 456\ \text{Hz}$

$$f_{P2} = 1{,}068\ \text{Hz}$$

Thus, $2 = \left(1 + \frac{f_{P1}^2}{f_L^2}\right)\left(1 + \frac{f_{P2}^2}{f_L^2}\right)$

$$\Rightarrow f_L = \underline{\underline{1.23\ \text{kHz}}}$$

High-Frequency Response



Thus, for $R_s = 0$: $\tau = 480 \times 0.34 + 13.9 \times 0.096 + 2 \times 1.9$

$$= 168.3\ \text{ms}$$

$$f_H \simeq \frac{1}{2\pi\tau} = \underline{\underline{0.95\ \text{MHz}}}$$

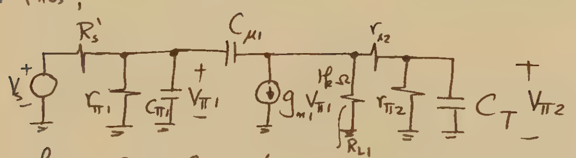
and for $R_s = 1\,\text{k}\Omega$: $\tau = 480 \times 0.34 + 13.9 \times 0.76 + 2 \times 12.6$

$$= 198.96\ \text{ms}$$

$$f_H \simeq \frac{1}{2\pi\tau} = \underline{\underline{0.8\ \text{MHz}}}$$

$V_o \simeq -g_{m2} R_{L2} V_{\pi 2} = -200 \times 1\, V_{\pi 2} = -200 V_{\pi 2}$

Thus,



where $C_T = C_{\pi 2} + (1 + 200)\, C_{\mu 2}$

$$= 78 + 201 \times 2 = 480\ \text{pF}$$

$$R_T = r_{\pi 2} \,//\, (r_{x2} + 1\,\text{k}\Omega)$$

$$= 0.5 \,//\, (0.1 + 1) = 0.34\ \text{k}\Omega$$

$$R_{\pi 1} = r_{\pi 1} \,//\, (R_s + r_x)$$

$$= 2.5 \,//\, (R_s + 0.1)$$

$= 96\ \Omega$ for $R_s = 0$, $0.76\ \text{k}\Omega$ for $R_s = 1\ \text{k}\Omega$

$$R_{\mu 1} = (r_{\pi 1} // R_s')\left\{1 + g_{m1}\left[R_{L1} \,//\, (r_{\pi 2} + r_{x2})\right]\right\} + \left[R_{L1} \,//\, (r_{\pi 2} + r_{x2})\right]$$

$$= 16 \times (2.5 // R_s') + 0.375$$

$= 1.9\ \text{k}\Omega$ for $R_s = 0$;

$12.6\ \text{k}\Omega$ for $R_s = 1\ \text{k}\Omega$.

CHAPTER 12—EXERCISES

12.1 (a) $\beta = \frac{R_1}{R_1 + R_2}$

(b) $A_f = 10 = \frac{A}{1 + A\beta} = \frac{10^4}{1 + 10^4 \beta}$

$1 + 10^4 \beta = 1000 \Rightarrow \beta = .0999 \Rightarrow \frac{R_2}{R_1} = \underline{\underline{9.01}}$

(c) Amount of feedback $= 20 \log_{10} (1 + A\beta) = \underline{\underline{60\ \text{dB}}}$.

(d) $V_s = 1\text{V}$; $V_o = A_f V_s = \underline{\underline{10\text{V}}}$; $V_f = \beta V_o = \underline{\underline{0.999\ \text{V}}}$;

$V_i = V_s - V_f = \underline{\underline{0.001\text{V}}}$.

(e) $A = 0.8 \times 10^4$ $\quad A_f = \frac{0.8 \times 10^4}{1 + 0.8 \times 10^4 \times 0.0999} = 9.9975$

Thus A_f decreases by about $\underline{\underline{0.02\ \%}}$.

12.2 $A_{f0} = \frac{A_0}{1 + A_0 \beta} = \frac{10^4}{1 + 10^4 \times \frac{R_1}{R_1 + R_2}} = \frac{10^4}{1 + 1000} = \underline{\underline{9.99\ \text{V/V}}}$

$f_{H_f} = f_H (1 + A_0 \beta) = 100 \times 1001 = \underline{\underline{100.1\ \text{kHz}}}$

12.3 Signal voltage at output $= V_s \times \frac{A_1 A_2}{1 + A_1 A_2 \beta}$

$$= 1 \times \frac{1 \times 100}{1 + 1 \times 100 \times 1} \simeq \underline{\underline{1\text{V}}}$$

Noise voltage at output $= V_n \times \frac{A_1}{1 + A_1 A_2 \beta} = 1 \times \frac{1}{1 + 100}$

$$\simeq \underline{\underline{0.01\text{V}}}$$

Thus the S/N ratio becomes 100 or 40 dB which is an improvement of $\underline{\underline{40\ \text{dB}}}$.

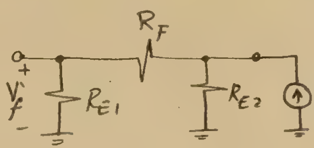
$$A \equiv \frac{I_o'}{V_i'} = \frac{\alpha_1\,[9 \,//\, r_{\pi 2}]}{r_{e1} + [R_{E1} \,//\, (R_F + R_{E2})]} \cdot g_{m2}\left[5 \,//\, (\beta_3+1)\left(r_{e3} + (R_{E2} \,//\, (R_F + R_{E1}))\right)\right] \cdot$$

$$\times \frac{1}{\{r_{e3} + [R_{E2} \,//\, (R_F + R_{E1})]\}}$$

Where $r_{e1} = 41.7\,\Omega$; $r_{e2} = 25\,\Omega$; $g_{m2} = 40\,\text{mA/V}$; $r_{\pi 2} = 2.5\,\text{k}\Omega$; $r_{e3} = 6.25\,\Omega$.

Thus $A \simeq \underline{20.8\,\text{A/V}}$

$$\beta \equiv \frac{V_f'}{I_o'} = \frac{R_{E2} \times R_{E1}}{R_{E2} + R_F + R_{E1}}$$



$$= \frac{100 \times 100}{100 + 640 + 100}$$

$\simeq \underline{12\,\Omega}$

$$A_f \equiv \frac{I_o}{V_s} = \frac{A}{1 + A\beta} = \frac{20.8}{1 + 20.8 \times 12} = \frac{20.8}{250.6}$$

$= \underline{0.083\,\text{A/V}}$

$$\frac{V_o}{V_s} = -\frac{I_o}{V_s} \times 600\,\Omega = -0.083 \times 600 = \underline{-49.8\,\text{V/V}}$$

$$R_i = (\beta_1 + 1)\{r_{e1} + [R_{E1} \,//\, (R_F + R_{E2})]\}$$

$= 13.1\,\text{k}\Omega$. $R_{if} = R_i(1 + A\beta) = \underline{3.28\,\text{M}\Omega}$

12.4 Since $1 + A\beta = 7$ then

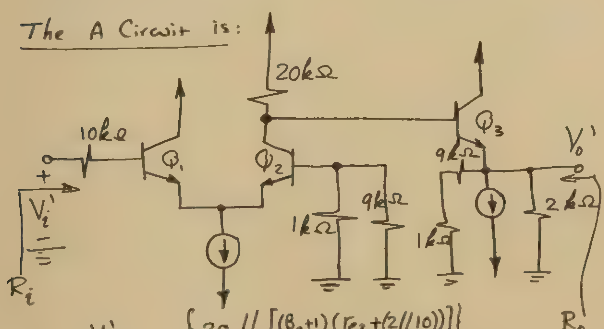
$f_{Hf} = 7 \times 1 = \underline{7\,\text{kHz}}$.

12.5 dc analysis: $I_{E1} = I_{E2} = 0.5\,\text{mA}$;

$V_{C2} \simeq 10.7 - 0.5 \times 20 = 0.7\,\text{V}$; $V_O = 0.7 - V_{BE3} = 0\,\text{V}$;

$I_{E3} = 5\,\text{mA}$. $r_{e1} = r_{e2} = 50\,\Omega$; $r_{e3} = 5\,\Omega$.

The A Circuit is:



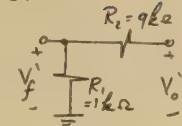
$$A = \frac{V_o'}{V_i'} \simeq \frac{\{20 \,//\, [(\beta_0+1)(r_{e3} + (2 \,//\, 10))]\}}{r_{e1} + r_{e2} + \frac{[10 + (1 \,//\, 9)]}{\beta_0 + 1}} \times \frac{(2 \,//\, 10)}{(2 \,//\, 10) + r_{e3}}$$

$$= \frac{(20 \,//\, 168.8)}{0.208} \times \frac{1.667}{1.667 + 0.005} = \underline{85.7\,\text{V/V}}$$

$$R_i = 10 + (1 \,//\, 9) + (\beta_0 + 1)(r_{e1} + r_{e2}) = 21\,\text{k}\Omega$$

$$R_o = 2 \,//\, 10 \,//\, \left[r_{e3} + \frac{20}{\beta_0 + 1}\right] = 181\,\Omega$$

$$\beta = \frac{R_1}{R_1 + R_2} = \underline{0.1\,\text{V/V}}$$



$$A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{85.7}{1 + 85.7 \times 0.1} = \underline{8.96\,\text{V/V}}.$$

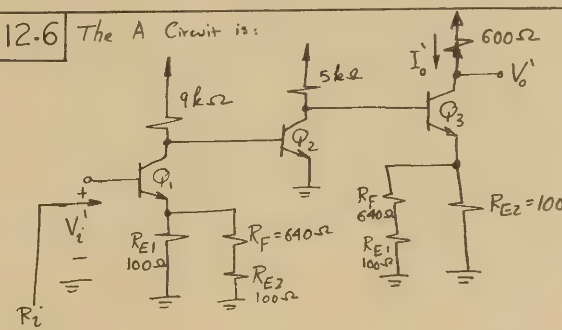
$$R_{if} = R_i(1 + A\beta) = 21 \times 9.57 = 201\,\text{k}\Omega$$

$$R_{if}' = R_{if} - R_s = 201 - 10 = \underline{191\,\text{k}\Omega}$$

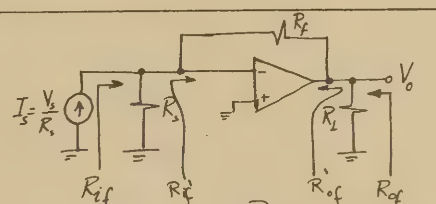
$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{181}{9.57} = 18.9\,\Omega$$

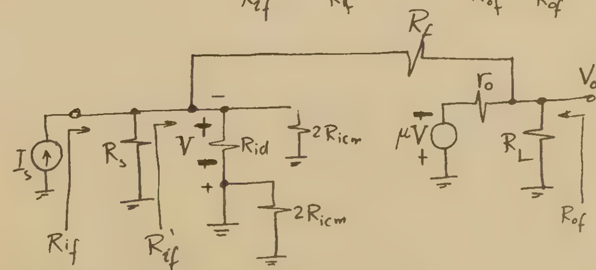
$$\frac{1}{R_{of}} = \frac{1}{R_{of}'} + \frac{1}{R_L} \Rightarrow R_{of}' = \underline{19.1\,\Omega}$$

12.6 The A Circuit is:

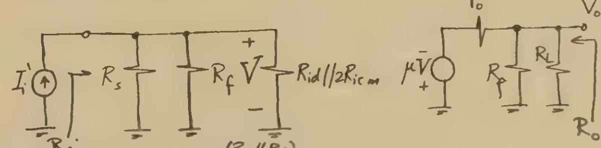


12.7 $A_f \equiv \frac{V_o}{I_s}$





The A circuit is



$$A \equiv \frac{V_o'}{I_i'} = \frac{-\mu V \frac{(R_L // R_f)}{(R_L // R_f) + r_o}}{I_i'} = -\mu\,(R_s // R_f // R_{id} // 2R_{icm}) \times \frac{(R_L // R_f)}{(R_L // R_f) + r_o}$$

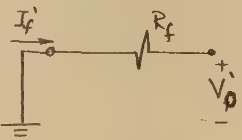
$$= -10^4\,(1 // 1000 // 100 // 20{,}000) \times \frac{(2 // 1000)}{(2 // 1000) + 1} = -6589\,\text{k}\Omega$$

$R_i = R_s // R_f // R_{id} // 2R_{icm} = 989.1\ \Omega$

$R_o = R_L // R_f // r_o = 2 // 1{,}000 // 1 = 666.7\ \Omega$

$\beta \equiv \frac{I_f'}{V_o'} = -\frac{1}{R_f}$

$= -10^{-6}\ \mho$



$1 + A\beta = 1 + 6589 \times 10^{3} \times 10^{-6}$

$= 7.589$

$A_f = \frac{V_o}{I_s} = \frac{A}{1+A\beta} = \frac{-6589}{7.589} \simeq -870\ k\Omega$

But $I_s = \frac{V_s}{R_s} = \frac{V_s}{1 k\Omega}$,

thus $\frac{V_o}{V_s} = \underline{\underline{-870\ V/V}}$

$R_{if} = \frac{R_i}{1+A\beta} = \frac{989.1}{7.589} = 130.3\ \Omega$

$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_{if}'} \Rightarrow R_{if}' \simeq \underline{\underline{150\ \Omega}}$

$R_{of} = \frac{R_o}{1+A\beta} = \frac{666.7}{7.589} = 87.85\ \Omega$

$\frac{1}{R_{of}} = \frac{1}{R_{of}'} + \frac{1}{R_L} \Rightarrow R_{of}' = \underline{\underline{92\ \Omega}}$

Node equation at X:

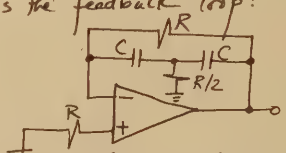
$$\frac{V_i - \frac{V_i}{3} + \frac{V_o - V_i/3}{sCR}}{R/2} + \frac{V_o - V_i/3}{R} + sC\left[V_o - \frac{V_i}{3} + \frac{V_o - V_i/3}{sCR}\right]$$

$$V_i\left(\frac{4}{3} - \frac{2}{3\,sCR} - \frac{1}{3} - \frac{sCR}{3} - \frac{1}{3}\right) = V_o\left(\frac{2}{sCR} + 1 + sCR + 1\right)$$

$$\frac{V_o}{V_i} = \frac{-\left[\frac{sCR}{3} - \frac{2}{3} + \frac{2}{3\,sCR}\right]}{sCR + 2 + \frac{2}{sCR}}$$

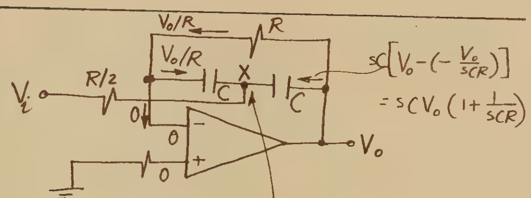
$$= -\frac{1}{3}\ \frac{s^2 - s\frac{2}{CR} + \frac{2}{(CR)^2}}{s^2 + s\frac{2}{CR} + \frac{2}{(CR)^2}}$$

Thus the poles of the second circuit are the roots of the polynomial $\left[s^2 + s\frac{2}{CR} + \frac{2}{(CR)^2}\right]$. We thus conclude that the two circuits have the same poles (or natural modes). This can be easily verified by short-circuiting the input voltage source in both circuits. The result in both case is the feedback loop:



Two circuits that have the same feedback loop have identical poles.

12.8



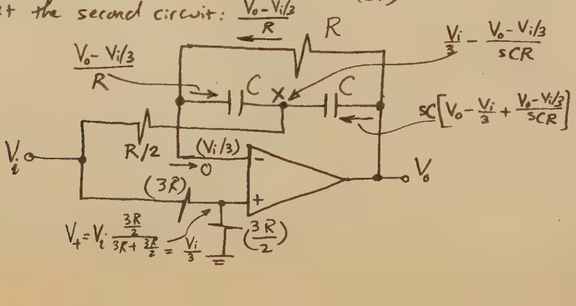
Node equation at X:

$$\frac{V_i - \left(-\frac{V_o}{sCR}\right)}{R/2} + \frac{V_o}{R} + sCV_o\left(1 + \frac{1}{sCR}\right) = 0$$

$$\frac{2}{R}V_i + V_o\frac{1}{R}\left(\frac{2}{sCR} + 1 + sCR + 1\right) = 0$$

$$\frac{V_o}{V_i} = \frac{-2}{sCR + 2 + \frac{2}{sCR}} = \frac{-2\frac{s}{CR}}{s^2 + 2\frac{s}{CR} + \frac{2}{(CR)^2}}$$

Thus the poles of the first circuit are the roots of the polynomial $\left[s^2 + s\frac{2}{CR} + \frac{2}{(CR)^2}\right]$. Consider next the second circuit:



12.9 $A(j\omega) = \left(\frac{10}{1 + j\omega/10^4}\right)^3$

$\phi = -3\ \tan^{-1}\left(\frac{\omega}{10^4}\right)$

A ω_{180}, $\phi = 180$; thus $\tan^{-1}\frac{\omega_{180}}{10^4} = 60°$

$\frac{\omega_{180}}{10^4} = \sqrt{3} \Rightarrow \omega_{180} = \underline{\sqrt{3} \times 10^4\ rad/s}$

The feedback amplifier will be stable if at ω_{180}, $|A\beta| < 1$. At the boundary $\beta = \beta_{cr}$,

$|A(j\omega_{180})|\ \beta_{cr} = 1$

$\beta_{cr} = 1/|A(j\omega_{180})|$

$= \left[\sqrt{1 + (\sqrt{3})^2}/10\right]^3 = \underline{\underline{0.008}}$

12.10 The closed-loop poles are obtained from

$1 + A(s)\,\beta = 0$

$1 + \frac{10^3\beta}{(1 + s/10^4)^3} = 0$

$\left(1 + \frac{s}{10^4}\right)^3 + 10^3\beta = 0$

$\frac{s^3}{10^{12}} + s^2\frac{3}{10^8} + s\frac{3}{10^4} + (1 + 1000\beta) = 0$

To simplify matters we shall normalize the complex frequency variable s by replacing $\frac{s}{10^4}$ by s_n; thus

$s_n^3 + 3s_n^2 + 3s_n + (1 + 1{,}000\beta) = 0$

The roots of this cubic equation are

$-(1+10\beta^{1/3})\ ,\ -1+5\beta^{1/3} \pm j\,\beta^{1/3}\,5\sqrt{3}$

It is easy to verify that these roots follow the locus shown in Fig E 12·10 (with the point at which all three roots coincide being at $-1+j0$).

The amplifier becomes unstable when the value of β is increased so that the complex conjugate pair of roots cross the $j\omega$-axis into the right-half of the s-plane. This happens at $\beta = \beta_{cr}$ where

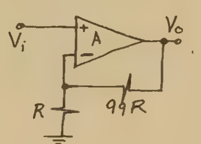
$10\,\beta_{cr}^{1/3} = 1/\cos 60^\circ = 2$

Thus, $\beta_{cr} = \underline{0.008}$.

12.11 $\beta = 0.01$

$A = \dfrac{A_0}{1 + j\omega/\omega_P} = \dfrac{A_0}{1 + jf/f_P}$

$= \dfrac{10^5}{1 + jf/10}$



$|A\beta| = \dfrac{10^5 \times 0.01}{\sqrt{1 + f^2/10^2}} = 1$ at

$1 + f^2/10^2 = 10^6 \Rightarrow f \simeq \underline{10^4\ \text{Hz}}$

At this frequency,

$\phi = -\tan^{-1}(10^4/10) \simeq -90^\circ$. Thus the phase margin is $\underline{90^\circ}$.

12.12 From page 577 of the Text book we have $|A_f(j\omega_1)| = \dfrac{1/\beta}{|1 + e^{-j\theta}|}$

where $1/\beta \simeq$ low-frequency gain, and

$\theta = 180 -$ phase margin.

For a phase margin of 30°, $\theta = 150^\circ$ and

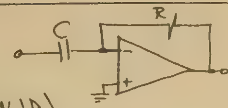
$|A_f(j\omega_1)| \,/\, (1/\beta) = \underline{1.93}$

For a phase margin of 60°, $\theta = 120^\circ$ and

$|A_f(j\omega_1)| / (1/\beta) = \underline{1}$

For a phase margin of 90°, $\theta = 90^\circ$ and $|A_f(j\omega_1)|/(1/\beta) = \underline{0.707}$

12.13 $\beta = \dfrac{1/sC}{R + \frac{1}{sC}} = \dfrac{1}{1 + sCR}$



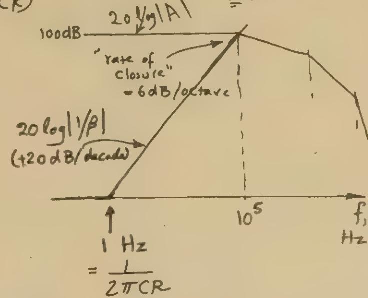
$|1/\beta| = \sqrt{1 + (\omega CR)^2}$

$\dfrac{1}{2\pi CR} \leq 1$ Hz

$CR \geq \dfrac{1}{2\pi}$ s

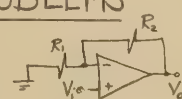
Thus,

$CR \geq \underline{0.159\ \text{s}}$



CHAPTER 12—PROBLEMS

12.1 $A_f = \dfrac{A}{1 + A\beta} = \dfrac{100}{1 + 100\,\frac{R_1}{R_1+R_2}}$



$= \dfrac{100}{1 + 100 \times 0.1} = \underline{9.09\ V/V}$.

To obtain $A_f = 10$ we must select $\frac{R_2}{R_1}$ so that

$10 = \dfrac{100}{1 + 100\,\frac{R_1}{R_1+R_2}}$

Thus, $\dfrac{R_1}{R_1+R_2} = 0.09 \Rightarrow \dfrac{R_2}{R_1} = \underline{10.11}$

In this case the loop gain is $\underline{9}$ (19.1dB) and the amount of feedback is $\underline{10}$ (i.e. 20dB)

12.2 $A_{f0} = \dfrac{100}{1 + 100 \times 0.1} = \underline{9.09\ V/V}$

$f_{Hf} = 10 \times (1 + A_0\beta) = 10 \times 11 = \underline{110\ kHz}$

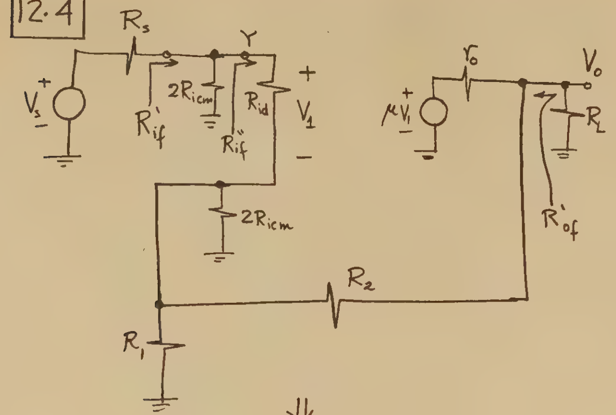
12.3 Originally $V_s = 1V$ and $V_n = V$; thus $S/N = 1$

After adding the preamplifier, the signal at the output becomes $1 \times \dfrac{A_1A_2}{1 + A_1A_2\beta} = 1 \times \dfrac{10}{1 + 10 \times 1} \simeq 1V$

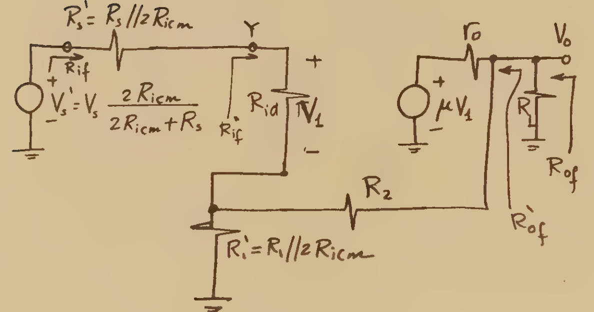
while the noise becomes $1 \times \dfrac{A_2}{1 + A_1A_2\beta} = 1 \times \dfrac{1}{1 + 10 \times 1} \simeq 0.01V$

Thus the $\frac{S}{N}$ ratio improves by a factor of 10 (or 20 dB).

12.4



$\Downarrow$



$\frac{V_o}{V_s} \simeq \frac{V_o}{V_s'} = \underline{\underline{85.5 \text{ V/V}}}$.

$R_{if} = R_i(1+A\beta) = 300 \times 6.5 = 1.95\,\text{M}\Omega$

$R_{if}'' = R_{if} - R_s' = 1.95 - 0.1 = 1.85\,\text{M}\Omega$

$R_{if}' = 2R_{icm} // R_{if}'' = 20 // 1.85 = \underline{1.69}\,\text{M}\Omega$

$R_{of} = \frac{R_o}{1+A\beta} = \frac{1.67}{6.5} = 257\,\Omega$

$R_{of}' = \left(\frac{1}{R_{of}} - \frac{1}{R_L}\right)^{-1} \simeq \underline{\underline{295\,\Omega}}$

12.5 $f_{Hf} = f_H(1+A_0\beta)$

$= 1 \times 6.5 = \underline{\underline{6.5 \text{ kHz}}}$

12.6 Refer to Fig. E12.5 with R_2 replaced by a 4 kΩ resistor.

dc analysis: $I_{E1} = I_{E2} = 0.5$ mA ; $V_{C2} \simeq 10.7 - 0.5 \times 20 = +0.7$ V ; $V_O = 0.7 - V_{BE3} = 0$ V ; $I_{E3} = 5$ mA.

$r_{e1} = r_{e2} = 50\,\Omega$; $r_{e3} = 5\,\Omega$.

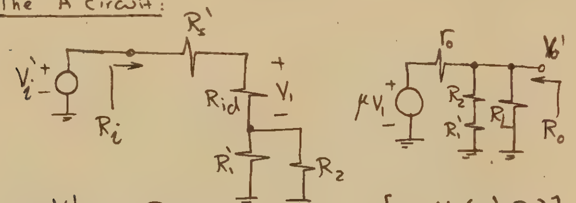
$\mu = 10^4$; $R_{id} = 100$ kΩ; $R_{icm} = 10$ MΩ ; $r_o = 10$ kΩ;

$R_L = 2$ kΩ ; $R_1 = 100$ kΩ; $R_2 = 10$ MΩ ; $R_s = 100$ kΩ;

$R_s' = R_s // 2R_{icm} = 100 // 20{,}000 \simeq 100$ kΩ ;

$R_1' = R_1 // 2R_{icm} \simeq 100$ kΩ ; $V_s' = V_s \frac{20{,}000}{20{,}000+100} \simeq V_s$.

The A circuit:



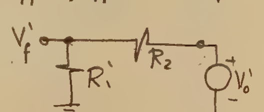
$A \equiv \frac{V_o'}{V_i'} = \frac{R_{id}}{R_s' + R_{id} + (R_1' // R_2)} \cdot \mu \frac{[R_L // (R_1'+R_2)]}{[R_L // (R_1'+R_2)] + r_o}$

$\simeq \frac{100}{100+100+100} \cdot 10^4 \cdot \frac{2}{2+10} = \underline{\underline{555.6 \text{ V/V}}}$

$R_i = R_s' + R_{id} + (R_1' // R_2) \simeq 300$ kΩ

$R_o = R_L // (R_1'+R_2) // r_o = 2 // 10{,}100 // 10 \simeq 1.67$ kΩ

$\beta = \frac{R_1'}{R_1' + R_2} = \frac{100}{100 + 10{,}000}$

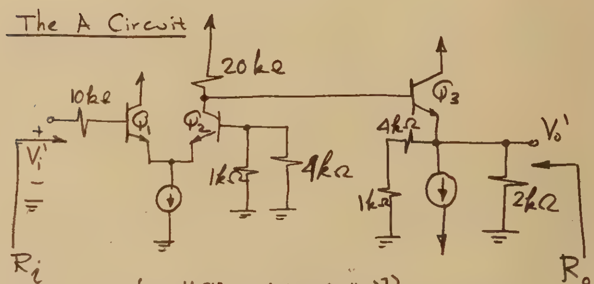


$= 0.0099$

$1 + A\beta = 1 + 555.6 \times 0.0099 = \underline{6.5}$

$A_f \equiv \frac{V_o}{V_s} = \frac{555.6}{6.5} = 85.5$ V/V

The A Circuit



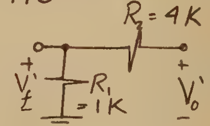
$A \equiv \frac{V_o'}{V_i'} \simeq \frac{\{20 // [(\beta_0+1)(r_{e3} + (2//5))]\}}{r_{e1} + r_{e2} + \frac{[10 + (1//4)]}{\beta_0+1}} \times \frac{(2//5)}{(2//5) + r_{e3}}$

$= \underline{\underline{84.6 \text{ V/V}}}$.

$R_i = 10 + r_{\pi 1} + r_{\pi 2} + (1//4) = 10 + 5 + 5 + 0.8 = 20.8$ kΩ

$R_o = 2 // 5 // \left\{ r_{e3} + \frac{20}{\beta_0+1} \right\} = 178\,\Omega$

$\beta = \frac{1}{1+4} = \underline{\underline{0.2 \text{ V/V}}}$



$A_f \equiv \frac{V_o}{V_s} = \frac{A}{1+A\beta}$

$= \frac{84.6}{1 + 84.6 \times 0.2} = \frac{84.6}{17.92} = \underline{\underline{4.72 \text{ V/V}}}$

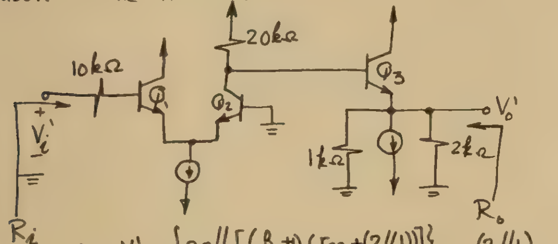
$R_{if} = R_i(1+A\beta) = 20.8 \times 17.92 = 372.7$ kΩ

$R_{if}' = R_{if} - R_s = 372.7 - 10 = \underline{\underline{362.7 \text{ k}\Omega}}$

$$R_{of} = \frac{R_o}{1+A\beta} = \frac{178}{17.92} = 9.93\,\Omega$$

$$R'_{of} = \left(\frac{1}{R_{of}} - \frac{1}{R_L}\right)^{-1} \simeq \underline{\underline{10\,\Omega}}.$$

12.7 Refer to Fig. E12.5 and assume that R_2 is replaced by a short circuit. The dc analysis is the same as in Problem 12.6 above. The A circuit is



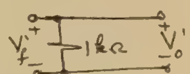
$$A \equiv \frac{V'_o}{V'_i} \simeq \frac{\{20 /\!/ [(\beta_0+1)(r_{e3} + (2/\!/1))]\}}{r_{e1} + r_{e2} + \frac{10}{\beta_0+1}} \cdot \frac{(2/\!/1)}{(2/\!/1)+r_{e3}}$$

$$= \underline{\underline{77\ V/V}}$$

$$R_i = 10 + 2r_\pi = 20\ k\Omega$$

$$R_o = 2/\!/1/\!/\left(r_{e3} + \frac{20}{101}\right) = 156\,\Omega$$

$\beta = 1$



$1 + A\beta = 78$

$$A_f \equiv \frac{V_o}{V_s} = \frac{77}{78} = \underline{\underline{0.987\ V/V}}$$

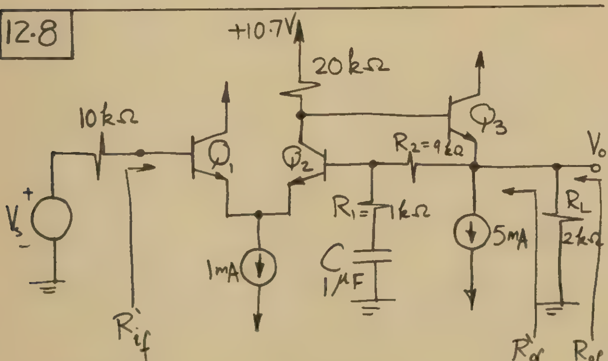
$$R_{if} = R_i(1+A\beta) = 20 \times 78 = 1.560\ M\Omega$$

$$R'_{if} = R_{if} - R_s = 1.560 - 0.01 = \underline{\underline{1.55\ M\Omega}}$$

$$R_{of} = \frac{R_o}{1+A\beta} = \frac{156}{78} = 2\,\Omega$$

$$R'_{of} \simeq R_o = \underline{\underline{2\,\Omega}}$$

12.8

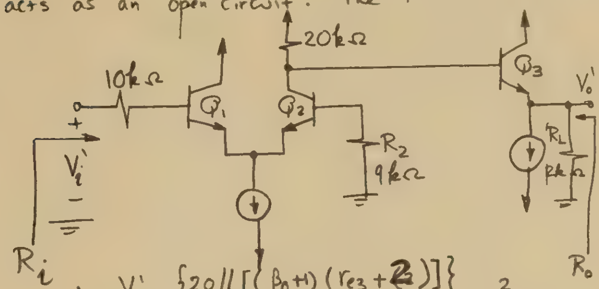


dc conditions are as in Exercise 12.5 and Problems 12.6 & 12.7.

(a) At relatively high frequencies where C acts as a short circuit the ac performance of the circuit becomes identical to that found in Exercise 12.5. Thus we have:

$A = \underline{\underline{85.7\ V/V}}$; $\beta = \underline{\underline{0.1\ V/V}}$; $A_f = \underline{\underline{8.96\ V/V}}$;
$R'_{if} = \underline{\underline{191\ k\Omega}}$; and $R'_{of} = \underline{\underline{19.1\,\Omega}}$.

(b) At relatively low frequencies the capacitor acts as an open circuit. The A circuit becomes



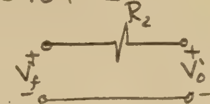
$$A \equiv \frac{V'_o}{V'_i} = \frac{\{20 /\!/ [(\beta_0+1)(r_{e3} + 2)]\}}{r_{e1} + r_{e2} + \frac{19}{\beta_0+1}} \cdot \frac{2}{2 + r_{e3}}$$

$$= \underline{\underline{63\ V/V}}$$

$$R_i = 19 + 2r_\pi = 29\ k\Omega$$

$$R_o = 2 /\!/ \left(r_{e3} + \frac{20}{\beta_0+1}\right) = 184\,\Omega$$

$\beta = \underline{\underline{1}}$



$1 + A\beta = 64$

$$A_f = \frac{63}{64} = \underline{\underline{0.984}}$$

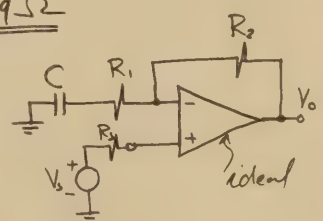
$$R_{if} = R_i(1+A\beta) = 29 \times 64 = 1.86\ M\Omega$$

$$R'_{if} = R_{if} - R_s = 1.86 - 0.01 = \underline{\underline{1.85\ M\Omega}}$$

$$R_{of} = \frac{R_o}{1+A\beta} = \frac{184}{64} = 2.9\,\Omega$$

$$R'_{of} \simeq R_{of} = \underline{\underline{2.9\,\Omega}}$$

By considering the amplifier to approximate an ideal op amp we can see from the circuit shown that



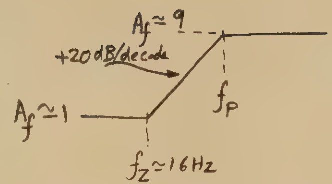
the gain $\frac{V_o}{V_s}$ will have a zero when $1 + \frac{R_2}{R_1 + \frac{1}{sC}} = 0$, i.e. at the value of s that give $R_1 + \frac{1}{sC} + R_2 = 0$,

$$s_Z = -\frac{1}{C(R_1+R_2)}$$

For our case $s_Z = -\frac{1}{10^{-6} \times 10 \times 10^3} = -100$ rad/s

i.e. the zero frequency is $\simeq 16$ Hz.

A sketch of the Bode plot for the closed loop gain is shown. If we assume $A_f \simeq 10$ then the pole frequency must be a decade higher than the zero frequency; thus



$$f_P \simeq \underline{160 \text{ Hz}}$$

$\boxed{12.9}$ Series - Series feedback.

$I_{C1} = 0.6$ mA ; $I_{C2} = 1$mA ;
$I_{C3} = 4$ mA ; $h_{fe} = 100$;
$g_{m1} = 24$ mA/V ; $r_{e1} = 41.7\,\Omega$;
$g_{m2} = 40$ mA/V ; $r_{e2} = 25\,\Omega$;
$g_{m3} = 160$ mA/V ; $r_{e3} = 6.25\,\Omega$

$A \equiv \dfrac{I_o}{V_s}$

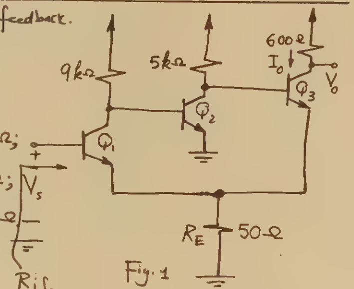


Fig. 1

The A circuit is shown in Fig. 2

$$A \equiv \frac{I_o'}{V_i'} = \frac{(9//r_{\pi 2})}{r_{e1}+R_E} \cdot g_{m2} \frac{5\,(\beta_0+1)}{5+(\beta_0+1)(r_{e3}+R_E)}$$

$$= \frac{(9//2.5)}{0.0917} \cdot 40 \cdot \frac{5 \times 101}{5+101 \times 0.05625}$$

$$= \underline{40.3 \text{ A/V}}$$

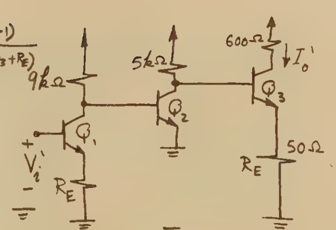


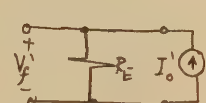
Fig. 2

$$R_i = (\beta_0+1)(r_{e1}+R_E)$$

$$= 101 \times 0.0917$$

$$= 9.26 \text{ k}\Omega$$

$$\beta \equiv \frac{V_f'}{I_o'} = R_E = \underline{\underline{50\,\Omega}}$$



$$1+A\beta = 1+40.3 \times 50 = 2016$$

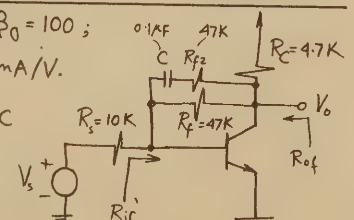
$$A_f \equiv \frac{I_o}{V_s} = \frac{40.3}{2016} = \underline{\underline{0.02 \text{ A/V}}}$$

$$\frac{V_o}{V_s} = -\frac{I_o}{V_s} \times 600 = -0.02 \times 600 = \underline{\underline{-12 \text{ V/V}}}$$

$$R_{if} = R_i\,(1+A\beta) = 9.26 \times 2016 = \underline{18.7 \text{ M}\Omega}$$

$\boxed{12.10}$ $I_C = 1.5$ mA ; $\beta_0 = 100$; $r_e = 16.7\,\Omega$; $g_m = 60$ mA/V.

(a) At low frequencies, C acts as an open circuit and the circuit behaves as the one analyzed in Example 12.3. Thus we have:



$$\frac{V_o}{V_s} = \underline{-4.16 \text{ V/V}} \; ; \; R_{if}' = \underline{\underline{165\,\Omega}} \text{ and } R_{of} = \underline{\underline{495\,\Omega}}.$$

(b) At high frequencies when the capacitor effectively acts as a short circuit, the circuit becomes like that analyzed in Example 12.3 except that the feedback resistance is halved, i.e. (47/2) kΩ. Thus the A circuit is that in Fig. 12.21d with $R_f = \frac{47}{2}$ kΩ,

$$A \equiv \frac{V_o'}{I_i'} = -\frac{g_m (R_f // R_C) V_\pi}{I_i}$$

$$= -g_m (R_f // R_C)(R_s // R_f // r_\pi)$$

$$= -60\left(\tfrac{47}{2} // 4.7\right) \cdot \left(10 // \tfrac{47}{2} // 1.67\right)$$

$$= -317 \text{ k}\Omega$$

$$R_i = (R_s // r_\pi // R_f) = 1.35 \text{ k}\Omega \; ; \; R_o = R_C // R_f = 3.92 \text{ k}\Omega$$

The β circuit is that shown in Fig. 12.21(e),

$$\beta \equiv \frac{I_f'}{V_o'} = -\frac{1}{R_f} = -\frac{2}{47} \text{ mA/V}$$

Thus, $1+A\beta = 14.5$

$$A_f \equiv \frac{V_o}{I_s} = -\frac{317}{14.5} = -21.9 \text{ k}\Omega$$

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = -\frac{21.9}{10} = \underline{\underline{-2.19 \text{ V/V}}}$$

$$R_{if} = \frac{R_i}{1+A\beta} = \frac{1.35}{14.5} = 93.1\,\Omega$$

$$R_{if}' = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{\underline{94\,\Omega}}$$

$$R_{of} = \frac{R_o}{1+A\beta} = \frac{3920\,\Omega}{14.5} = \underline{\underline{270\,\Omega}}$$

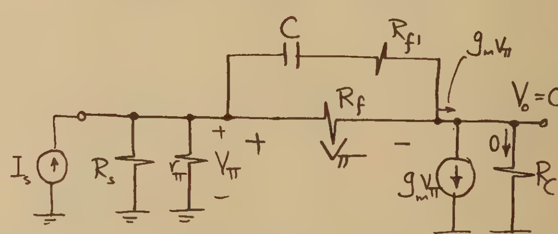


Fig. 2

Fig. 2 shows the equivalent circuit of the closed-loop amplifier prepared for finding the frequency s_Z of the real zero. Note that at $s = s_Z$, $V_o = 0$ V, thus the voltage across the feedback network is equal to V_π and the current through it is $g_m V_\pi$. We can write

$$g_m V_\pi = \left(\frac{1}{R_f} + \frac{1}{R_{f1} + (1/s_Z C)}\right) V_\pi$$

which leads to

$$R_{f1} + \frac{1}{s_Z C} = \frac{1}{g_m - \frac{1}{R_f}} \simeq \frac{1}{g_m}$$

$$\frac{1}{s_Z C} = \frac{1}{g_m} - R_{f1} \simeq -R_{f1}$$

$$s_Z \simeq -\frac{1}{R_{f1} C}$$

For our case $C = 0.1\ \mu F$ and $R_{f1} = 47\ k\Omega$; thus

$$f_Z = \frac{1}{2\pi \times 47 \times 10^3 \times 0.1 \times 10^{-6}} = \quad Hz \quad (212.8\ rad/s)$$

Fig. 3 shows a sketch of the closed-loop gain versus frequency. The closed-loop transfer function can be expressed as

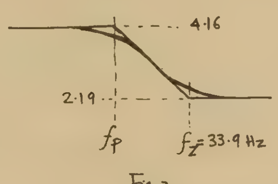


Fig. 3

$$A_f(s) = K \frac{s + \omega_Z}{s + \omega_P}$$

The value of K can be found from the value of the high-frequency gain; $K = 2.19$. As s approches zero, $A_f(s)$ approaches the low-frequency gain magnitude (2.19); thus

$$K \frac{\omega_Z}{\omega_P} = 4.16$$

$$\omega_P = \omega_Z \frac{K}{4.16} = \omega_Z \frac{2.19}{4.16} = 212.8 \times \frac{2.19}{4.16}$$

$$= \underline{\underline{112\ rad/s}} \quad \underline{\underline{(17.8\ Hz)}}$$

12.11

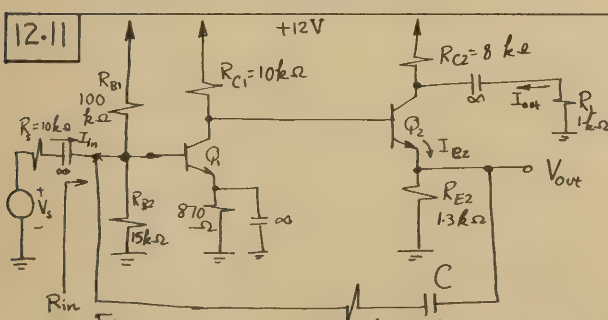
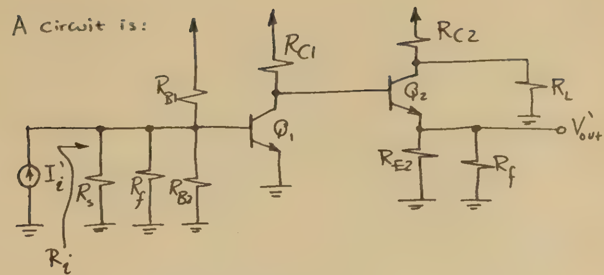


Fig. 1

-dc conditions are the same as in Example (12.4): $I_{C1} = 1\,mA$; $I_{C2} = 1\,mA$; $r_{e1} = r_{e2} = 25\,\Omega$;
$g_{m1} = g_{m2} = 40\ mA/V$; $r_{o1} = r_{o2} = 100\ k\Omega$;
$r_{\pi 1} = r_{\pi 2} = 2.5\ k\Omega$.

The feedback is of the shunt-shunt type and the

A circuit is:



$$R_i = R_s // R_f // R_{B1} // R_{B2} // r_{\pi 1}$$

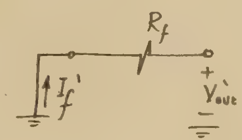
$$= 10 // 10 // 100 // 15 // 2.5 = 1.48\ k\Omega$$

$$A \equiv \frac{V_{out}'}{I_i'} = R_i \times -g_m \left\{ R_{C1} // r_{o1} // [(\beta_0 + 1)(r_{e2} + (R_{E2} // R_f))] \right\} \times \frac{(R_{E2} // R_f)}{(R_{E2} // R_f) + r_{e2}}$$

$$= -1.48 \times 40 \left\{ 10 // 100 // [101 \times (0.025 + (1.3 // 10))] \right\} \times \frac{(1.3 // 10)}{(1.3 // 10) + 0.025}$$

$$= -489.3\ k\Omega$$

$$\beta \equiv \frac{I_f'}{V_{out}'} = -\frac{1}{R_f} = -0.1\ mA/V$$



$$1 + A\beta = 49.93 \simeq 50$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1.48}{50} = 29.6\,\Omega$$

$$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s} \right)^{-1} = \underline{29.7\,\Omega}$$

$$A_f \equiv \frac{V_{out}}{I_s} = \frac{A}{1 + A\beta} = \frac{-489.3}{50} = -9.8\ k\Omega$$

Since $I_{in} = I_s \dfrac{R_s}{R_s + R_{in}} = I_s \dfrac{10}{10 + 0.0297} \simeq I_s$,

then $\dfrac{V_{out}}{I_{in}} \simeq \underline{\underline{-9.8\ k\Omega}}$

To calculate $\dfrac{I_{out}}{I_{in}}$ refer to Fig. 1

$$I_{E2} = \frac{V_{out}}{R_{E2}} + \frac{V_{out} - V_{b1}}{R_f}$$

$$= \frac{V_{out}}{1.3} + \frac{V_{out} - I_{in} R_{in}}{10}$$

$$= V_{out} \left(\frac{1}{1.3} + \frac{1}{10} \right) - I_{in} \times 0.0297$$

$$I_{out} = I_{C2} \frac{R_{C2}}{R_{C2} + R_L} \simeq I_{E2} \frac{R_{C2}}{R_{C2} + R_L}$$

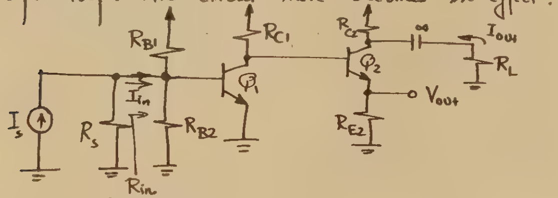
$$= \left[V_{out} \left(\frac{1}{1.3} + \frac{1}{10} \right) - 0.0297\, I_{in} \right] \frac{8}{8+1}$$

Thus, $I_{out} = \frac{8}{9} \times \left(\frac{1}{1.3} + \frac{1}{10} \right) \times (-9.8\, I_{in}) - \frac{8}{9} \times 0.0297\, I_{in}$

$$\frac{I_{out}}{I_{in}} = \underline{\underline{-7.6}}$$

Note that the results obtained here are very close to those found in Example 12-4; differences are due to the different approximations made.

$\boxed{12.12}$ At very low frequencies the capacitor acts as open circuit and the amplifier operates open-loop. The circuit then becomes in effect:



$R_{in} = R_s // R_{B1} // R_{B2} // r_{\pi 1} = 1.73\ k\Omega$

$R_{in} = 2.1\ k\Omega$

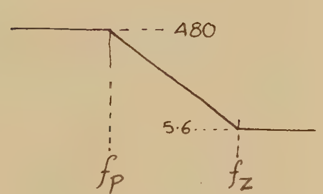
$$\frac{I_{out}}{I_{in}} = R_{in} \times -g_{m1} \times \left[R_{C1} // r_{o1} // (\beta_0+1)(r_{e2}+R_{E2})\right]\left(\frac{1}{r_{e2}+R_{E2}}\right)\times\left(\frac{R_{C2}}{R_{C2}+R_L}\right)$$

$$= 2.1 \times -40 \times \left[10 // 100 // 101 \times (0.025+1.3)\right]\left(\frac{1}{0.025+1.3}\right)\times\left(\frac{8}{8+1}\right)$$

$$= \underline{\underline{-480\ A/A}}$$

At high frequencies the capacitor acts as a short circuit and the circuit reduces to that analyzed in Problem 12.11 (and in Example 12.4) with $\frac{I_{out}}{I_{in}} = \underline{\underline{-5.6\ A/A}}$.

The frequency response of the amplifier will have the shape shown.



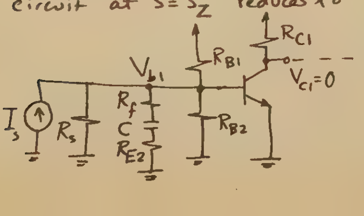
It can be descibed by

$$\frac{I_{out}}{I_{in}}(s) = -K\ \frac{s+\omega_Z}{s+\omega_P}$$

where K is the magnitude of high-frequency gain; that is, $K = 5.6$. The low-frequency gain is $K\frac{\omega_Z}{\omega_P}$, thus

$$5.6\ \frac{\omega_Z}{\omega_P} = 480 \Rightarrow \frac{\omega_Z}{\omega_P} = 8.57$$

To find the frequency of the zero, refer to Fig. 1 in the solution to Problem 12.11. I_{out} will be zero when I_{E2} is zero. This will happen when $V_{c1} = 0$ and $V_{b1} = 0$. Since $I_{E2} = 0$, the circuit at $s = s_Z$ reduces to



V_{b1} will be zero when the branch that includes the capacitor C behaves as a short circuit; that is

$$R_f + R_{E2} + \frac{1}{s_Z C} = 0$$

Thus $$s_Z = -\frac{1}{C(R_f+R_{E2})}$$

and $$\omega_Z = \frac{1}{C(R_f+R_{E2})} = \frac{1}{0.1\times10^{-6}\times 11.3\times10^{3}}$$

$$= 884.96\ \text{rad/s or } \underline{\underline{141\ Hz}}$$

We can now find the pole frequency as

$$\omega_P = \frac{\omega_Z}{8.57} = 103.3\ \text{rad/s} \quad \text{or } \underline{\underline{16.4\ Hz}}$$

$\boxed{12.13}$ Refer to the solution to Exercise 12.7.

$A = -658.9\,k\Omega$; $R_i = 989.1\ \Omega$; $R_o = 666.7\ \Omega$; $\beta = -10^{-6}\ ℧$.

Thus $1 + A\beta = 1.66$

$$A_f = -\frac{658.9}{1.66} = -397\ k\Omega$$

$$\frac{V_o}{V_s} = \underline{\underline{-397\ V/V}}$$

$$R_{if} = 989.1 \times 1.66 = 596\ \Omega$$

$$R'_{if} = \left(\frac{1}{596} - \frac{1}{1000}\right)^{-1} = \underline{1.5\ k\Omega}$$

$$R_{of} = 666.7/1.66 = 401.6\ \Omega \qquad R'_{of} = \underline{\underline{502.5\ \Omega}}$$

$\boxed{12.14}$ $-\phi = \tan^{-1}\left(\frac{\omega}{10^4}\right) + 2\tan^{-1}\left(\frac{\omega}{10^5}\right)$

Thus, $\underline{\omega_{180} \simeq 1.1\times10^5\ \text{rad/s}}$

$$|A(j\omega_{180})| = \frac{1000}{\sqrt{1+11^2}\ \left(\sqrt{1+1.1^2}\ \right)^2} = 40.966$$

For oscillations to start $|A(j\omega_{180})|\ \beta \geqslant 1$

Thus $$\beta_{cr} = \frac{1}{|A(j\omega_{180})|} = 0.0244$$

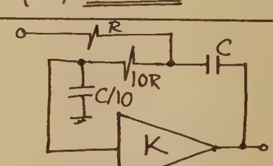
Critical amount of feedback $= 1 + A_0\,\beta_{cr}$

$$= 1 + 1000 \times 0.0244 = \underline{25.4} \text{ or } \underline{28\ dB}.$$

For the amount of feedback reduced from this critical value by 20 dB, i.e. to 8 dB, the maximum value of β ~~becomes~~ is found from

$$20\log(1 + A_0\,\beta_{max}) = 8 \Rightarrow \beta_{max} = \underline{1.5\times10^{-3}}$$

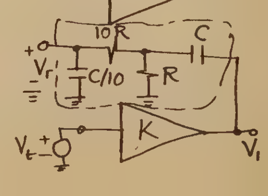
$\boxed{12.15}$ Characteristic equation is $1 + L(s) = 0$



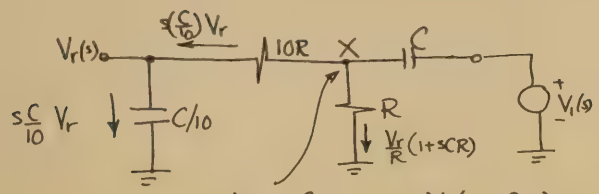
where $$L(s) = -\frac{V_r(s)}{V_t(s)}$$

$$= -K\ \frac{V_r(s)}{V_1(s)}$$

where $\frac{V_r(s)}{V_1(s)}$ is the transfer function of the



RC network inside the dotted box.



$$V_r + s(\tfrac{C}{10})V_r \times 10R = V_r(1+sCR)$$

Node equation at X:

$$s(\tfrac{C}{10})V_r + \tfrac{V_r}{R}(1+sCR) + sC\left[V_r(1+sCR) - V_1\right] = 0$$

$$V_r\left[s\tfrac{C}{10} + \tfrac{1}{R} + sC + sC + s^2C^2R\right] = sC\,V_1$$

$$\frac{V_r}{V_1} = \frac{s\,(1/CR)}{s^2 + s\,(2.1/CR) + (1/CR)^2}$$

Thus the characteristic equation is

$$1 - \frac{s\,(K/CR)}{s^2 + s(2.1/CR) + (1/CR)^2} = 0$$

$$\Rightarrow \quad s^2 + s\,\frac{2.1-K}{CR} + \left(\frac{1}{CR}\right)^2 = 0$$

$$\omega_0 = \frac{1}{CR} \qquad Q = \frac{1}{2.1-K}$$

The circuit becomes unstable at

$$K = 2.1$$

12.16 The closed-loop poles are obtained from

$$1 + A(s)\,\beta(s) = 0$$

$$1 + \frac{1000\,\beta}{(1+\frac{s}{10^4})^3} = 0$$

$$\left(1+\tfrac{s}{10^4}\right)^3 + 1000\,\beta = 0$$

To simplify matters replace $\frac{s}{10^4}$ by s_n (n for normalized); thus

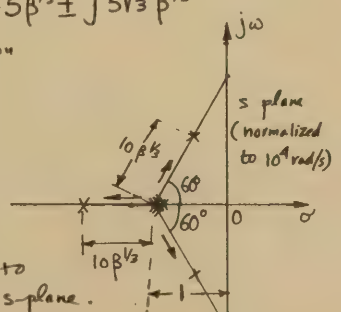
$$(1+s_n)^3 + 1000\,\beta = 0$$

$$s_n^3 + 3s_n^2 + 3s_n + (1+1000\beta) = 0$$

The roots of this cubic equation are

$$-(1+10\,\beta^{1/3}),\ -1+5\beta^{1/3} \pm j\,5\sqrt{3}\,\beta^{1/3}$$

The root locus is shown in the Figure. The loop becomes unstable when the pair of complex conjugate poles crosses the $j\omega$-axis to the righ-half of the s-plane.



This happens when $\beta = \beta_{cr}$ where $10\,\beta_{cr}^{1/3} = 1/\cos 60°$ $\Rightarrow$ $\beta_{cr} = 0.008$

12.17 (a) For $\beta(s) = \beta/[1+\frac{s}{10^3}]$

$$A(s)\,\beta(s) = \frac{1{,}000\,\beta}{(1+\frac{s}{10^4})^2\,(1+\frac{s}{10^3})}$$

$$-\phi = \tan^{-1}\frac{\omega}{10^3} + 2\tan^{-1}\frac{\omega}{10^4}$$

$\phi = 180°$ at $\omega \simeq 1.1 \times 10^4$ rad/s.

$$|A(j\omega_{180})\,\beta(j\omega_{180})| = \frac{1{,}000\,\beta}{\sqrt{1+11^2}\,(\sqrt{1+1.1^2})^2} = 40.966\beta$$

Instability results when $|A(j\omega_{180})\beta(j\omega_{180})| \geq 1$; thus the critical value of β is: $\beta_{cr} = \frac{1}{40.966} = 0.0244$

The characteristic equation with $\frac{s}{10^3}$ replaced by s_n is $1 + A(s_n)\,\beta(s_n) = 0$, thus

$$(1+0.1\,s_n)^2\,(1+s_n) + 1{,}000\,\beta = 0$$

$$0.01\,s_n^3 + 0.21\,s_n^2 + 1.2\,s_n + 1 + 1{,}000\beta = 0$$

$$s_n^3 + 21\,s_n^2 + 120\,s_n + 100(1+1000\beta) = 0 \qquad (1)$$

To obtain a pair of complex conjugate roots with $Q = 0.707$ we must be able to factor this polynomial in the form

$$(s_n + a)\,(s_n^2 + \sqrt{2}\,\omega_0 s_n + \omega_0^2) = 0 \qquad (2)$$

Equating the coefficents of like powers of s in (1) and (2) provides:

$$a + \sqrt{2}\,\omega_0 = 21 \qquad (3)$$

$$\sqrt{2}\,a\omega_0 + \omega_0^2 = 120 \qquad (4)$$

$$a\,\omega_0^2 = 100(1+1{,}000\beta) \qquad (5)$$

Combining (3) and (4) yields $\omega_0 = 4.82$ and $a = 14.18$.

These values must be denormalized ~~retain~~ with the factor 10^3. Thus $\omega_0 = 4.82 \times 10^3$ rad/s. The corresponding value of β can be found from (5),

$$14.18 \times 4.82^2 = 100(1+1000\,\beta) \Rightarrow \beta = 2.3 \times 10^{-3}$$

The corresponding value of low-frequency gain is

$$A_f(0) = \frac{A(0)}{1+A(0)\,\beta(0)} = \frac{1000}{1+1000 \times 2.3 \times 10^{-3}} = 303\ \text{V/V}$$

(b) For $\beta(s) = \beta/(1+\frac{s}{10^5})$

$$A(s)\,\beta(s) = \frac{1000\,\beta}{(1+\frac{s}{10^4})^2\,(1+\frac{s}{10^5})}$$

$$-\phi = 2\tan^{-1}\left(\frac{\omega}{10^4}\right) + \tan^{-1}\left(\frac{\omega}{10^5}\right)$$

Thus, $\omega_{180} = 4.6 \times 10^4$ rad/s

$$|A(j\omega_{180})\,\beta(j\omega_{180})| = \frac{1000\,\beta}{(1+4.6^2)\sqrt{1+0.46^2}} = 40.997\,\beta$$

Thus, $\beta_{cr} = \underline{0.0244}$.

The characteristic equation with $\frac{s}{10^4} = s_n$ is

$$(1+s_n)^2\,(1+0.1 s_n) + 1{,}000\,\beta = 0$$

$$0.1\, s_n^3 + 1.2\, s_n^2 + 2.1\, s_n + 1 + 1000\beta = 0$$

$$s_n^3 + 12\, s_n^2 + 21\, s_n + 10(1+1000\,\beta) = 0$$

$$(s_n + a)\,(s_n^2 + \sqrt{2}\,\omega_0 s_n + \omega_0^2) = 0$$

$$a + \sqrt{2}\,\omega_0 = 12 \qquad (6)$$

$$\sqrt{2}\,a\omega_0 + \omega_0^2 = 21 \qquad (7)$$

$$a\,\omega_0^2 = 10(1+1000\beta) \qquad (8)$$

Combing (6) &(7) yields $\omega_0 = 1.34$ and $a = 10.1$.

Denormalizing, we obtain $\underline{\omega_0 = 1.34 \times 10^4 \text{ rad/s}}$

The corresponding value of β is found from

$$10(1+1000\,\beta) = 10.1 \times 1.34^2 \Rightarrow \beta = 0.8 \times 10^{-3}$$

and the corresponding value of low-frequency gain is

$$A_f(0) = \frac{1000}{1+1000 \times 0.8 \times 10^{-3}} = \underline{555.6 \text{ V/V}}$$

12.18 $1 + \frac{R_2}{R_1} = 100 \Rightarrow \beta = 0.01$

$$A\beta = \frac{10^5 \times 0.01}{\left(1+\frac{s}{2\pi \times 10}\right)\left(1+\frac{s}{2\pi \times 10^4}\right)}$$

$$|A\beta| = \frac{1000}{\sqrt{1+\frac{f^2}{100}}\,\sqrt{1+\frac{f^2}{10^8}}}$$

$|A\beta| = 1$ at $\underline{f_1 \simeq 0.786 \times 10^4 \text{ Hz}}$

At f_1, $-\phi = \tan^{-1} 786 + \tan^{-1} 0.786 = 128°$

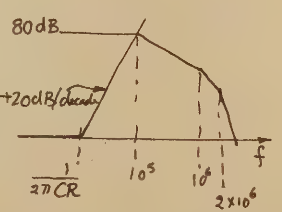
Thus, phase margin $= \underline{52°}$.

Note: The answers given in the book are gross approximations based on a sketch of the Bode diagram.

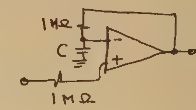
12.19

$$\frac{1}{2\pi CR} = \frac{10^5}{10^4} = 10$$

$$CR = \frac{1}{2\pi \times 10} = \underline{15.9 \text{ ms}}$$



12.20 $f_D = \frac{10^5}{10^4} = \underline{10 \text{ Hz}}$



$$\frac{1}{RC} = 2\pi f_D$$

$$C = \frac{1}{2\pi \times 10^6 \times 10} = \underline{0.016\ \mu F}$$

12.21 $f_{P1} = 10^5 = \frac{1}{2\pi C_1 R_1} = \frac{1}{2\pi R_1 \times 150 \times 10^{-12}} \Rightarrow R_1 = 10.61\ k\Omega$

$$f_{P2} = 10^6 = \frac{1}{2\pi C_2 R_2} = \frac{1}{2\pi \times R_2 \times 5 \times 10^{-12}} \Rightarrow R_2 = 31.83\ k\Omega$$

If we ignore pole splitting then we assume that connecting C_f causes the pole f_{P1} to move to a new frequency f_{P1}' while f_{P2} and f_{P3} remain constant,

$$f_{P1}' \simeq \frac{1}{2\pi\, g_m R_2 C_f R_1} = \frac{1}{2\pi \times 40 \times 31.83 \times C_f \times 10.61 \times 10^3}$$

$$= \frac{1}{8.5 \times 10^7 C_f}$$

We must place this new pole at $\frac{f_{P2}}{10^4} = 100$ Hz; thus

$$100 = \frac{1}{8.5 \times 10^7 C_f} \Rightarrow C_f = \underline{117.6 \text{ pF}}$$

If we take pole splitting into account we assume that the pole at f_{P2} assumes a much higher frequency. This permits to place the new dominant pole at $\frac{f_{P3}}{10^4}$; thus

$$\frac{2 \times 10^6}{10^4} = \frac{1}{8.5 \times 10^7 C_f} \Rightarrow C_f = \underline{58.8 \text{ pF}}$$

We must now verify that f_{P2}' is indeed very high. Using Eqn. 12.29,

$$f_{P2}' \simeq \frac{g_m C_f}{2\pi\,[C_1 C_2 + C_f(C_1 + C_2)]}$$

$$= \frac{40 \times 10^{-3} \times 58.8 \times 10^{-12}}{2\pi\,[150 \times 5 \times 10^{-24} + 58.8 \times 155 \times 10^{-24}]}$$

$$\simeq 38 \text{ MHz}$$

which is indeed much greater than f_{P3}.

12.22 The feedback is of the series-shunt type. The A circuit is shown in Fig. 2.

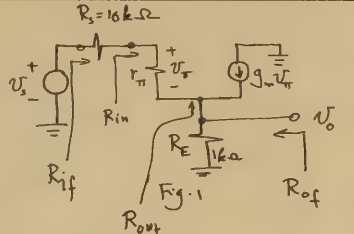


Fig. 1

$$A \equiv \frac{V_o'}{V_i'} = \frac{r_\pi}{r_\pi + R_s} \cdot g_m R_E$$

$$= \frac{2.5}{2.5+10} \cdot 40 \times 1$$

$$= 8.$$

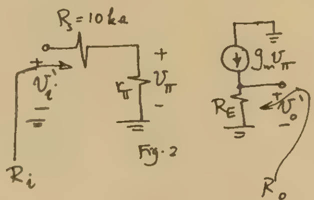


Fig. 2

The β circuit is in Fig. 3. $\beta = 1$

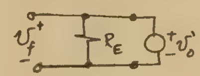


Fig. 3

Thus $1 + A\beta = 9$

$$A_f \equiv \frac{V_o}{V_s} = \frac{8}{9} = \underline{0.89 \text{ V/V}}$$

$$R_{if} = R_i(1+A\beta) = (R_s + r_\pi) \times 9$$

$$= 9 \times 12.5 = 112.5 \text{ k}\Omega$$

$$R_{in} = R_{if} - R_s = \underline{102.5 \text{ k}\Omega}$$

$$R_{of} = R_o/(1+A\beta) = R_E/(1+A\beta) = \frac{1}{9} \text{ k}\Omega = 111\,\Omega$$

$$R_{out} = \left(\frac{1}{R_{of}} - \frac{1}{R_E}\right)^{-1} = \underline{125\,\Omega}$$

__Direct Analysis__

Refer to the circuit diagram in Fig. P12.22

$$R_{in} = r_\pi + (\beta_0 + 1) \times R_E = 2.5 + 101 \times 1 = \underline{103.5 \text{ k}\Omega}$$

$$\frac{V_o}{V_s} = \frac{R_{in}}{R_{in} + R_s} \cdot \frac{R_E}{R_E + r_e} = \frac{103.5}{113.5} \cdot \frac{1}{1.025} = \underline{0.89 \text{ V/V}}$$

$$R_{out} = r_e + \frac{R_s}{\beta_0 + 1} = 25 + \frac{10{,}000}{101} \simeq \underline{125\,\Omega}$$

__Note__: In such simple circuits, direct analysis is certainly faster than analysis using feedback techniques.

12.23 The feedback is of the series-series type. The A circuit is shown in Fig. 1.

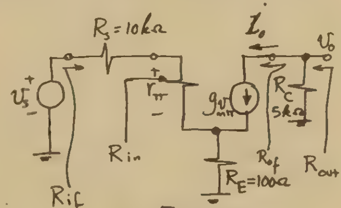


Fig. 1

$$A \equiv \frac{I_o'}{V_i'} = \frac{r_\pi}{r_\pi + R_s + R_E} \times g_m$$

$$= \frac{100}{2.5 + 10 + 0.1} = 7.94 \text{ mA/V}$$

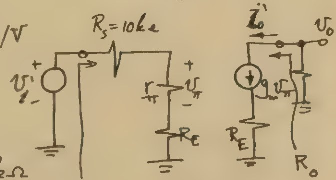


Fig. 2

The β circuit is shown in Fig. 3,

$$\beta \equiv \frac{V_f'}{I_o'} = R_E = 0.1 \text{ k}\Omega$$

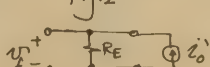


Fig. 3

$$1 + A\beta \simeq 1.8$$

$$A_f \equiv \frac{I_o}{V_s} = \frac{A}{1+A\beta} = \frac{7.94}{1.8} = 4.41 \text{ mA/V}$$

$$\frac{V_o}{V_s} = -\frac{I_o R_C}{V_s} = -A_f R_C = -4.41 \times 5 = \underline{-22 \text{ V/V}}$$

$$R_{if} = R_i(1+A\beta) = 1.8 \times (10 + 2.5 + 0.1) = 22.7 \text{ k}\Omega$$

$$R_{in} = R_{if} - R_s = 22.7 - 10 = \underline{12.7 \text{ k}\Omega}$$

$$R_{of} = R_o(1+A\beta) = \infty \times (1+A\beta) = \infty$$

$$R_{out} = R_{of} /\!/ R_C = \underline{5 \text{ k}\Omega}$$

__Direct Analysis__

Refer to the circuit diagram in Fig. P12.23

$$R_{in} = r_\pi + (\beta_0 + 1) R_E = 2.5 + 101 \times 0.1 = \underline{12.6 \text{ k}\Omega}$$

$$\frac{V_o}{V_s} = -\frac{R_{in}}{R_{in} + R_s} \cdot \frac{\alpha \times 5}{0.025 + 0.1} \simeq \underline{-22.1 \text{ V/V}}$$

$$R_{out} = R_C = \underline{5 \text{ k}\Omega}$$

12.24 The feedback is of the shunt-shunt type. The A circuit is shown in Fig. 2 and the β circuit is in Fig. 3.

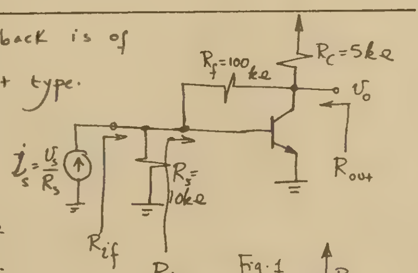


Fig. 1

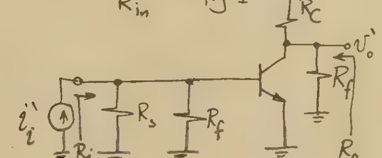


Fig. 2

$$A \equiv \frac{V_o'}{I_i'}$$

$$= (R_s /\!/ R_f /\!/ r_\pi) \times -g_m (R_C /\!/ R_f)$$

$$= (10 /\!/ 100 /\!/ 2.5) \times -40 \times (5 /\!/ 100)$$

$$= -373.5 \text{ k}\Omega$$

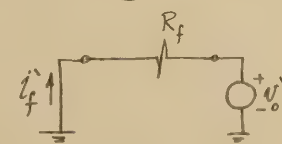


Fig. 3

$$R_i = R_s /\!/ R_f /\!/ r_\pi = 1.96 \text{ k}\Omega$$

$$R_o = R_f /\!/ R_C = 4.76 \text{ k}\Omega$$

$$\beta \equiv \frac{I_f'}{V_o'} = -\frac{1}{R_f} = -0.01 \text{ mA/V}$$

$$1 + A\beta = 4.735$$

$$A_f \equiv \frac{V_o}{I_s} = -\frac{373.5}{4.735} = -78.9 \text{ k}\Omega$$

Thus, $\frac{V_o}{V_s} = -\frac{78.9}{10} = \underline{-7.89 \text{ V/V}}$

$R_{if} = R_i/(1+A\beta) = 1.96/4.735 = 414\,\Omega$

$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{432\,\Omega}$

$R_{out} = R_{of} = \frac{R_o}{1+A\beta} = \frac{4.76}{4.735} = \underline{1\,k\Omega}$

Direct Analysis

Refer to the circuit diagram in Fig. P12.24.

The small-signal equivalent circuit is shown in Fig. 4.

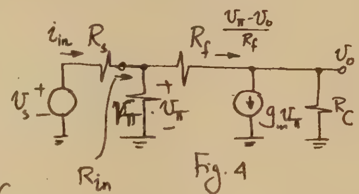


Fig. 4

$v_o = \left(\frac{v_\pi - v_o}{R_f} + g_m v_\pi\right) R_C$

$\frac{v_o}{v_\pi} = -\frac{(g_m - \frac{1}{R_f}) R_C}{1 + \frac{R_C}{R_f}} = -38.1 \times 5 = -190.5$

Using Miller's theorem:

$R_{in} = r_\pi \,//\, \frac{R_f}{1-(-190.5)} = \underline{432\,\Omega}$

$\frac{v_o}{v_s} = \frac{R_{in}}{R_{in}+R_s} \times -190.5 = \underline{-7.89\text{ V/V}}$

The circuit for finding the output resistance is shown

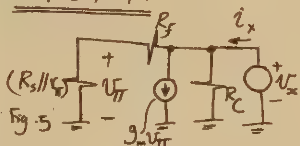


Fig. 5

in Fig. 5; $R_{out} \equiv v_x/i_x$.

$i_x = \frac{v_x}{R_C} + g_m v_\pi + \frac{v_x}{R_f + (R_s//r_\pi)}$

$= \frac{v_x}{R_C} + g_m \frac{v_x (R_s//r_\pi)}{R_f + (R_s//r_\pi)} + \frac{v_x}{R_f + (R_s//r_\pi)}$

Thus,

$R_{out} = R_C \,//\, \frac{R_f + (R_s//r_\pi)}{1 + g_m (R_s//r_\pi)}$

$= 5 // \frac{100 + (10//2.5)}{1 + 40 \times (10//2.5)} = \underline{1\,k\Omega}$.

$\boxed{12.25}$ $I_D = \frac{1}{2}\beta (V_{GS} - V_T)^2 = \frac{10 - V_{GS}}{6}$

$\Rightarrow V_{GS} = 4\text{ V}$; $I_D = 1\text{ mA}$; $g_m = \beta(V_{GS} - V_T) = 1\text{ mA/V}$.

The feedback is of the shunt-shunt type.

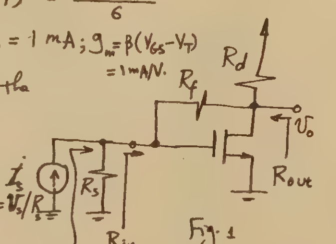


Fig. 1

The A circuit is shown in Fig. 2

$R_i = R_s // R_f = 10 // 1000$
$= 9.9\,k\Omega$

$A \equiv \frac{v_o'}{i_i'} = R_{in} \times -g_m (R_d // R_f)$
$= -9.9 \times 1 \times (6//1000)$
$= -59\,k\Omega$

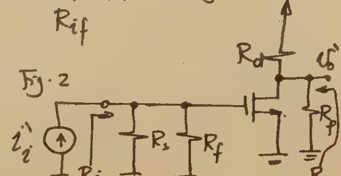


Fig. 2

$R_o = R_d // R_f = 5.96\,k\Omega$

$\beta = -\frac{1}{R_f} = -10^{-6}\text{ A/V}$

$1 + A\beta = 1 + 59 \times 10^3 \times 10^{-6} = 1.059$

$A = \frac{v_o}{i_s} = \frac{A}{1+A\beta} = -\frac{59}{1.059} = -55.7\,k\Omega$

$\frac{v_o}{v_s} = -\frac{55.7}{10} = \underline{-5.57\text{ V/V}}$

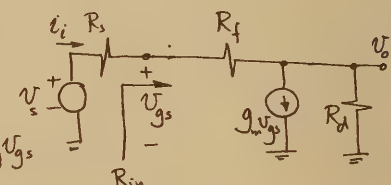
$R_{if} = \frac{R_i}{1+A\beta} = \frac{9.9}{1.059} = 9.35\,k\Omega$

$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{144\,k\Omega}$

$R_{out} = R_{of} = \frac{R_o}{1+A\beta} = \frac{5.96}{1.059} = \underline{5.63\,k\Omega}$

Direct Analysis

$i_i = \frac{v_{gs} - v_o}{R_f}$



$v_o = (i_i - g_m v_{gs}) R_d$

$= -(g_m - \frac{1}{R_f}) R_d v_{gs}$

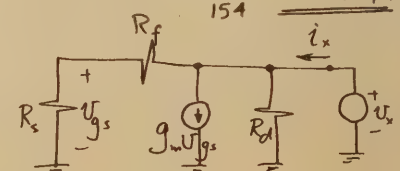
$- v_o \frac{R_d}{R_f}$

$\frac{v_o}{v_{gs}} = -\frac{g_m - (1/R_f)}{1 + \frac{R_d}{R_f}} R_d = -5.96$

$R_{in} = \frac{v_{gs}}{i_i} = \frac{R_f}{1-(-5.96)} = \underline{144\,k\Omega}$

$\frac{v_o}{v_s} = \frac{R_{in}}{R_{in}+R_s} \times -5.96 = -5.96 \times \frac{144}{154} = \underline{-5.57\text{ V/V}}$

$R_{out} \equiv \frac{v_x}{i_x}$



$i_x = \frac{v_x}{R_d} + g_m v_{gs} + \frac{v_x}{R_f + R_s}$

$= \frac{v_x}{R_d} + g_m \frac{R_s}{R_s + R_f} v_x + \frac{v_x}{R_f + R_s}$

$R_{out} = R_d \,//\, \frac{R_f + R_s}{1 + g_m R_s} = \underline{5.63\,k\Omega}$.

$\boxed{12.26}$ There is no quick way of using feedback analysis to determine f_L. This is due to the fact that the capacitor C_E appears in the feedback network making both A and β functions of frequency. We must determine $A(s)$ and $\beta(s)$ to find the frequency response of $A_f(s)$. To do this refer to the solution to Problem 12.23 and replace R_E by $Z_E = R_E + \frac{1}{sC_E}$. We find that:

$A(s) \equiv \frac{I_o'(s)}{V_i'(s)} = \frac{r_\pi}{r_\pi + R_s + Z_E} \times g_m = \frac{\beta_0}{r_\pi + R_s + Z_E}$

$$\beta(s) = Z_E$$

Thus, $$A_f(s) \equiv \frac{I_o}{V_s} = \frac{A(s)}{1 + A(s)\beta(s)}$$

$$= \frac{\beta_0/(r_\pi + R_s + Z_E)}{1 + \beta_0 Z_E/(r_\pi + R_s + Z_E)}$$

$$= \frac{\beta_0}{r_\pi + R_s + Z_E(\beta_0 + 1)}$$

$$= \frac{\beta_0}{[r_\pi + R_s + R_E(\beta_0+1)] + \frac{1}{sC_E/(\beta_0+1)}}$$

We thus see that the closed-loop response has a pole with frequency f_L given by

$$f_L = \frac{1}{2\pi C_E \frac{[r_\pi + R_s + R_E(\beta_0+1)]}{(\beta_0+1)}} \quad \cdots\cdots (1)$$

We could have arrived at this result directly from examination of the circuit in Fig. P12.23: The resistance seen by C_E is $R_E + r_e + \frac{R_s}{(\beta_0+1)}$. The numerical value of f_L is

$$f_L = \frac{1}{2\pi \times 10^{-6} \times \left(\frac{2.5 + 10 + 10.1}{101}\right) \times 10^3} = \underline{\underline{711.3 \text{ Hz}}}$$

12.27 dc conditions are the same as in Problem 12.25. Thus $I_D = 1\,mA$ and $g_m = 1\,mA/V$.

* At high frequencies the 0.01-μF behaves as a short circuit and for ac the circuit reduces to that in Fig 1.

Note the absence of feedback.

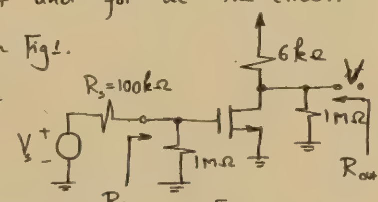


Fig. 1

$R_{in} = \underline{1 M\Omega}$

$R_{out} = 6k\Omega // 1M\Omega \simeq \underline{6k\Omega}$

$$\frac{V_o}{V_s} = \frac{1}{1+0.1} \times -g_m (6k\Omega // 1M\Omega) \simeq \underline{\underline{-5.45 \text{ V/V}}}$$

* At low frequencies the 0.01-μF capacitor behaves as an open circuit and the circuit reduces to that in Fig. 2. This is a feedback amplifier of the shunt-shunt type. The A circuit is shown in Fig 3

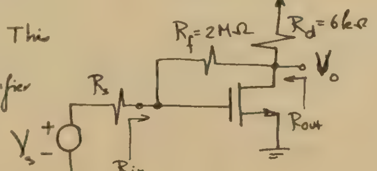


Fig. 2

$R_i = R_s // R_f$

$= 100k\Omega // 2M\Omega$

$= 95.2\ k\Omega$

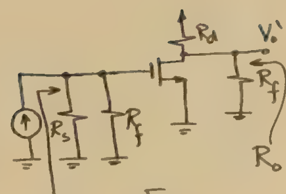


Fig. 3

$$A \equiv \frac{V_o'}{I_i'} = R_i \times -g_m(R_d // R_f)$$

$$\simeq -571.2\ k\Omega$$

$$\beta = \frac{-1}{R_f} = -0.5 \times 10^{-6}\ \mho$$

$$1 + A\beta \simeq 1.3$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{-571.2}{1.3} = -439.4\ k\Omega$$

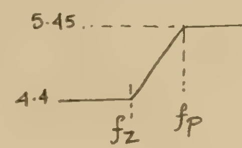
$$\frac{V_o}{V_s} = \frac{-439.4}{100} = \underline{\underline{-4.4\ \text{V/V}}}$$

$$R_{if} = \frac{R_i}{1+A\beta} = \frac{95.2}{1.3} = 73.2\ k\Omega$$

$$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{\underline{273.6\ k\Omega}}$$

$$R_{out} = R_{of} = \frac{R_o}{1+A\beta} = \frac{R_d // R_f}{1.3} \simeq \underline{\underline{4.6\ k\Omega}}$$

A sketch of the low-frequency response of the amplifier is shown. It can



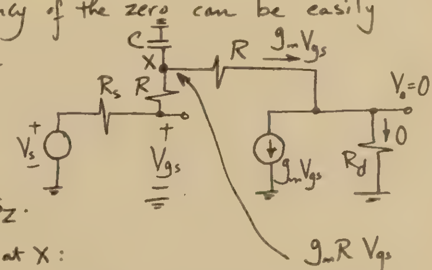
be described by

$$\frac{V_o}{V_s}(s) = 5.45\ \frac{s + \omega_Z}{s + \omega_P}$$

$$\frac{V_o}{V_s}(0) = 5.45\ \frac{\omega_Z}{\omega_P} = 4.4$$

Thus $$\frac{\omega_P}{\omega_Z} = \frac{5.45}{4.4} = 1.24$$

The frequency of the zero can be easily found from the equivalent circuit shown which applies at $s = s_Z$.



Node equation at X:

$$g_m V_{gs} + \frac{g_m R V_{gs} - V_{gs}}{R} + s_Z C g_m R V_{gs} = 0$$

$$\Rightarrow s_Z \simeq -\frac{2}{CR}$$

Thus $$\omega_Z = \frac{2}{0.01 \times 10^{-6} \times 10^6} = 200 \text{ rad/s or } \underline{(31.8 \text{ Hz})}$$

& $\omega_P = 248$ rad/s $\underline{\underline{(\text{or } 39.5 \text{ Hz})}}$

12.28 * At very low frequencies the 1-μF capacitor acts as an open circuit and the circuit reduces to that shown in Fig. 1.

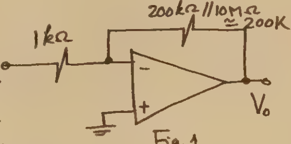


Fig. 1

Thus the low-frequency gain is constant at approximately $\frac{200}{1} = \underline{200 \text{ V/V}}$

* At medium frequencies, i.e. frequencies that are sufficiently high for the 1-μF capacitor to act as a short circuit but sufficiently low so that the amplifier limited high frequency response can be neglected, the circuit reduces to that shown in Fig. 2

This is a feedback circuit with the feedback being of the shunt-shunt type.

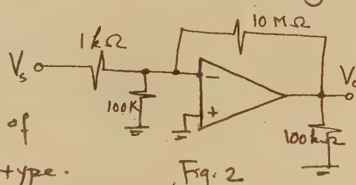


Fig. 2

It can be manipulated to the form shown in Fig. 3.

$A \equiv \frac{V_o'}{I_i'} = (R_s // R_f) \times -\mu_0$

$= -0.99 \times 10^6$ kΩ

$\beta = -\frac{1}{R_f} = -10^{-7}$ ℧

$1 + A\beta = 100$

$A_f = \frac{V_o}{I_s} = \frac{-0.99 \times 10^6}{100}$

$= -9900$ kΩ

$\frac{V_o}{V_s} = \underline{-9900 \text{ V/V}}$

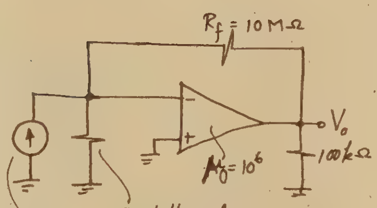


Fig. 3

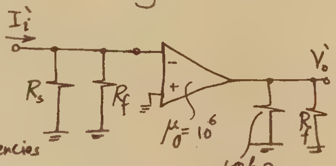


Fig. 4

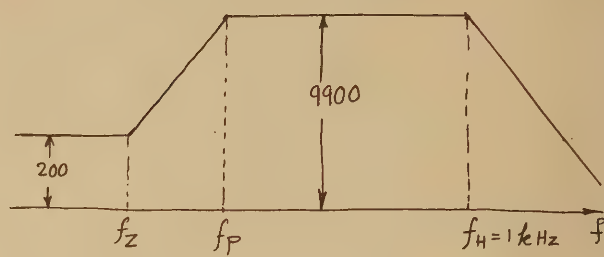
* At high frequencies the gain falls off at −20 dB/decade due to the finite bandwidth of the op amp. The high-frequency pole has a frequency f_H given by

$f_H = f_b\,(1 + A\beta)$

where f_b is the 3-dB frequency of the open-loop amplifier; $f_b = \frac{f_t}{\mu_0} = \frac{10^7}{10^6} = 10$ Hz.

Thus, $f_H = 10 \times 100 = \underline{1 \text{ kHz}}$

From the above we arrive at the following sketch of the frequency response of the circuit

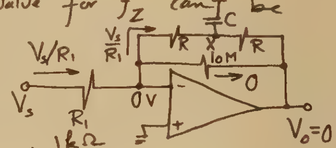


The low-frequency response can be described by $\left.\frac{V_o}{V_s}(s)\right|_{\text{Low-frequency}} = 9{,}900\,\frac{s + \omega_Z}{s + \omega_P}$

Thus we have, $200 = 9{,}900\,\frac{\omega_Z}{\omega_P}$

or, $\frac{f_P}{f_Z} = \frac{9900}{200} = 49.5$

An approximate value for f_Z can be found from the circuit shown. Here we assume the op amp to be ideal ($\mu_0 = \infty$).



$V_x = -\frac{R}{R_1} V_s$.

Node equation at X:

$\frac{V_s}{R_1} + \frac{V_s}{R_1} + s_Z C V_s \frac{R}{R_1} = 0$

$\Rightarrow \quad s_Z = -\frac{2}{CR}$

$f_Z = \frac{2}{2\pi CR} = \frac{1}{\pi \times 10^{-6} \times 100 \times 10^3} = \underline{3.2 \text{ Hz}}$

$f_P = 3.2 \times 49.5 = \underline{158.4 \text{ Hz}}$

The circuit can function as a differentiator with a time constant $\tau = \frac{1}{2\pi f_Z} = \underline{0.05 \text{ s}}$

12.29 dc bias

$I_E = \frac{10 - 0.7}{1 + \frac{10}{101}}$

$= 8.5$ mA

$r_e \simeq 3\ \Omega$

$r_\pi \simeq 0.3$ kΩ

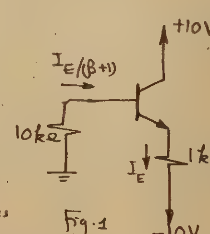


Fig. 1

At relatively high frequencies the 0.1-μF capacitor acts as a short circuit

and the circuit reduces to that shown in Fig. 2. Note that the feedback supplied by R_f is of the shunt-shunt type and hence it is convenient to transform the source to the Norton representation, as shown in Fig. 3.

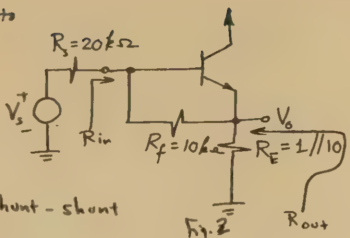


Fig. 2

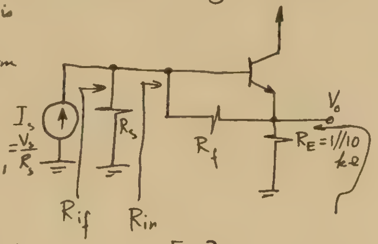


Fig. 3

Next observe that the feedback here is positive since V_o and V_b are in phase. This positive feedback has the effect (in this particular topology) of raising the input resistance of the follower (relative to the case with R_f grounded).

Direct Analysis

Fig. 4 shows the small-signal equivalent circuit of the circuit of Fig. 2.

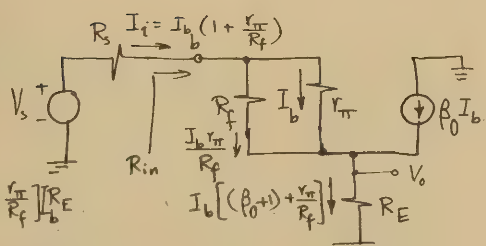


Fig. 4

$$V_o = \left[(\beta_0+1)+\frac{r_\pi}{R_f}\right] I_b R_E$$

$$V_b = I_b\left[r_\pi + (\beta_0+1) R_E + \frac{r_\pi}{R_f} R_E\right]$$

$$I_i = I_b\left(1+\frac{r_\pi}{R_f}\right)$$

Thus,
$$R_{in} \equiv \frac{V_b}{I_i} = \frac{r_\pi + \left(\beta_0+1+\frac{r_\pi}{R_f}\right) R_E}{1+\frac{r_\pi}{R_f}}$$

$$= \frac{0.3 + \left(101+\frac{0.3}{10}\right)(1//10)}{1+\frac{0.3}{10}} = \underline{\underline{89.5\ k\Omega}}$$

Note that this value is much greater than that would have obtained if R_f were returned to ground.

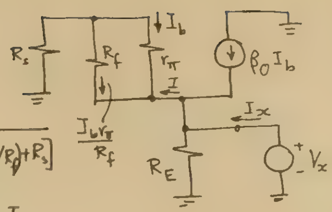
$$\frac{V_b}{V_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{89.5}{89.5+20} = 0.817$$

$$\frac{V_o}{V_b} = \frac{\left[(\beta_0+1)+(r_\pi/R_f)\right] R_E}{r_\pi + \left[(\beta_0+1)+(r_\pi/R_f)\right] R_E} \simeq 1$$

Thus $\frac{V_o}{V_s} = \underline{0.817\ V/V}$

To find the output resistance consider the circuit in Fig. 5.

$$I = \frac{V_x}{(r_\pi//R_f)+R_s}$$



$$= -I_b\left(1+\frac{r_\pi}{R_f}\right)$$

Thus,
$$I_b = \frac{-V_x}{\left(1+\frac{r_\pi}{R_f}\right)\left[(r_\pi//R_f)+R_s\right]}$$

But,
$$I_x = \frac{V_x}{R_E} - \beta_0 I_b + I$$

$$= \frac{V_x}{R_E} + \frac{\beta_0 V_x}{\left(1+\frac{r_\pi}{R_f}\right)\left[(r_\pi//R_f)+R_s\right]} + \frac{V_x}{\left[(r_\pi//R_f)+R_s\right]}$$

Thus,
$$R_{out} \equiv \frac{V_x}{I_x} = R_E \,//\, \frac{\left[(r_\pi//R_f)+R_s\right]}{\left[\frac{\beta_0}{1+\frac{r_\pi}{R_f}}+1\right]}$$

$$= 1\,//\,10\,//\left[\frac{(0.3//10)+20}{\frac{100}{1+0.03}+1}\right] = \underline{\underline{168.5\ \Omega}}$$

Feedback Analysis

The A circuit is in Fig. 5.

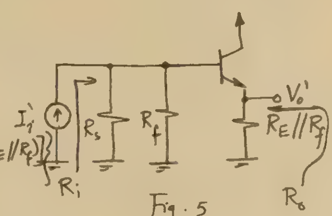


Fig. 5

$$R_i = R_s // R_f // \left\{(\beta_0+1)\left[r_e + (R_E//R_f)\right]\right\}$$
$$= 6.18\ k\Omega$$

$$A \equiv \frac{V_o'}{I_i'} = R_i \cdot \frac{(R_E//R_f)}{r_e + (R_E//R_f)} = 6.18 \times 0.996 = 6.16\ k\Omega$$

$$R_o = R_E // R_f // \left\{r_e + \frac{R_f//R_s}{\beta_0+1}\right\}$$
$$= 63.7\ \Omega$$

$$\beta = -\frac{1}{R_f} = -0.1\ mA/V$$

$$1+A\beta = 1 - 6.16 \times 0.1 = 0.384$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{6.16}{0.384} = 16.04\ k\Omega$$

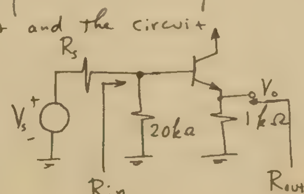
$$\frac{V_o}{V_s} = \frac{16.04}{20} \simeq \underline{\underline{0.8\ V/V}}$$

$$R_{if} = \frac{R_i}{1+A\beta} = \frac{6.18}{0.384} = 16.09\ k\Omega$$

$$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{\underline{82.4\ k\Omega}}$$

$$R_{out} = R_{of} = \frac{R_o}{1+A\beta} = \frac{63.7}{0.384} = \underline{\underline{166\ \Omega}}$$

* At very low frequencies the capacitor acts as an open circuit and the circuit reduces to that shown in Fig. 6.

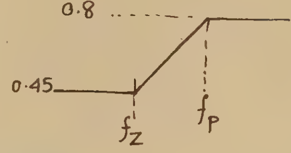


$$R_{in} = 20 \,//\, 101 \times 1.003$$
$$= \underline{16.7\ k\Omega}$$

$$\frac{V_o}{V_s} = \frac{16.7}{16.7+20}\ \frac{1}{1.003} = 0.45\ \text{V/V}$$

$$R_{out} = 1 \,//\, \left(0.003 + \frac{20//20}{101}\right) = 92.6\,\Omega$$

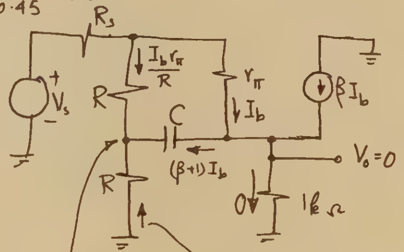
A sketch of the gain magnitude is shown,



$$\frac{V_o(s)}{V_s(s)} = 0.8\ \frac{s+\omega_Z}{s+\omega_P}$$

where $\dfrac{\omega_P}{\omega_Z} = \dfrac{0.8}{0.45} = 1.8$

The zero frequency can be determined from the circuit shown.



$$(\beta+1)I_b + \frac{I_b r_\pi}{R} + \frac{(\beta+1)I_b}{s_Z CR} = 0 \qquad -\frac{(\beta+1)I_b}{s_Z C} \qquad \frac{(\beta+1)I_b}{s_Z CR}$$

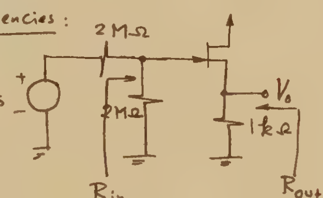
$$\Rightarrow\ s_Z = -\frac{1}{CR}\ \frac{(\beta+1)}{\beta+1+\frac{r_\pi}{R}}$$

Thus $\omega_Z \simeq \underline{\underline{1000\ \text{rad/s}}}$ $\qquad \omega_P = \underline{\underline{1{,}800\ \text{rad/s}}}$

12.30 $I_D = I_{DSS} = 8\ \text{mA}$; thus $V_{GS} = 0$ and the dc voltage at the output is 0V. $g_m = \frac{2\times 8}{2} = 8\ \text{mA/V}$

* At very low frequencies:

$R_{in} = \underline{2\ \text{M}\Omega}$

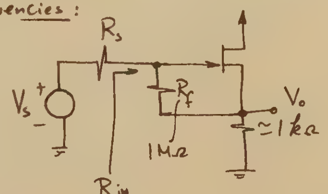


$$\frac{V_o}{V_s} = \frac{2}{2+2}\ \frac{1}{1+\frac{1}{g_m}} = 0.5 \times \frac{1}{1+\frac{1}{8}} = \underline{\underline{0.44\ \text{V/V}}}$$

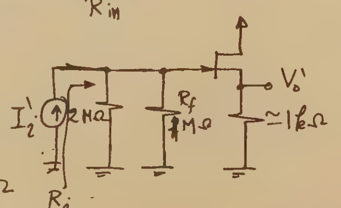
$$R_{out} = 1 \,//\, (1/g_m) = \underline{\underline{111.1\,\Omega}}$$

* At very high frequencies:

The circuit is as shown. Positive feedback of the shunt–shunt type.



The A circuit is →



$R_i = 2\text{M}\Omega // 1\text{M}\Omega = \frac{2}{3}\ \text{M}\Omega$

$$A \equiv \frac{V_o'}{I_i'} = R_i \cdot \frac{1}{1+\frac{1}{8}} = 0.59\ \text{M}\Omega$$

$$\beta = -\frac{1}{R_f} = -10^{-6}\ \mho$$

$$1 + A\beta = 1 - 0.59 = 0.41$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{0.59}{0.41} = 1.44\ \text{M}\Omega$$

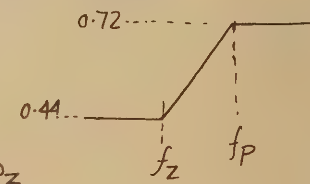
$$\frac{V_o}{V_s} = \frac{1.44}{2} = \underline{\underline{0.72\ \text{V/V}}}$$

$$R_{if} = \frac{R_i}{1+A\beta} = \frac{2/3}{0.41} = 1.626\ \text{M}\Omega$$

$$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{\underline{8.7\ \text{M}\Omega}}$$

$$R_{out} = \frac{R_o}{1+A\beta} = \frac{111.1}{0.41} = \underline{\underline{271\,\Omega}}$$

A sketch of the gain magnitude is shown,



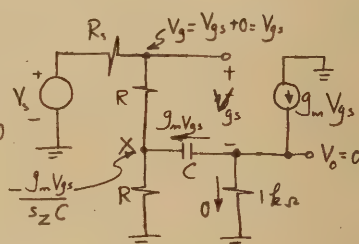
$$\frac{V_o(s)}{V_s(s)} = 0.72\ \frac{s+\omega_Z}{s+\omega_P}$$

where $\dfrac{\omega_P}{\omega_Z} = \dfrac{0.72}{0.44} = 1.64$

The frequency of the zero can be determined by setting $V_o = 0$ in the equivalent circuit shown

Node equation at X:

$$g_m V_{gs} + \frac{g_m V_{gs}}{s_Z CR} + \frac{V_{gs} + g_m V_{gs}/s_Z C}{R} = 0$$



$$s_Z = -\frac{2}{CR\left(1+\frac{1}{g_m R}\right)} \simeq -\frac{2}{CR}$$

$$\omega_Z = \frac{2}{10^{-8}\times 10^{6}} = \underline{\underline{200\ \text{rad/s}}}$$

$$\omega_P = \underline{\underline{328\ \text{rad/s}}}$$

12.31 dc analysis

See Figure 1.

$I_{E1} \simeq 0.02\ \text{mA}$

$I_{E2} \simeq 1\ \text{mA}$

$r_{e1} = 1.25\ k\Omega$

$r_{e2} = 25\,\Omega$

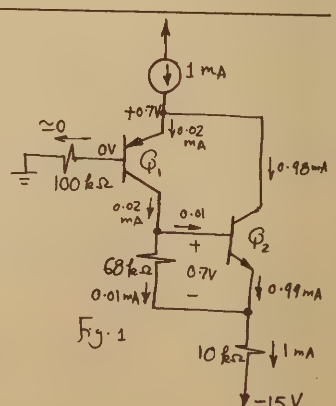


Fig. 1

This is a rather interesting circuit. It embodies negative feedback of the series–shunt type: Resistor R_2 samples the output voltage V_o and provides a current V_o/R that is proportional to V_o. Most of this current flows through the emitter and collector of Q_2 and thus develops across R_1 a voltage (the feedback signal) that is proportional to V_o. The voltage across R_1 is mixed with V_s in the loop containing the emitter–base junction of Q_1.

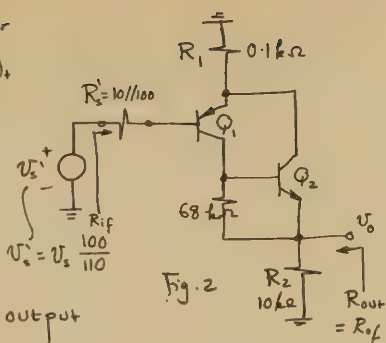


Fig. 2

To break the feedback loop we simply have to disconnect the collector of Q_2 from the emitter of Q_1. The A circuit is shown in Fig. 3

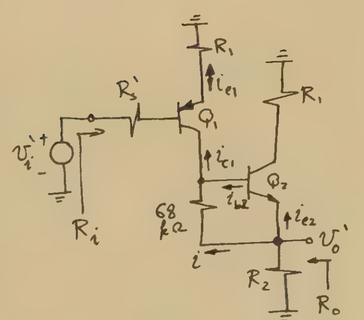
$$A \equiv \frac{V_o'}{V_i'}$$

$$i_{e1} = \frac{V_i'}{R_1 + r_{e1} + \frac{R_s'}{\beta_1+1}}$$

$$i_{c1} \simeq i_{e1}$$

$$i_{b2} = i_{c1}\,\frac{68}{68 + r_{\pi 2}}$$

$$i = i_{c1}\,\frac{r_{\pi 2}}{68 + r_{\pi 2}}$$



$$i_{e2} = (\beta_2+1)\,i_{b2} = (\beta_2+1)\,i_{c1}\,\frac{68}{68+r_{\pi 2}}$$

$$V_o' = -(i_{e2}+i)R_2 = -R_2\left[(\beta_2+1)\,i_{c1}\,\frac{68}{68+r_{\pi 2}} + i_{c1}\,\frac{r_{\pi 2}}{68+r_{\pi 2}}\right]$$

$$= -R_2\,\frac{V_i'}{R_1 + r_{e1} + \frac{R_s'}{\beta_1+1}}\left[(\beta_2+1)\,\frac{68}{68+r_{\pi 2}} + \frac{r_{\pi 2}}{68+r_{\pi 2}}\right]$$

Thus, $$A = -\frac{R_2}{R_1 + r_{e1} + \frac{R_s'}{\beta_1+1}}\;\frac{(\beta_2+1)\times 68 + r_{\pi 2}}{68 + r_{\pi 2}}$$

$$= -\frac{10}{0.1 + 1.25 + 0.09}\;\frac{101\times 68 + 2.5}{68 + 2.5}$$

$$= -676.8 \text{ V/V}$$

$$\beta \simeq -\frac{R_1}{R_2} = -\frac{0.1}{10} = -0.01$$

$$1 + A\beta = 7.77$$

$$A_f \equiv \frac{V_o}{V_s'} = -\frac{676.8}{7.77} = -87.1 \text{ V/V}$$

Since $V_s' = V_s\,\frac{100}{110}$, then

$$\frac{V_o}{V_s} = \frac{V_o}{V_s'}\,\frac{V_s'}{V_s} = -87.1 \times \frac{100}{110} = \underline{\underline{-79.2 \text{ V/V}}}$$

$$R_i = R_s' + (\beta_1+1)(R_1 + r_{e1})$$

$$= 9.09 + 101\times(0.1 + 1.25) = 145.4\ k\Omega$$

$$R_{if} = R_i(1 + A\beta) = 1.13\ M\Omega$$

$$R_{in}\,(\text{into the base of } Q_1) = 1.13 - 0.009 = 1.12\ M\Omega$$

$$R_{in} = 100\,k\Omega \,/\!/\, 1.12\,M\Omega = \underline{\underline{92\ k\Omega}}$$

$$R_o = R_2 = 10\ k\Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta} = \frac{10}{7.77} = \underline{\underline{1.3\ k\Omega}}$$

12.32 This is a challenging problem! First please note the following changes to the problem statement (from what appeared in the first printing of the text).

– Change the emitter resistance from $100\,\Omega$ to $1\,\Omega$ (A $100\text{-}\Omega$ emitter resistance may cause the circuit to become unstable.

– Let the circuit be fed with a source having a resistance $R_s = 250\,k\Omega$ and evaluate the voltage gain V_o/V_s (not V_o/V_i).

We shall now present a detailed solution.

dc Analysis

For simplicity we shall assume $\beta = \infty$ and we shall also neglect the small current in the overall-feedback network. Now that $0.4\,V_o \simeq 0.7\text{V}$

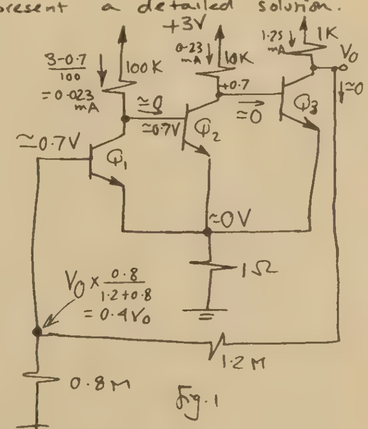


Fig. 1

Thus $V_o = \frac{0.7}{0.4} = \underline{\underline{1.75\text{ V}}}$

$$I_{C3} \simeq \frac{3 - 1.75}{1} = 1.25\text{ mA}\;;\quad I_{C2} = \frac{3-0.7}{10} = 0.23\text{ mA}$$

and $I_{C1} = 0.023$ mA

Thus, $r_{e3} = 20\,\Omega$; $r_{\pi 3} \simeq 2\,k\Omega$; $r_{e2} \simeq 100\,\Omega$; $r_{\pi 2} \simeq 10\,k\Omega$; $r_{e1} \simeq 1\,k\Omega$; $r_{\pi 1} \simeq 100\,k\Omega$.

Analysis of the internal amplifier

Next we shall analyze the internal amplifier shown in Fig 2. This amplifier embodies feedback of the series-series type. To find its gain $A_f = \frac{I_o}{V_i}$, input resistance R_{if} and output resistance R_{of} we shall make an approximation that considerably simplifies the analysis: We shall assume that the collector resistances (of Q_1 & Q_2) are so large that ~~the~~ most of the collector currents flow through the bases of succeeding transistors. This assumption enables us to eliminate the collector resistances from the ac equivalent circuit as shown in Fig. 3

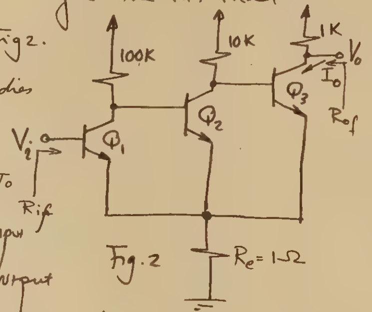


Fig. 2

Thus, $A_f \equiv \frac{I_o}{V_i} = \frac{A}{1+A\beta} = \frac{10}{11} = 0.91$ A/V

We can use this value to determine the voltage gain $\frac{V_o}{V_i}$ since $V_o = -I_o R_L$, thus

$\frac{V_o}{V_i} = -A_f R_L = -910$ V/V

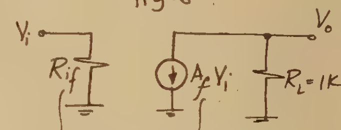
Note that $\frac{V_o}{V_i}$ is equal approximately to the ratio of R_L to R_e. This comes about because negative feedback causes the base-emitter voltage ($v_{\pi 1}$) of Q_1 to be approximately zero, thus $V_i \simeq V_e$, but $V_e \simeq I_o R_e$.

$R_{if} = R_i(1+A\beta) = 100 \times 11 = \underline{1.1\ M\Omega}$

Finally, note that $R_{of} = 1\,k\Omega$ (since r_o of the transistors is assumed infinite). We can now draw the following equivalent circuit for the internal amplifier.

In the following we shall use this equivalent circuit to simplify analysis of the overall amplifier.

Fig. 6



For a discussion of the need for such an approximation see Reference G8, pages 653-656

We can now obtain the A circuit shown in Fig 4. From this circuit we find $A \equiv \frac{I_o'}{V_i'}$ as

$A = \left(\frac{1}{r_{\pi 1} + R_e}\right) \times (\beta_1) \times (\beta_2) \times \beta_3$

$\simeq \frac{10^6}{100}$ mA/V

$= 10^4$ mA/V

$R_i = r_{\pi 1} + R_e \simeq 100\,k\Omega$

The circuit for determining β is shown in Fig. 5

$\beta \equiv \frac{V_f'}{I_o'} = R_e = 1\,\Omega$

Thus, $A\beta = 10$ and $1+A\beta = 11$

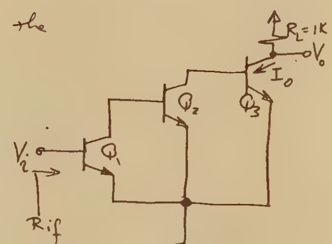


Fig. 3

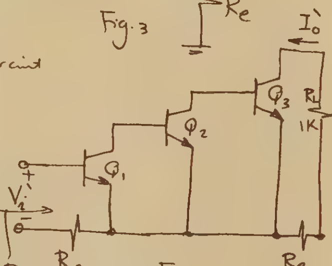


Fig. 4

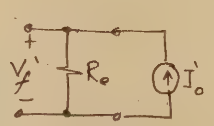


Fig. 5

Case (a) $C_1 = \infty$

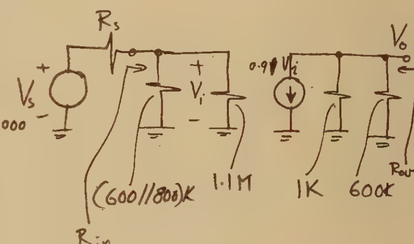
$R_{in} = 261\,k\Omega$

$\frac{V_o}{V_s} = \frac{261}{261+250} \times -0.91 \times 1000$

$= \underline{\underline{-466.4}}$ V/V

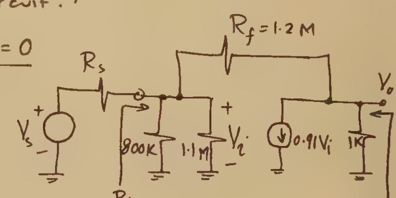
$R_{out} = 1\,k\Omega$

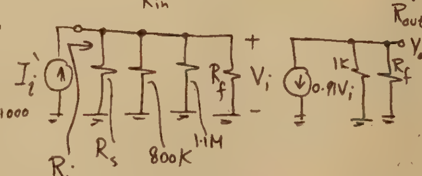
This case would represent the circuit at relatively high frequencies where C_1 acts as a short circuit.



Case (b): $C_1 = 0$

The amplifier embodies shunt-shunt feedback. The analysis is illustrated.





$A \equiv \frac{V_o'}{I_i'} \simeq -R_i \times 0.91 \times 1000$

$R_i = 143\,k\Omega$

Thus $A = -130\,M\Omega$

$\beta = -\frac{1}{R_f} = -\frac{1}{1.2\,M\Omega} \Rightarrow 1+A\beta = 109.3$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1+A\beta} = -\frac{130}{109.3} = -1.189\ M\Omega$$

$$\frac{V_o}{V_s} \equiv \frac{V_o}{V_s} = \frac{A_f}{R_s} = \frac{-1.189}{0.25} = \underline{\underline{-4.76\ V/V}}$$

$$R_{if} = \frac{R_i}{1+A\beta} = 1.3\ k\Omega$$

$$R_{in} \simeq R_{if} = \underline{\underline{1.3\ k\Omega}}$$

$$R_{out} = \frac{1000}{109.3} \simeq \underline{\underline{9\ \Omega}}$$

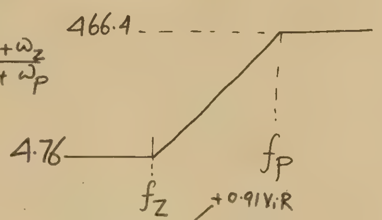
Note that in this case the gain is approximately equal to $-\frac{R_f}{R_s}$.

With $C = 1\,\mu F$:

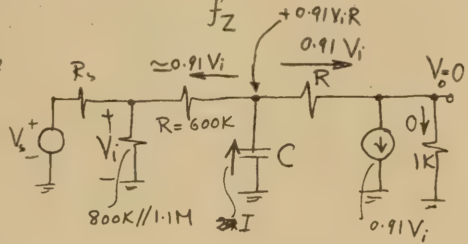
$$\frac{V_o}{V_s}(s) = -466.4\ \frac{s+\omega_Z}{s+\omega_P}$$

where

$$\frac{\omega_P}{\omega_Z} = \frac{466.4}{4.76} = 98$$



To determine ω_Z we use the



equivalent circuit shown which applies at $s = s_Z$. We find that

$$I = -s_Z C \times 0.91\,V_i\,R = 2 \times 0.91\,V_i.$$

Thus

$$s_Z = \frac{-2}{CR}$$

$$\omega_Z = \frac{2}{CR} = \frac{2}{10^{-6} \times 600 \times 10^3} = 3.33\ rad/s$$

$$f_Z = 0.53\ Hz$$

$$\omega_P = 326.34\ rad/s$$

$$f_P = 52\ Hz$$

12.33 (a) Solution of Problem 10.10 using feedback techniques. Shunt-Shunt feedback. The A circuit is shown in Fig. 2.

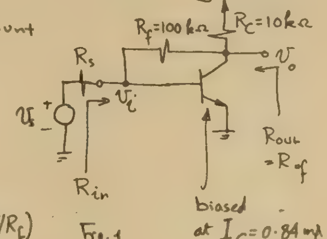
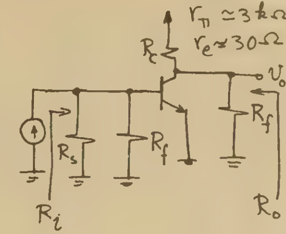


Fig. 1

$$A \equiv \frac{v_o'}{i_i} = R_i \times -g_m(R_C // R_f)$$

Use $R_s = 10\ k\Omega$.

$$R_i = R_s // R_f // r_\pi$$
$$= 10 // 100 // 3$$
$$= 2.26\ k\Omega$$



$$A = -2.26 \times 33.6 \times 9.09$$
$$= -690.3\ k\Omega$$

$$\beta = -\frac{1}{R_f} = -10^{-5}\ \mho$$

$$1 + A\beta = 7.9$$

$$A_f \equiv \frac{v_o}{i_s} = \frac{A}{1+A\beta} = -87.4\ k\Omega$$

$$\frac{v_o}{v_s} = -\frac{87.4}{10} = -8.74\ V/V$$

$$R_{if} = R_i/(1+A\beta) = \frac{2.26}{7.9} = 286\ \Omega$$

$$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1}$$
$$= \underline{295\ \Omega}$$

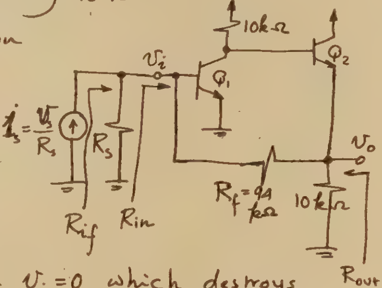
Thus,
$$\frac{v_i}{v_s} = \frac{0.295}{10.295} = 0.0286$$

$$\frac{v_o}{v_i} = \frac{v_o}{v_s}\,\frac{v_s}{v_i} = -\frac{8.74}{0.0286} = \underline{\underline{-305}}$$

$$R_o = R_C // R_f = 9.09\ k\Omega$$

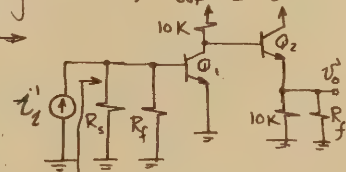
Since in this circuit it is required to find the output resistance with $v_i = 0$; setting v_i to zero destroys the feedback and $R_{of} = R_o = 9.09\ k\Omega$. Note that in solving this problem we assumed that the source has a finite value of R_s; the value chosen for R_s, however, has no effect on $\frac{v_o}{v_i}$ and R_{in}.

(b) Problem 10.12. DC calculations as in the solution to Problem 10.12: $I_{E1} = 0.86$ mA; $I_{E2} = 1.07$ mA; $r_{e1} = 29\,\Omega$; $r_{e2} = 23.4\,\Omega$. With $C = \infty$ the overall feedback is destroyed and the results are as in the solution to Problem 10.12. With C removed the circuit has feedback of the shunt-shunt type. To solve the problem using feedback techniques we assumes that the signal source has a finite resistance of arbitrary value, say 10 kΩ. Thus we have the circuit in Fig. 1. Please note at the outset that the value of R_{out} required is with $v_i = 0$ which destroys the feedback. Thus R_{out} will be the same



value found without feedback; $R_{out} = 23.3\,\Omega$.

The A circuit is →



$R_i = R_s // R_f // r_{\pi 1}$

$= 10//94//2.9$

$= 2.2$ kΩ

$$A \equiv \frac{v_o'}{i_i'} = \underbrace{R_i}_{} \times g_{m1} \left\{10 // \left[(\beta_2+1)(r_{e2} + (10//R_f))\right]\right\} \frac{(10//R_f)}{(10//R_f) + r_{e2}}$$

$= -2.2 \times 34.4 \times 9.89 \times 0.997 = -746.6$ kΩ

$\beta = -\frac{1}{R_f} = -\frac{1}{94\text{ k}\Omega}$

$1 + A\beta = 8.94$

$A_f \equiv \frac{v_o}{i_s} = \frac{A}{1+A\beta} = -\frac{746.6}{8.94} = -83.5$ kΩ

$R_{if} = \frac{R_i}{1+A\beta} = \frac{2.2}{8.94} = 246\,\Omega$

$v_i = i_s \times R_{if} = i_s \times 246\,\Omega$

Thus $\frac{v_o}{v_i} = \frac{v_o}{i_s} \times \frac{i_s}{v_i} = -83.5 \times 10^3 \times \frac{1}{246}$

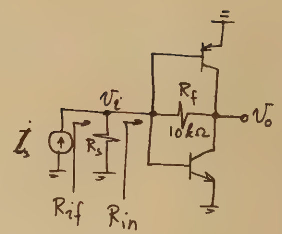
$\underline{\underline{\simeq -340 \text{ V/V}}}$

$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{252\,\Omega}$

(c) Problem 10.13

No-Load Case

Use a source with an arbitrary R_s, say 10 kΩ. Shunt-shunt feedback.



$A \equiv \frac{v_o'}{i_i'} = R_i \times -2g_m R_f$

where $R_i = R_s // R_f // \frac{r_\pi}{2}$

$= 10//10// \frac{2.5}{2}$

$= 1$ kΩ

Thus, $A = -2 \times 40 \times 1 \times 10$

$= -800$ kΩ

$\beta = -\frac{1}{R_f} = -0.1$ mA/V

$1 + A\beta = 81$

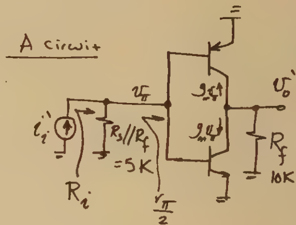
$A_f \equiv \frac{v_o}{i_s} = \frac{-800}{81} = -9.88$ kΩ

$R_{if} = \frac{R_i}{1+A\beta} = \frac{1}{81}$ kΩ

$\frac{v_o}{v_i} = \frac{v_o}{i_s} \frac{1}{R_{if}} = \frac{-800}{81} \times 81 = \underline{\underline{-800 \text{ V/V}}}$

$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{\underline{12.4\,\Omega}}$

A circuit



With a 1-kΩ load:

$R_i = 5 // \frac{r_\pi}{2} = 1$ kΩ

$A \equiv \frac{v_o'}{i_i'} = R_i \times -2g_m (1//10)$

$= -2 \times 40 \times 1 \times \frac{1 \times 10}{11}$

$= -72.7$ kΩ

$\beta = -\frac{1}{R_f} = -0.1$ mA/V

$1 + A\beta = 8.27$

$A \equiv \frac{v_o}{i_s} = \frac{-72.7}{8.27}$

$= -8.8$ kΩ

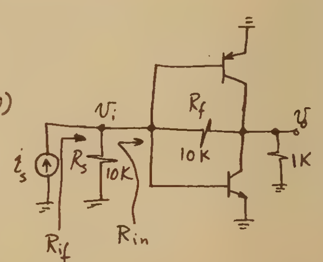
$R_{if} = \frac{1}{8.27}$ kΩ

$v_i = i_s \times R_{if} = \frac{i_s}{8.27}$

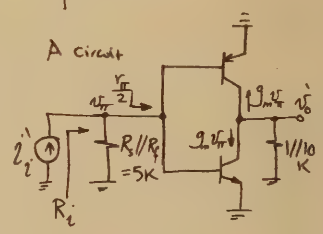
Thus $\frac{v_o}{v_i} = -8.8 \times 8.27 = \underline{\underline{-72.7 \text{ V/V}}}$

$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1}$

$= \underline{122.4\,\Omega}$

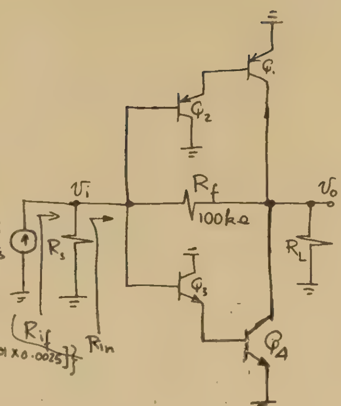


A circuit



(d) Problem 10.14

Use a source having an arbitrary but finite R_s, say $10k\Omega$.



$C_1 = C_2 = \infty$

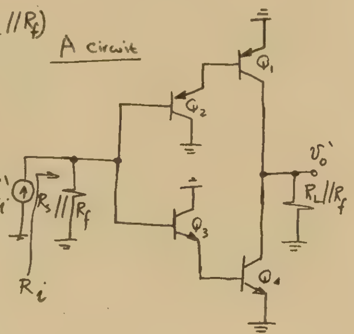
$R_i = R_s // R_f // \frac{1}{2}\{(\beta_2+1)[r_{e2} + (\beta_1+1) r_{e1}]\}$

$= 10//100//\frac{1}{2}\{101[0.25 + 101 \times 0.0025]\}$

$= 6.7\ k\Omega$

$A \equiv \frac{v_o'}{i_i'} = -R_i \times \frac{1}{2} g_{m1} \times 2 \times (R_L // R_f)$

A circuit



For $R_L = \infty$

$A = -6.7 \times 400 \times 100$

$= -266.7\ M\Omega$

$\beta = \frac{-1}{R_f} = -\frac{1}{100}\ mA/V$

$1 + A\beta = 2667.7$

$A_f \equiv \frac{v_o}{i_s} = -0.1\ M\Omega$

$R_{if} = \frac{R_i}{1+A\beta} = 2.5\ \Omega$

$\frac{v_o}{v_i} = \frac{v_o}{i_s}\frac{1}{R_{if}} = \frac{-0.1 \times 10^6}{2.5} = \underline{\underline{-4 \times 10^4\ V/V}}$

$R_{in} \simeq R_{if} = \underline{\underline{2.5\ \Omega}}$

For $R_L = 200\ \Omega$

$A \simeq -6.7 \times 400 \times 0.2 = -536\ k\Omega$

$\beta = -\frac{1}{100}\ mA/V$

$1 + A\beta = 6.36$

$A_f \equiv \frac{v_o}{i_s} = -84.3\ k\Omega$

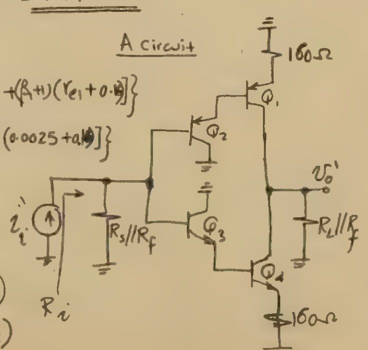
$R_{if} = \frac{6.7}{6.36} = 1.05\ k\Omega$

$\frac{v_o}{v_i} = -\frac{84.3}{1.05} = \underline{-80\ V/V}$

$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{\underline{1.17\ k\Omega}}$

$C_1 = C_2 = 0$

A circuit



$R_i = R_s // R_f // \frac{1}{2}\{(\beta_2+1)[r_{e2} + (\beta_1+1)(r_{e1} + 0.16)]\}$

$= 10//100//\frac{1}{2}\{101[0.25 + 101(0.0025 + 0.16)]\}$

$= 8.99\ k\Omega$

$A \equiv \frac{v_o'}{i_i'} = R_i \cdot \frac{160}{160 + 2.5 + \frac{250}{\beta+1}} \times \frac{1}{0.16} \times 2 \times (R_L // R_f)$

$= -109.0 \times (R_L // R_f)$

$\beta = -\frac{1}{R_f} = -0.01\ mA/V$

For $R_L = \infty$:

$A = -109 \times 100 = -10.9\ M\Omega$

$1 + A\beta = 110$

$A_f \equiv \frac{v_o}{i_s} = \frac{-10.9}{110} = -0.1\ M\Omega$

$R_{if} = \frac{R_i}{1+A\beta} = \frac{8.99}{110} = \underline{\underline{81.8\ \Omega}}$

$\frac{v_o}{v_i} = \frac{v_o}{i_s} \times \frac{1}{R_{if}} = \frac{-0.1 \times 10^6}{81.8} = \underline{\underline{-1222\ V/V}}$

$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{\underline{82.5\ \Omega}}$

For $R_L = 200\ \Omega$

$A = -109 \times 0.2 = -21.8\ k\Omega$

$1 + A\beta = 1.218$

$A_f = -17.9\ k\Omega$

$R_{if} = 7.38\ k\Omega$

$\frac{v_o}{v_i} = \frac{-17.9}{7.38} = \underline{\underline{-2.4\ V/V}}$

$R_{in} = \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right)^{-1} = \underline{\underline{28.2\ k\Omega}}$

CHAPTER 13—EXERCISES

13.1 $V_{BE} = V_T \ln(I_C / I_S)$

$= 0.025 \ln(10^{-3}/10^{-14}) = \underline{0.633\ V}$

$g_m = I_C / V_T = \underline{40\ mA/V}$

$r_e = V_T / I_E \simeq \underline{25\ \Omega}$

$r_\pi = \beta / g_m = \underline{5\ k\Omega}$

$r_o = \mu / g_m = \underline{125\ k\Omega}$

$r_\mu = 10 \beta r_o = \underline{250\ M\Omega}$

13.2 $V_{BE1} = V_T \ln(I_1 / I_{S1})$

$V_{BE2} = V_T \ln(I_1 / I_{S2})$

$V_{BE1} + V_{BE2} = V_T \ln\left(\frac{I_1^2}{I_{S1} I_{S2}}\right)$

Similarly, $V_{BE3} + V_{BE4} = V_T \ln(I_3^2 / I_{S3} I_{S4})$

But $V_{BE1} + V_{BE2} = V_{BE3} + V_{BE4}$, thus

$V_T \ln(I_1^2 / I_{S1} I_{S2}) = V_T \ln(I_3^2 / I_{S3} I_{S4})$

$\Rightarrow \quad I_3 = I_1 \sqrt{\frac{I_{S3} I_{S4}}{I_{S1} I_{S2}}}$

13.3 Eqn. (13.1) $\Rightarrow \quad V_T \ln \frac{I_{REF}}{I_{C10}} = I_{C10} R_4$

Thus, $0.025 \ln \frac{10^{-3}}{10^{-5}} = 10^{-5} R_4 \quad \Rightarrow R_4 = \underline{\underline{11.5\ k\Omega}}$

$V_{BE11} = \underline{0.7\,V}$

$V_{BE10} = 0.7 + 0.025 \ln\left(\frac{0.01}{1}\right) = \underline{0.585V}$

$\boxed{13.4}$ Refer to Fig. 13.1. The upper limit is determined by Q_1 and Q_2 leaving the active mode. Since their collectors are at $15-0.6 = \underline{+14.4V}$; this value can be taken as the upper limit of the common-mode range. The lower limit is determined by Q_5 and Q_6 leaving the active mode; thus the lower limit is

$-15 + V_{BE5} + V_{BE7} + V_{EB3} + V_{BE1} = -15 + 2.4 = \underline{-12.6V}$.

$\boxed{13.5}$ Using the result of Exercise 13.2,

$$I_{14} = 0.25\, I_{REF} \sqrt{\frac{I_{S14} I_{S15}}{I_{SX} I_{SY}}}$$

where X and Y are the two diode-connected transistors. Substituting $I_{REF} = 0.73$ mA,

$I_{S14} = I_{S15} = 3 I_{SX} = 3 I_{SY}$ (because the area of each of the output devices is three times the area of a standard device) we obtain

$I_{14} = 0.25 \times 0.73 \times 3 = \underline{0.548\ mA}$

$\boxed{13.6}$ Refer to Fig. 13.7.

(a) $v_{b6} = i_e (R_2 + r_{e6}) = i_e(1+2.63) = \underline{3.63\,k\Omega \times i_e}$

(b) $i_{e7} = \frac{v_{b6}}{R_3} + \frac{2 i_e}{\beta+1} = \frac{3.63 \times i_e}{50} + \frac{2 i_e}{201} = \underline{0.08\, i_e}$

(c) $i_{b7} = \frac{i_{e7}}{\beta+1} \simeq \underline{0.0004\, i_e}$

(d) $v_{b7} = v_{b6} + i_{e7} r_{e7} = 3.63\, i_e + 0.08\, i_e \times \frac{25}{10.5} = \underline{3.82 k\Omega \times i_e}$

(e) $R_{in} = \frac{v_{b7}}{\alpha i_e} \simeq \underline{3.82\ k\Omega}$

$\boxed{13.7}$ Assume β very large so that we can neglect base current (our purpose here is to investigate the effect of resistance mismatch on the current gain of the mirror).

Thus $v_b = i_i\,(r_e + R)$

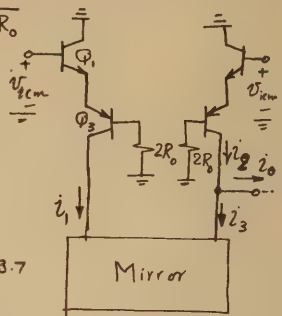
where r_e here is the resistance of the diode. We now can find the emitter current of the transistor as

$$i_e = \frac{v_b}{r_e + R + \Delta R} = i_i\, \frac{r_e + R}{r_e + R + \Delta R}$$

Finally, $i_o = \alpha i_e \simeq i_e = i_i \frac{r_e + R}{r_e + R + \Delta R}$

Thus, $\frac{i_o}{i_i} = \frac{1}{1 + \frac{\Delta R}{r_e + R}} \simeq 1 - \frac{\Delta R}{r_e + R}$ for $\Delta R \ll r_e + R$.

$\boxed{13.8}$ $i_2 = i_4 = \frac{v_{icm}\, \beta_P}{(r_{e1} + r_{e3})(\beta_P + 1) + 2R_o}$



But $r_{e1} = r_{e3}$,

thus $i_2 = i_4 = \frac{v_{icm}\, \beta_P}{2[r_e(\beta_P + 1) + R_o]}$

Now if $R_o \gg r_e(\beta_P + 1)$,

then $i_2 = i_4 \simeq \frac{v_{icm}\, \beta_P}{2 R_o}$

From the result of Exercise 13.7 we have

$$i_3 = i_1 \left(1 - \frac{\Delta R}{R + r_e}\right)$$

Now,

$$i_o = i_2 - i_3 = i_1 - i_1\left(1 - \frac{\Delta R}{R + r_e}\right) = i_1 \frac{\Delta R}{R + r_e}$$

$$= v_{icm} \frac{\beta_P}{2 R_o} \frac{\Delta R}{R + r_e}$$

Thus

$$G_{mcm} \equiv \frac{i_o}{v_{icm}} = \underline{\frac{\beta_P}{2R_o} \frac{\Delta R}{R + r_e}}$$

$\boxed{13.9}$ (a) $R_o = r_{o9} // [R_{out}|_{10}]$

where $R_{out}|_{10}$ can be found from Eqn (13.5),

$$R_{out}|_{10} = r_o \left(\frac{1 + R_E/r_e}{1 + R_E/r_\pi}\right) // r_\mu$$

For our case, $r_o = \frac{\mu}{g_{m10}} = \frac{5000}{40 \times 19} = 6.6\ M\Omega$

$r_e = \frac{25}{19} = 1.3\ k\Omega$

$r_\pi = \frac{200}{g_m} = 260\ k\Omega$

$R_E = 5\ k\Omega$

$r_\mu = 10\beta_o r_o = 6600\ M\Omega$

Thus, $R_{out}|_{10} = 6.6 \left(\frac{1 + \frac{5}{1.3}}{1 + \frac{5}{260}}\right) // 6600 \simeq 31.4\ M\Omega$

Now since $r_{o9} = \frac{2000}{g_{m9}} = \frac{2000}{40 \times 19} = 2.63 M\Omega$

we have $R_o = 2.6 // 31.4 = \underline{2.4\ M\Omega}$

(b) $G_{mcm} = \frac{50}{2 \times 2.4 \times 10^{+6}} \cdot \frac{0.02}{1 + \frac{2.63}{1}} = \underline{0.058\ \mu A/V}$

$$CMRR = 20 \log \frac{G_{m1}}{G_{mcm}}$$

$$= 20 \log \frac{(1/5.26) \times 10^{-3}}{0.058 \times 10^{-6}} = \underline{70\ dB}$$

$\boxed{13.10}$ (a) Average current drawn from the positive supply is $I = 10$ mA. Thus $P_+ = 10 \times 15 = 150$ mW. The current from the negative supply is constant at 10mA. Thus $P_- = 10 \times 15 = 150$ mW. Thus,

$P_S = P_+ + P_- = 300\ mW$

(b) $v_{CE1} = V_{CC} - \hat{V}_o \sin\theta = 15 - 5\sin\theta$

$i_{C1} = I + \dfrac{\hat{V}_o \sin\theta}{R_L} = 10 + 5\sin\theta$

$P_{C1} = v_{CE1} \cdot i_{C1} = (15 - 5\sin\theta)(10 + 5\sin\theta)$

$= 150 - 25\sin^2\theta + 25\sin\theta$

$\dfrac{\partial P_{C1}}{\partial\theta} = -50\sin\theta\cos\theta + 25\cos\theta = 0$ at

$\sin\theta = \frac{25}{50} = 0.5$, i.e. $\theta = 30°$. At this

point $P_{C1} = 150 - 25 \times \frac{1}{4} + 25 \times \frac{1}{2} = \underline{156.25\ mW}$

(c) $\eta = \dfrac{1}{4}\dfrac{\hat{V}_o^2}{IR_L V_{CC}} = \dfrac{1}{4}\dfrac{25}{10 \times 1 \times 15} = \underline{4.2\ \%}$

$\boxed{13.11}$ (a) $P_S = \dfrac{2}{\pi}\dfrac{\hat{V}_o}{R_L}V_{CC} = \dfrac{2}{\pi} \times \dfrac{5}{1} \times 15 = \underline{47.7\ \ mW}$

(b) $v_{CEN} = 15 - 5\sin\theta$

$i_{CN} = \dfrac{5\sin\theta}{R_L} = 5\sin\theta$

$P_{CN} = (15 - 5\sin\theta)\,5\sin\theta$

$= 75\sin\theta - 25\sin^2\theta$

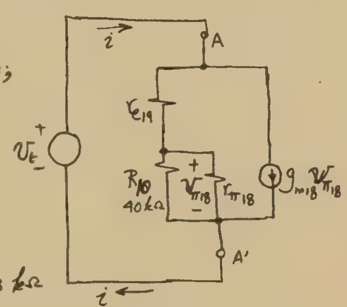
$\dfrac{\partial P_{CN}}{\partial\theta} = 75\cos\theta - 50\sin\theta\cos\theta = 0$

at $\theta = 90°$, thus

$P_{CN\,max} = \underline{50\ mW}$

(c) $\eta = \dfrac{\pi}{4}\dfrac{\hat{V}_o}{V_{CC}} = \dfrac{\pi}{4}\dfrac{5}{15} = \underline{26.2\ \%}$

$\boxed{13.12}$ Q_{19} can be simply represented by a resistance; r_{e19}.



$r_{e19} = \dfrac{25 \times 10^{-3}}{16 \times 10^{-6}}$

$= 1.56\ k\Omega$

$r_{\pi 18} = \dfrac{200}{40 \times 0.165} = 30.3\ k\Omega$

$v_{\pi 18} = v_t \dfrac{(R_{10} \,//\, r_{\pi 18})}{r_{e19} + (R_{10} \,//\, r_{\pi 18})} = 0.92\ v_t$

$i = \dfrac{v_t}{r_{e19} + (R_{10}//r_{\pi 18})} + g_m \times 0.92\ v_t$

$= v_t \times 6.125$

Thus $R = \dfrac{v_t}{i} = \underline{163\ \Omega}$

$\boxed{13.13}$ $R_o = \left[\left(\dfrac{R_{02}}{\beta_{23}+1} + r_{e23} + R\right) / (\beta_{14}+1)\right] + r_{e14}$

$= \left[\left(\dfrac{81000}{51} + 139 + 63\right) / 201\right] + r_{e14}$

$= 9.4 + r_{e14}$

For an output current of $5\ mA$, $r_{e14} = 5\,\Omega$ and $R_o = \underline{14.4\,\Omega}$

$\boxed{13.14}$ For $v_O = 10\sin\omega t$

$\dfrac{dv_O}{dt} = \omega \times 10\cos\omega t$

$SR = \left.\dfrac{dv_O}{dt}\right|_{max} = 10 \times \omega_M = 20\pi f_M$

$f_M = \dfrac{SR}{20\pi} = \dfrac{0.63 \times 10^6}{20\pi} = \underline{10\ kHz}$

$\boxed{13.15}$ $SR = \dfrac{2I}{C_C}$ and $\omega_t = \dfrac{G_{m1}}{C_C}$

Thus $SR = \dfrac{2I}{G_{m1}}\omega_t \qquad (1)$

With a resistance R_E included in each of the emitter leads of Q_3 and Q_4, G_{m1} becomes

$G_{m1} = 2 \times \dfrac{1}{4r_e + 2R_E} = \dfrac{1}{2r_e + R_E}$

$= \dfrac{I}{2Ir_e + IR_E} = \dfrac{I/2}{Ir_e + IR_E/2}$

$= \dfrac{I/2}{V_T + \frac{IR_E}{2}}$

Thus $\dfrac{I}{G_{m1}} = 2\left(V_T + \dfrac{IR_E}{2}\right)$

Substituting in Eqn. (1) yields

$\underline{SR = 4\,(V_T + IR_E/2)\,\omega_t}$

For a given ω_t we can double the slew rate by including R_E whose value provides

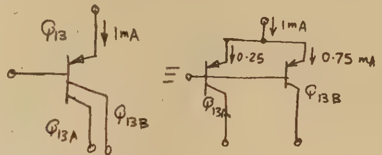
$IR_E/2 = V_T$

i.e. $R_E = \dfrac{2V_T}{I} = 2 \times 2.63 = \underline{5.26\ k\Omega}$

Note that this value of R_E reduces G_{m1} by half and C_C has to be reduced to half its original value. (I, of course, remains constant)

CHAPTER 13 — PROBLEMS

13.1



$V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right)$

$= \underline{633 \text{ mV}}$

$g_{mA} = \frac{I_{CA}}{V_T} = \frac{0.25\times10^{-3}}{0.025} = \underline{10 \text{ mA/V}}$

$g_{mB} = \frac{I_{CB}}{V_T} = \frac{0.75\times10^{-3}}{0.025} = \underline{30 \text{ mA/V}}$

$r_e\big|_{\text{combined}} = \frac{V_T}{I_E} = \underline{25\,\Omega}$

$r_\pi\big|_{\text{combined}} = (\beta+1)\, r_e = 51\times25 = \underline{1275\,\Omega}$

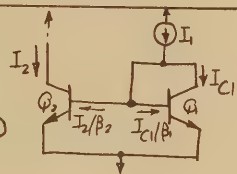
$r_{oA} = \frac{\mu}{g_{mA}} = \frac{2000}{10} = \underline{200\text{ k}\Omega}$

$r_{oB} = \frac{\mu}{g_{mB}} = \frac{2000}{30} = \underline{66.7\text{ k}\Omega}$

$r_{\mu A} = 10\beta_0\, r_{oA} = 10\times50\times200 = \underline{100\text{ M}\Omega}$

$r_{\mu B} = 10\beta_0\, r_{oB} = 10\times50\times66.7 = \underline{33.3\text{ M}\Omega}$

13.2 Node equation at base:



$I_1 = I_{C1}\left(1+\frac{1}{\beta_1}\right) + \frac{I_2}{\beta_2}$

Thus, $I_{C1} = \frac{1}{1+(1/\beta_1)}\left[I_1 - \frac{I_2}{\beta_2}\right]$ (1)

Now, $I_2 = I_{S2}\, e^{V_{BE}/V_T}$,

and $I_{C1} = I_{S1}\, e^{V_{BE}/V_T}$

Thus, $I_2 = \left(\frac{I_{S2}}{I_{S1}}\right) I_{C1}$ (2)

Substituting for I_{C1} from (1) results in

$$I_2 = \frac{(I_{S2}/I_{S1})}{(1+1/\beta_1)}\left[I_1 - \frac{I_2}{\beta_2}\right]$$

Thus, $I_2\left[1 + \frac{I_{S2}/I_{S1}}{1+1/\beta_1}\,\frac{1}{\beta_2}\right] = \frac{(I_{S2}/I_{S1})}{1+1/\beta_1}\, I_1$

$$\frac{I_2}{I_1} = \frac{I_{S2}/I_{S1}}{1+1/\beta_1 + \left(\frac{I_{S2}}{I_{S1}}\right)\left(\frac{1}{\beta_2}\right)}$$

(a) For Q_1 having twice as great a junction area as Q_2, $I_{S1} = 2I_{S2}$ and

$$\frac{I_2}{I_1} = \frac{0.5}{1+\frac{1}{100}+\frac{0.5}{100}} = \underline{0.493}$$

(b) For Q_1 having half as great a junction area as Q_2, $I_{S1} = 0.5 I_{S2}$ and

$$\frac{I_2}{I_1} = \frac{2}{1+\frac{1}{100}+\frac{2}{100}} = \underline{1.942}$$

13.3 Refer to Fig. 13.2. $I_{REF} = 1\text{mA}$

(a) For $I_{C10} = 1\,\mu A$,

$$I_{C10}R_4 = V_T \ln\frac{I_{REF}}{I_{C10}}$$

$$R_4 = \frac{25\times10^{-3}}{10^{-6}}\ln\left(\frac{10^{-3}}{10^{-6}}\right) = \underline{172.7\text{ k}\Omega}$$

$$V_{BE10} = 0.7 + 0.025\ln\left(\frac{10^{-6}}{10^{-3}}\right)$$

$$= \underline{527.3\text{ mV}}$$

(b) For $I_{C10} = 100\,\mu A$,

$$R_4 = \frac{25\times10^{-3}}{10^{-4}}\ln\left(\frac{10^{-3}}{10^{-4}}\right) = \underline{575.6\,\Omega}$$

$$V_{BE10} = 0.7 + 0.025\ln\left(\frac{10^{-4}}{10^{-3}}\right) = \underline{642.4\text{ mV}}$$

13.4 Refer to Fig. E13.2 and use the result of Exercise 13.2, namely

$$I_3 = I_1\sqrt{\frac{I_{S3}\,I_{S4}}{I_{S1}\,I_{S2}}} \quad (1)$$

for our case $I_{S3} = I_{S4} = 3\times10^{-14}$ A, $I_1 = 180\,\mu A$ and $I_3 = 154\,\mu A$. Substituting in (1) provides

$$154 = 180\sqrt{\frac{9\times10^{-28}}{I_{S1}\,I_{S2}}}$$

Since $I_{S1} = I_{S2}$ we have: $I_{S1} = I_{S2} = \underline{3.5\times10^{-14}}$!

Thus the devices required would be 3.5 times the area of a standard device!

13.5 Refer to Fig. 13.5. The voltage between the bases of Q_{14} and Q_{20} is

$$V_{BB} = V_T\ln\frac{I_{C14}}{I_{S14}} + V_T\ln\frac{I_{C20}}{I_{S20}}$$

but $I_{C14} = I_{C20}$ and $I_{S14} = I_{S20}$, thus

$$V_{BB} = 2V_T\ln\frac{I_{C14}}{I_{S14}}$$

Now to reduce I_{C14} from 154 μA to 77 μA, i.e. by a factor of 0.5, we need to reduce V_{BB} by $2V_T\ln 2 \simeq 35$ mV.

This change can be effected by increasing the value of R_{10} so that the current through Q_{19} decreases from 15.8 μA to $15.8\, e^{-\frac{35}{25}} = 3.9\,\mu A$. We note that the current in Q_{18} will increase from 165 μA to approx 177 μA. Thus V_{BE18} will increase from 588 mV to $\simeq$ 590 mV; a negligible change. The base current of Q_{18} is 177/201 = 0.9 μA. Thus the current through R_{10} must be 3.9 − 0.9 = 3 μA. The value of R_{10} should therefore be $\frac{590\text{ mV}}{3\,\mu A} = \underline{197\text{ k}\Omega}$

13.6 dc bias

I_{REF}, I_{C10}, I, I_{C1}, I_{C2}, I_{C3}, I_{C4}, I_{C5} & I_{C6} remain unchanged. V_{BE6} changes to $V_T \ln \frac{I}{2\times10^{-14}}$; thus it decreases by 35 mV to 482 mV. Substituting in Eqn. 13.2 yield the new value of I_{C7} as

$$I_{C7} = \frac{2I}{\beta_N} + \frac{V_{BE6} + IR_2}{R_3}$$

$$= \frac{2\times9.5}{200} + \frac{482 + 9.5\times0.5}{25} = 19.6\ \mu A$$

We shall next ~~consid~~ solve the problem in

Exercise 13.6 :

(a) $v_{b6} = i_e\,(R_2 + r_{e6}) = i_e\,(0.5 + 2.63) = \underline{3.13\,k\Omega \times i_e}$

(b) $i_{e7} = \frac{v_{b6}}{R_3} + \frac{2i_e}{\beta+1} = \frac{3.13\times i_e}{25} + \frac{2i_e}{201} = \underline{0.14\ i_e}$

(c) $i_{b7} = i_{e7}/\beta+1 = \underline{0.0007\ i_e}$

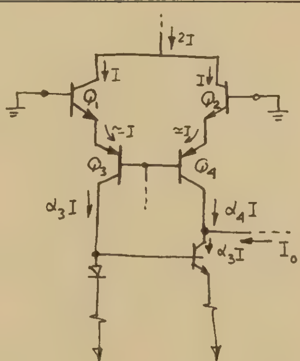
(d) $v_{b7} = v_{b6} + i_{e7}\,r_{e7} = 3.13\,i_e + 0.14\,i_e \times \frac{25}{19.6}$

$= \underline{3.3\ k\Omega \times i_e}$

(e) $R_{in} \equiv \frac{v_{b7}}{\alpha i_e} \simeq \underline{3.3\ k\Omega}$

13.7

To find the input offset voltage we grand the two input terminals and find the output current of the first stage, I_o. Dividing this current by the transconductance



of the input stage, G_{m1}, gives the input voltage which when applied reduces I_o to zero. This by definition is V_{off}.

From the Figure we see that

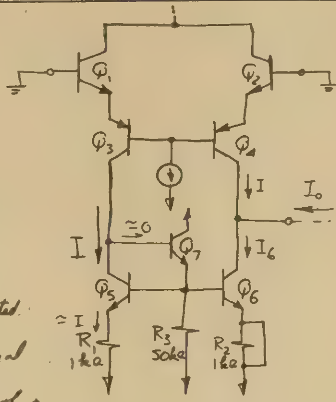
$$I_o = (\alpha_3 - \alpha_4)\,I = \left(\frac{50}{51} - \frac{25}{26}\right)\times 9.5\ \mu A$$

Thus, $V_{off} \equiv \frac{|I_o|}{G_{m1}} = \frac{\left(\frac{50}{51} - \frac{25}{26}\right)\times9.5}{(1/5.26)}$ mV

$= \underline{0.94\ mV}$

13.8

To find the maximum offset voltage that can be compensated by short circuiting R_1 or R_2 we find the offset voltage generated by assuming an ideal input stage except that



one of the two resistors (R_1 or R_2) is short circuited. This offset voltage can be found by analyzing the circuit shown and then dividing I_o by G_{m1}.

$$IR_1 + V_{BE5} = V_{BE6}$$

Thus, $V_{BE6} - V_{BE5} = IR_1 = 9.5\,\mu A \times 1\,k\Omega = 9.5\ mV$

$$\frac{I_6}{I} = e^{\left(\frac{V_{BE6}-V_{BE5}}{V_T}\right)} = 1.46$$

Thus, $I_o \equiv I_6 - I = 0.46\ I$,

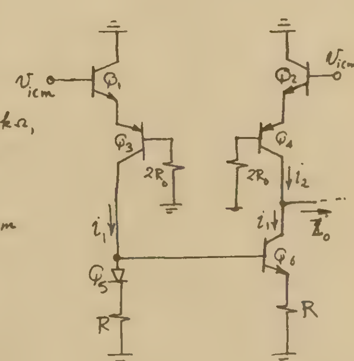
and $V_{off} = \frac{0.46\times9.5\times10^{-6}}{(1/5.26)\times10^{-3}} = \underline{23\ mV}$

13.9

Reducing $\frac{\Delta R}{R}$ by a factor of 10 reduces G_{mcm} by a factor of 10 (see the equation for G_{mcm} in Exercise 13.8) and thus increases the common-mode rejection ratio by a factor of 10 or $\underline{20\ dB}$.

13.10

$$i_1 = \frac{v_{icm}}{2r_e + \frac{2R_o}{\beta_3+1}} \times \frac{\beta_3}{\beta_3+1}$$



where $r_e = r_{e1} = r_{e3} = 2.63\ k\Omega$,

$R_o = 2.4\ M\Omega$, and

$\beta_3 = 50$

Thus, $i_1 = 9.86\times10^{-6}\ v_{icm}$

$$i_2 = \frac{v_{icm}}{2r_e + \frac{2R_o}{\beta_4+1}} \times \frac{\beta_4}{\beta_4+1}$$

where $\beta_4 = 60$

Thus, $i_2 = 11.7\times10^{-6}\ v_{icm}$

$$i_o = i_2 - i_1 = 1.84\times10^{-6}\ v_{icm}$$

$$G_{mcm} \equiv \frac{i_o}{v_{icm}} = 1.84\ \mu A/V$$

$$CMRR \equiv 20\log\frac{G_{m1}}{G_{mcm}} = 20\log\frac{1}{5.26\times10^{3}\times1.84\times10^{-6}}$$

$= \underline{40.3\ dB}$

13.11 (a) Short circuiting R_1

First we must find the effect on the dc operating currents of Q_5 and Q_6. Q_5 will continue to carry a current equal to I (i.e. $9.5\,\mu A$). V_{BE5} will continue to be 517 mV. This voltage now appears across the series combination of V_{BE6} and R_2. Thus we can write

$$V_T \ln \frac{9.5\,\mu A}{I_{C6}} = I_{C6} R_2$$

The solution to this equation is $I_6 \simeq 7.14\,\mu A$

Now refer to Fig. 13.7 and let $R_1 = 0$,

$i_{e5} \simeq i_e$, thus $v_{b6} = i_e r_{e5} = i_e \times 2.63\,k\Omega$

$$i_{e6} = \frac{v_{b6}}{r_{e6} + R_2} = \frac{i_e \times 2.63}{3.5 + 1} = 0.58\, i_e$$

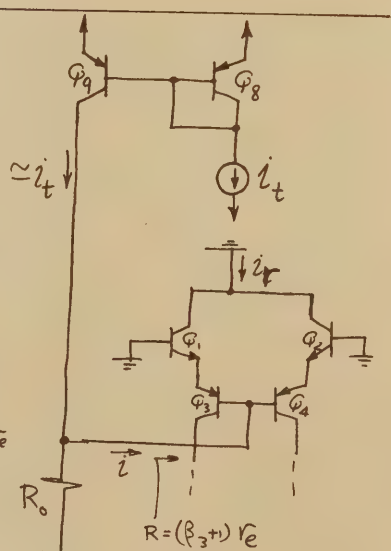
$$i_o = \alpha i_e + \alpha \times 0.58\, i_e = \alpha \times 1.58\, i_e$$

Thus the gain decreases by a factor of $\frac{1.58}{2} = \underline{0.79}$ i.e. the new gain = old gain $\times$ 0.79.

(b) Short circuiting R_2

The effect on dc bias is as follows: Q_5 still conducts $9.5\,\mu A$ and $V_{BE5} = 517$ mV. V_{BE6} becomes

13.12



$$i = i_t \frac{R_o}{R_o + (\beta_3+1) r_e}$$

$$i_r \simeq -\beta_3 i$$

$$= -i_t \frac{\beta_3 R_o}{R_o + (\beta_3+1) r_e}$$

Loop Gain $\equiv A\beta$

$$\equiv -\frac{i_r}{i_t}$$

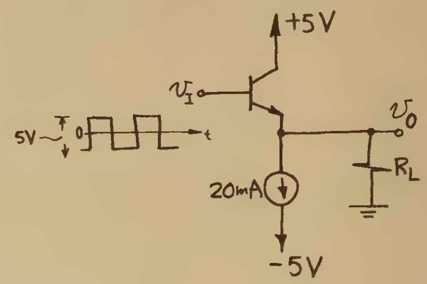
$$= \frac{\beta_3 R_o}{R_o + (\beta_3+1) r_e}$$

$$= \frac{50 \times 2.4}{2.4 + 51 \times 2.63 \times 10^{-3}}$$

$$= \underline{47.4}$$

Thus $1 + A\beta = 48.4$ or 33.7 dB. Therefore the common-mode gain will be reduced by 33.7 dB. Correspondingly the CMRR increases by 33.7 dB; from 40.3 dB (from Problem 13.10) to $\underline{74\text{ dB}}$

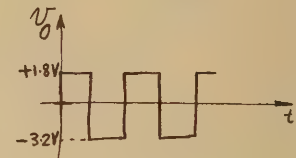
13.13



The lowest resistance load that can be driven is determined from

$$\frac{3.2\,V}{R_L} = 20\text{ mA}$$

$$\Rightarrow R_L = \frac{3.2\,V}{20\text{ mA}} = \underline{160\,\Omega}$$



Output power is

$$P_O = \frac{1}{2}\left(\frac{1.8^2}{R_L} + \frac{3.2^2}{R_L}\right) \simeq \underline{42\text{ mW}}$$

$$P_{S+} = \left(\frac{1.8}{0.16} + 20\right) 5 \times \tfrac{1}{2} + 0 \times 5 \times \tfrac{1}{2} \simeq 78\text{ mW}$$

$$P_{S-} = 20 \times 5 = 100\text{ mW}$$

Thus $P_S = \underline{178\text{ mW}}$

Average Power Lost in Follower $= 178 - 42 = \underline{136\text{ mW}}$

Efficiency $= \frac{42}{178} \times 100 = \underline{23.6\,\%}$

$$V_{BE6} = V_{BE5} + 9.5\,\mu A \times 1\,k\Omega$$

$$= 526.5\text{ mV}$$

i.e. $V_{BE6} - V_{BE5} = 9.5$ mV which causes I_{C6} to become, $I_{C6} = 9.5\; e^{\frac{9.5}{25}} = 13.9\,\mu A$.

Now refer to Fig. 13.7 and let $R_2 = 0$,

$$v_{b6} = v_{b5} = i_e (r_{e5} + R_1) = 3.63\, i_e$$

Thus, $i_{e6} = \frac{v_{b6}}{r_{e6}} = \frac{3.63\, i_e}{1.8} = 2\, i_e$,

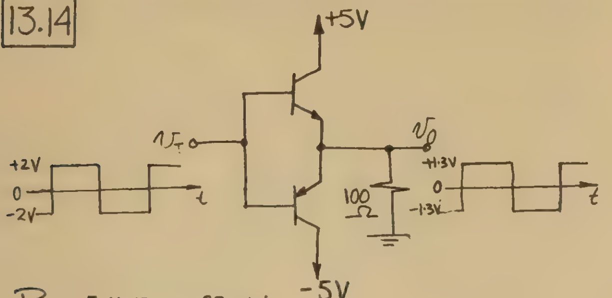
and $i_o = \alpha i_e + \alpha\, 2\, i_e = 3\alpha i_e$

Thus the gain increases by a factor of $\underline{1.5}$, i.e. new gain = old gain $\times$ 1.5.

(c) Short circuiting R_1 and R_2

The dc bias and gain remain $\underline{\text{unchanged}}$.

13.14



$P_S = 5 \times 13 = \underline{65\text{ mW}}$

$P_0 = \frac{(1.3)^2}{0.1} = 16.9\text{ mW}$

$\eta = \frac{P_0}{P_S} \times 100 = \underline{26\%}$

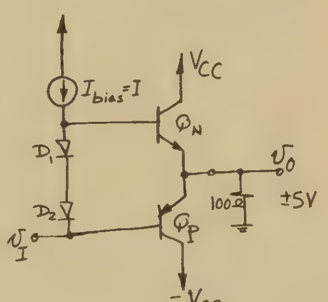
* For 6-V peak-to-peak input the output is a symmerical −2.3 V → +2.3 V square wave, thus

$P_S = 5 \times 23 = 115\text{ mW}$

$P_0 = \frac{2.3^2}{0.1} = 52.9\text{ mW}$

$\eta = \underline{46\%}$

13.15



* The quiescent current in Q_N and Q_P is $10I$.

* The maximum current in each of Q_N and Q_P is $\frac{5}{0.1} = 50$ mA

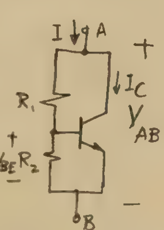
Thus we can write

$$\frac{50}{10I} = e^{\frac{0.1}{0.025}}$$

$\Rightarrow I = \underline{0.09\text{ mA}}$

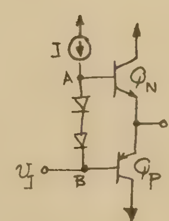
13.16

The circuit shown is known as a V_{BE} multiplier; it provides a voltage drop equal approximately to $V_{BE}\left(1 + \frac{R_1}{R_2}\right)$ and thus can be set by selecting an appropriate value for R_1/R_2. This is based on the assumption that the transistor β is large and thus its base current is negligibly



small. We should also note that although the voltage drop V_{AB} is rather insensitive to the value of I, it nevertheless depends on the value of I because V_{BE} depends on I_C which is usually quite close to I. The advantage of using this circuit in biasing the class AB output stage is that by the proper selection of $\frac{R_1}{R_2}$ one can arrange for any desired value of quiescent bias current in the output transistors. This current can be made much smaller than the current I supplied to to the biasing network. This is in ~~conta~~ contrast to the the case where two diode-connected transistors are used to bias the class AB transistors where the quiescent output stage current will be equal to I, ~~or to~~ ~~th~~ or even greater than I if the output devices are larger than the biasing devices.

To be specific consider first the case of biasing using two diode-connected transistors. Furthermore assume that the output devices are similar to the biasing devices. If $I = 1\text{mA}$ then the quiescent current in the output devices will also be 1mA which is rather large. If $I_S = 10^{-14}$ A then in the quiesent state $V_{AB} = 2 \times 0.025 \ln \frac{10^{-3}}{10^{-14}} = \underline{1.266\text{V}}$

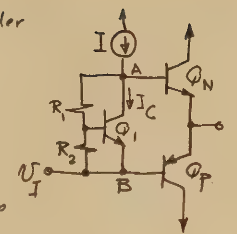


The incremental resistance of the biasing network will be $2 \times r_d = 2 \times 25 = \underline{50\,\Omega}$.

When Q_N is supplying its maximum current and its base current is 0.9 mA, only 0.1 mA will be left for the biasing network. Thus V_{AB} becomes $2 \times 0.025 \ln \frac{10^{-4}}{10^{-14}} = \underline{1.151\text{V}}$. The incremental resistance becomes $2 \times 250 = \underline{500\,\Omega}$.

Consider next the case of using the V_{BE} multiplier circuit for biasing the Class AB stage. The circuit is as follows:

Again let $I = 1\,mA$ and consider the quiescent state. If we use $0.05\,mA$ through the R_1, R_2 divider then $I_C = 0.95\,mA \simeq 1\,mA$. Let us further neglect the base current of Q_1. Since Q_1 is carrying a current of $1\,mA$ its V_{BE} will be $0.025 \ln \frac{10^{-3}}{10^{-14}} = 0.633\,V$. Thus,



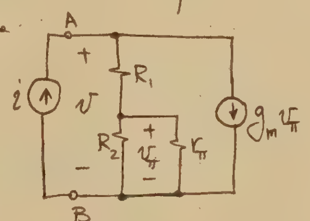
$V_{AB} = 0.633\left(1+\frac{R_1}{R_2}\right)$ where $R_2 = \frac{0.633}{0.05} \simeq 12.7\,k\Omega$

Let us assume that we wish to bias Q_N and Q_P at say $0.1\,mA$. It follows that V_{AB} must be set to $2 \times 0.025 \ln \frac{10^{-4}}{10^{-14}} = 1.151\,V$. We can now find the required value of R_1 from

$1.151 = 0.633\left(1+\frac{R_1}{12.7}\right) \Rightarrow \underline{R_1 = 10.4\,k\Omega}$

In the extreme case when the base current of Q_N increases to $0.9\,mA$, only $0.1\,mA$ will flow through the biasing network. With a couple of iterations we can show that this $0.1\,mA$ divides as follows: $0.044\,mA$ through the resistive divider and $\simeq 0.056\,mA$ through the transistor collector. Thus V_{AB} reduces to $0.044\,(12.7+10.4) = \underline{1.016\,V}$.

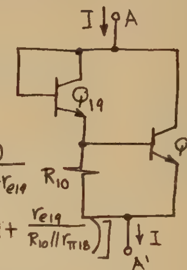
To find the incremental resistance of the biasing network we use a simple model for Q_1 as shown. The resistance r_{AB} can be shown to be



$$r_{AB} = \frac{1+\frac{R_1}{R_2}+\frac{R_1}{r_\pi}}{\frac{1}{r_e}+\frac{1}{R_2}}$$

For our case we can substitute numerical values in this expression and obtain: r_{AB} in the quiescent state $\simeq \underline{150\,\Omega}$, and r_{AB} in the extreme state $\simeq \underline{960\,\Omega}$. We note that the change in the value of incremental resistance (an increase by a factor of about 6) is less than the corresponding increase in the case of the diode biasing network (a factor of 10).

13.17 From the small-signal model shown we have

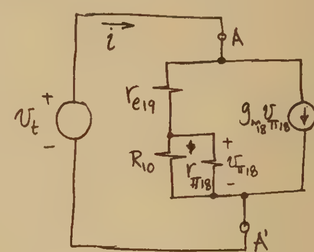


$$v_{\pi 18} = v_t \frac{(R_{10} // r_{\pi 18})}{(R_{10} // r_{\pi 18}) + r_{e19}}$$

$$i = \frac{v_t}{r_{e19} + (R_{10} // r_{\pi 18})} + g_{m18} v_t \frac{(R_{10}//r_{\pi 18})}{(R_{10}//r_{\pi 18}) + r_{e19}}$$

$$r_{AA'} = \frac{v_t}{i} = \left[r_{e19} + (R_{10} // r_{\pi 18})\right] // \left[\frac{1}{g_{m18}}\left(1+\frac{r_{e19}}{R_{10}//r_{\pi 18}}\right)\right]$$

(a) $\underline{R_{10} = 10\,k\Omega}$



Assume $V_{BE18} \simeq 0.6\,V$

then $I_{R_{10}} = \frac{0.6}{10} = 0.06\,mA$

$I_{C19} \simeq I_{R_{10}} = 60\,\mu A$

$V_{BE19} = 0.025 \ln \frac{60 \times 10^{-6}}{10^{-14}}$

$= 0.563\,V$

$I_{C18} = 180 - 60 = 120\,\mu A$

$V_{BE18} = 0.025 \ln \frac{120 \times 10^{-6}}{10^{-14}} = 0.580\,V$

Thus, $V_{AA'} = 0.563 + 0.580 = \underline{1.143\,V}$

$g_{m18} = 4.8\,mA/V$, $r_{\pi 18} = 20.8\,k\Omega$, $r_{e19} = 416.7\,\Omega$

$r_{AA'} = \underline{214.6\,\Omega}$

(b) $\underline{R_{10} = 80\,k\Omega}$

Assume $V_{BE18} \simeq 0.6\,V$, then $I_{R_{10}} = \frac{0.6}{80} = 7.5\,\mu A$

$I_{C19} \simeq I_{R_{10}} = 7.5\,\mu A$. $V_{BE19} = 0.025 \ln \frac{7.5 \times 10^{-6}}{10^{-14}} = 0.511\,V$

$I_{C18} = 180 - 7.5 = 172.5\,\mu A$. $V_{BE18} = 0.589\,V$.

Since $I_{B18} = \frac{172.5}{100} \simeq 1.7\,\mu A$ we should take it into account. Thus a better estimate of I_{C19} is $I_{R10} + I_{B18} = \frac{0.589}{0.080} + 1.7 \simeq 9\,\mu A$. Thus,

$V_{BE19} = 0.515\,V$. $I_{C18} = 180 - 9 = 171\,\mu A$. $V_{BE18} = 0.589\,V$

Thus $V_{AA'} = \underline{1.104\,V}$.

$g_{m18} = 6.84\,mA/V$, $r_{\pi 18} = 14.62\,k\Omega$, $r_{e19} = 2.78\,k\Omega$

$r_{AA'} = \underline{251\,\Omega}$.

13.18 dc Bias:

$$I_{REF} = \frac{V_{CC} - V_{EB12} - V_{BE11} - (-V_{EE})}{R_5}$$

$$= \frac{9 - 0.7 - 0.7 - (-9)}{39} = 0.43\,mA$$

$V_T \ln \frac{I_{REF}}{I_{C10}} = I_{C10} R_4$

$0.025 \ln \frac{0.43}{I_{C10}} = I_{C10} \times 5 \Rightarrow I_{C10} \simeq 16\,\mu A$

$I = \frac{16}{2} = 8\,\mu A$

$I_{C1} = I_{C2} = I_{C3} = I_{C4} = 8\,\mu A$

$I_{C6} \simeq 8\ \mu A$

$I_{C5} \simeq 8\ \mu A$

$V_{BE6} = V_T \ln \frac{I}{I_S} = 0.025 \ln \frac{8\times10^{-6}}{10^{-14}} = 512.5\ mV$

$$I_{C7} \simeq I_{E7} = \frac{2I}{\beta_N} + \frac{V_{BE6}+IR_2}{R_3} = \frac{16}{200} + \frac{512.5+8\times1}{50} = 10.5\ \mu A$$

$I_{C13B} = 0.75 I_{REF} = 0.75\times0.43 = 0.323\ mA$

$I_{C17} \simeq 323\ \mu A$.

$V_{BE17} = V_T \ln\frac{I_{C17}}{I_S} = 605\ mV$

$$I_{C16} \simeq I_{E16} = I_{B17} + \frac{I_{E17}R_8+V_{BE17}}{R_9} = 14.4\ \mu A$$

$I_{C23}\simeq I_{E23} \simeq 0.25 I_{REF} = 108\ \mu A$

Assume that $V_{BE18}\simeq 0.6V$, then $I_{R_{10}} = \frac{0.6}{40} = 15\ \mu A$.

$I_{E18} = 108-15 = 93\ \mu A$, $I_{C18}\simeq I_{E18} = 93\ \mu A$

$V_{BE18} = V_T \ln \frac{93\times10^{-6}}{10^{-14}} = 574\ mV$

$I_{B18} = \frac{93}{200} = 0.5\ \mu A$

$I_{C19} = 15.5\ \mu A$

$V_{BE19} = V_T\ln\frac{I_{C19}}{I_S} = 529\ mV$

$R_{o4} = 12.3\ M\Omega$

$$R_{o6} = r_{o6}\left(\frac{1+R_2/r_{e6}}{1+R_2/r_{\pi6}}\right) // r_{\mu6} \simeq 15.63\left(\frac{1+1/3.125}{1+1/159.4}\right)$$

$\simeq 20.5\ M\Omega$

$R_{o1} = R_{o4}//R_{o6} = 7.7\ M\Omega$

Small-signal analysis of the second stage:

$$R_{i2} = (\beta_{16}+1)\left[r_{e16} + R_9 // (\beta_{17}+1)(r_{e17}+R_8)\right]$$
$$= 201\left[1.74 + 50//201\times(0.077+0.1)\right]$$
$$= 4.5\ M\Omega$$

$R_{i17} = (\beta_{17}+1)(r_{e17}+R_8) = 201\times(0.077+0.1) = 35.6\ k\Omega$

$$v_{b17} = v_{i2}\frac{(R_9//R_{i17})}{(R_9//R_{i17})+r_{e16}} = v_{i2}\frac{(50//35.6)}{(50//35.6)+1.74} = 0.92\, v_{i2}$$

$$i_{c17} = \frac{\alpha\, v_{b17}}{r_{e17}+R_8} \simeq \frac{0.92\, v_{i2}}{0.077+0.1} = 5.2\, v_{i2}$$

$G_{m2} = 5.2\ mA/V$

$R_{o13B} = r_{o13B} = \frac{2000}{12.92} = 154.8\ k\Omega$

$$R_{o17} = r_{o17}\left(\frac{1+R_8/r_{e17}}{1+R_8/r_{\pi17}}\right)//r_{\mu17}$$
$$\simeq 387\left(\frac{1+0.100/0.077}{1+0.1/15.5}\right) = 884\ k\Omega$$

$R_{o2} = R_{o13B}//R_{o17} = 154.8//884 \simeq 132\ k\Omega$

$V_{BB} = V_{BE18} + V_{BE19} = 574 + 529 = 1.103\ V$

$$V_{BB} = V_T\ln\frac{I_{C14}}{I_{S14}} + V_T \ln\frac{I_{C20}}{I_{S20}}$$
$$= V_T \ln\frac{I_{C14}^2}{I_{S14}^2} = 2\times0.025\ln\frac{I_{C14}}{3\times10^{-14}}$$

$\Rightarrow I_{C14} = I_{C20} = 114\ \mu A$

Summary: DC operating currents in μA

Q_1	8	Q_8	16	Q_{13B}	323	Q_{19}	15.5
Q_2	8	Q_9	16	Q_{14}	114	Q_{20}	114
Q_3	8	Q_{10}	16	Q_{15}	0	Q_{21}	0
Q_4	8	Q_{11}	430	Q_{16}	14.4	Q_{22}	0
Q_5	8	Q_{12}	430	Q_{17}	323	Q_{23}	108
Q_6	8	Q_{13A}	108	Q_{18}	93	Q_{24}	0
Q_7	10.5						

Small-signal analysis of input stage

$r_e = \frac{25\,mV}{8\,\mu A} = 3.125\ k\Omega$

$R_{id} = 4r_\pi = 4(\beta_N+1)r_e = 2.5\ M\Omega$

$G_{m1} = \frac{\alpha}{2r_e} \simeq \left(\frac{1}{6.25}\right) mA/V$

$$R_{o4} = r_{o4}\left(\frac{1+r_{e2}/r_{e4}}{1+r_{e2}/r_{\pi4}}\right)//r_{\mu4} \simeq 6.25\times\frac{1+1}{1+3.125/159.4}$$

Analysis of the output stage:

$R_{i3}\simeq \beta_{23}\times100\ k\Omega = 5\ M\Omega$

$$A_2 = -G_{m2}R_{o2}\frac{R_{i3}}{R_{i3}+R_{o2}} = -5.2\times132\ \frac{5}{5+0.132}$$
$$= -668.7$$

$\mu\simeq1$

$R_{o23} = \frac{R_{o2}}{\beta_{23}+1} + r_{e23} = \frac{132}{51}+0.231 = 2.82\ k\Omega$

$R_o = \frac{R_{o23}}{\beta_{20}+1} + r_{e20} = \frac{2.82\times10^3}{51}+5 \simeq 60\ \Omega$

Adding R_7, $R_o' = 60+27 = 87\ \Omega$.

Small-Signal Gain

$$\frac{v_o}{v_i} = -G_{m1}(R_{o1}//R_{i2})(-G_{m2}R_{o2})\,\mu\,\frac{R_L}{R_L+R_o}$$
$$= -\frac{1}{6.25}\times(7.7//4.5)\times10^3\times(-5.2\times132)\times1\times\frac{2}{2+0.087}$$
$$= 298{,}915\ V/V \quad \text{or} \quad \underline{109.5\ dB}$$

Frequency Response

$C_i = C_C(1+|A_2|) = 30\times669.7 = 20{,}091\ pF$

$R_t = R_{o1}//R_{i2} = 2.84\ M\Omega$

$$f_P = \frac{1}{2\pi C_i R_t} = \frac{1}{2\pi\times20.091\times10^{-9}\times2.84\times10^6}$$
$$= 2.8\ Hz$$

Thus, $f_{3dB} = 2.8\ Hz$

$f_t = A_0 f_{3dB} = 298,915 \times 2.8 = \underline{834 \text{ kHz}}$

Another way for calculating f_t is

$$f_t = \frac{G_{m1}}{2\pi C_C} = \frac{10^{-3}}{2\pi \times 6.25 \times 30 \times 10^{-12}}$$

$$= \underline{849 \text{ kHz}}$$

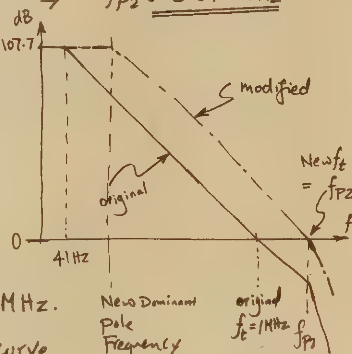
which is close to the value previously found.

$$SR = \frac{2I}{C_C} = \frac{2 \times 8 \times 10^{-6}}{30 \times 10^{-12}} = \underline{0.53 \text{ V}/\mu s}$$

13.19 Let the second pole be at f_{P2}. Since it produces $10°$ of phase at $f_t = 1$ MHz we can write

$$\tan^{-1}\frac{f_t}{f_{P2}} = 10° \Rightarrow f_{P2} = \underline{5.67 \text{ MHz}}$$

To reduce the phase margin (at unity gain) to $45°$ we much choose a new value of C_C that places f_t at f_{P2} i.e. new f_t = 5.67 MHz. The modified gain curve



will be as shown. Note that the dominant pole frequency will be (4.1×5.67) Hz. Thus the 3 dB frequency of the modified op amp will be $\underline{23.3 \text{ Hz}}$. The new value of C_C will be $30 \text{ pF}/5.67 = \underline{5.3 \text{ pF}}$. Finally the open-loop gain at 1000 Hz will increase from 1,000 to 5,670 V/V, i.e. by $\underline{15 \text{ dB}}$.

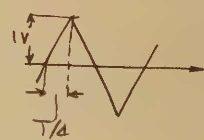
13.20 We have to determine whether the bandwidth is determined by the small-signal frequency limitations by by the large signal dynamics (i.e. the limited slew rate of the op amp). If the small-signal dynamics were the determining factor then the 3 dB bandwidth of the follower would be f_t, i.e. 1 MHz. On the other hand, if slew rate is the limiting factor then the bandwidth can be determined from

$v_O = 1 \sin \omega t \quad \frac{dv_O}{dt} = \omega \cos \omega t \quad \left.\frac{dv_O}{dt}\right|_{max} = \omega$

Thus, $\omega_{bandwidth} = SR = 0.63 \times 10^6$ V/s,

leading to a bandwidth of $\frac{0.63 \times 10^6}{2\pi}$ Hz or 100 kHz. It follows that for ± 1V output sinusoid the follower bandwidth is $\underline{100 \text{ kHz}}$. (Notice that this is ten times the bandwidth for ± 10V output sinusoid).

A ± 1V symmetrical triangle wave will be faithfully reproduced up to the frequency $f = 1/T$ determined from



$$\frac{1}{T/4} = SR = 0.63 \times 10^6 \text{ V/s}$$

$$\Rightarrow T = \frac{4}{0.63 \times 10^6} \quad \text{or} \quad f = \frac{0.63}{4} = \underline{157.5 \text{ kHz}}$$

CHAPTER 14 — EXERCISES

14.1 Eqn. (14.13) $\Rightarrow m \equiv \frac{R_3}{R_4} = 4Q^2 = 4$

Eqn. (14.14) $\Rightarrow CR = \frac{2Q}{\omega_0} = \frac{2 \times 1}{10^4} = 0.2$ ms

$C_1 = C_2 = 1$nF $\Rightarrow C = 1$nF $\Rightarrow R = \frac{0.2 \times 10^{-3}}{10^{-9}} = 200$ kΩ

Thus, $R_3 = \underline{200 \text{ k}\Omega}$ and $R_4 = \frac{200}{4} = \underline{50 \text{ k}\Omega}$

14.2 From Fig. 14.5(a), the poles of the RC network are the roots of the denominator polynomial,

$$D(s) = s^2 + s\left(\frac{1}{C_1R_3} + \frac{1}{C_2R_3} + \frac{1}{C_1R_4}\right) + \frac{1}{C_1C_2R_3R_4}$$

$$= s^2 + s\left(\frac{1}{CR} + \frac{1}{CR} + \frac{4}{CR}\right) + \frac{4}{(CR)^2}$$

$$= s^2 + s\,\frac{6}{0.2 \times 10^{-3}} + \frac{4}{(0.2 \times 10^{-3})^2}$$

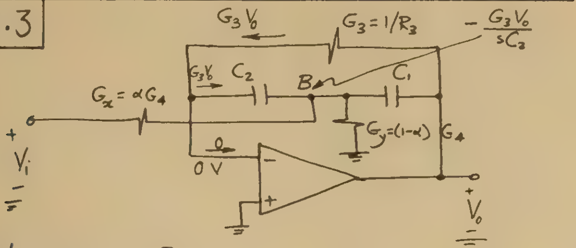
$$= s^2 + s\,3 \times 10^4 + 1 \times 10^8$$

Thus the poles are

$$s = \frac{-3 \times 10^4 \pm \sqrt{9 \times 10^8 - 4 \times 10^8}}{2}$$

$$= \underline{-0.382 \times 10^4} \text{ and } \underline{-2.618 \times 10^4} \text{ rad/s}$$

14.3



Node equation at B:

$$G_3V_o + sC_1\left(V_o + \frac{G_3}{sC_2}V_o\right) + \left(V_i + \frac{G_3}{sC_2}V_o\right)G_x + G_y\frac{G_3}{sC_2}V_o = 0$$

Thus, $$V_o\left[G_3 + sC_1 + \frac{C_1}{C_2}G_3 + \frac{G_xG_3}{sC_2} + \frac{G_yG_3}{sC_2}\right] = -V_iG_x$$

$$\frac{V_o}{V_i}(s) = \frac{-G_x}{sC_1 + G_3\left(1+\frac{C_1}{C_2}\right) + \frac{(G_x+G_y)G_3}{sC_2}}$$

$$= \frac{-sG_x/C_1}{s^2 + s\frac{G_3}{C_1}\left(1+\frac{C_1}{C_2}\right) + \frac{(G_x+G_y)G_3}{C_1C_2}}$$

$$= \frac{-s\alpha/C_1R_4}{s^2 + s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}$$

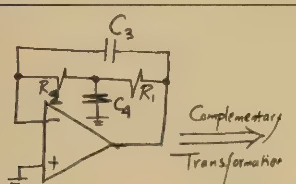
which is a bandpass function having poles coincident with the zeros of $t(s)$ in Fig. 14.5(a).

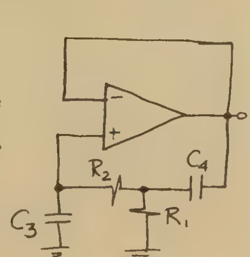
The center-frequency gain is

$$\frac{V_o}{V_i}(j\omega_0) = -\alpha\Big/\left[\left(1+\frac{C_1}{C_2}\right)\left(\frac{R_4}{R_3}\right)\right] = -\alpha\Big/\left(2\times\tfrac{1}{4}\right) = -2\alpha$$

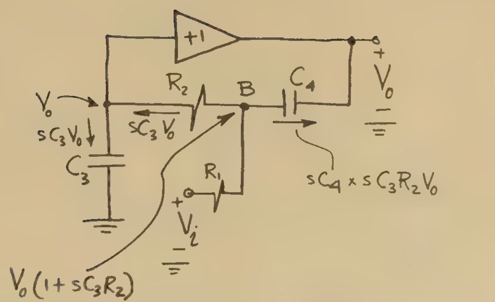
For unity gain, $\alpha = 0.5$. Thus, $R_x = R_y = \frac{R_4}{0.5} = \underline{\underline{100\ k\Omega}}$.

14.4





We shall now show that by injecting the input signal to the grounded end of R_1 we will be able to realize a second-order low-pass function. The analysis is illustrated below. Note that we have replaced the op amp and its negative feedback with a unity-gain amplifier.



Node equation at B: $sC_3V_o + s^2C_3C_4R_2V_o + \frac{1}{R_1}\left[V_o(1+sC_3R_2) - V_i\right] = 0$

$$\Rightarrow V_o\left[\left(s^2C_3C_4R_2\right) + s\left(C_3 + C_3\frac{R_2}{R_1}\right) + \frac{1}{R_1}\right] = \frac{V_i}{R_1}$$

$$\frac{V_o}{V_i} = \frac{1/C_3C_4R_1R_2}{s^2 + s\frac{1}{C_4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) + \frac{1}{C_3C_4R_1R_2}}$$

which is a low-pass function having poles identical to the zeros of $t(s)$ given in Fig. 14.5(b).

14.5 Eqn. (14.26) $\Rightarrow$ $2\pi \times 10^4 = \frac{1}{CR}$

Thus, $CR = \frac{10^{-4}}{2\pi}$ s. For $R = 10\ k\Omega$, $C = \underline{\underline{1.59\ nF}}$.

Eqn. (14.27) $\Rightarrow$ $20 = \frac{R_7}{R}$, thus $R_7 = 20R$.

For $R = 10\ k\Omega$, $R_7 = \underline{\underline{200\ k\Omega}}$.

To find the center-frequency gain, i.e. the gain at resonance, refer to Fig. 14.12. At the resonance frequency $V_1 = V_i$. Now since $V_{o1} = \frac{R_4+R_5}{R_5}V_2$ where V_2 is the voltage at the positive input terminal of op-amp A_2, and because of the virtual short circuits at the inputs of A_2 and A_1 we see that $V_{o1} = \frac{R_4+R_5}{R_5}V_1 = 2V_1$. Thus at $\omega = \omega_0$ we have $V_{o1} = 2V_i$, i.e. the center-frequency gain is 2.

14.6 $\omega_0 = 1/CR \Rightarrow CR = \frac{1}{2\pi \times 10^4}$

For $R = 10\ k\Omega$, $C = \frac{1}{2\pi \times 10^8} = \underline{\underline{1.59\ nF}}$

$R_d = QR = 20 \times 10 = \underline{\underline{200\ k\Omega}}$

From Eqn. (14.28) and Fig. 14.15 we find that

$$\frac{V_{bp}}{V_i}(s) = \frac{-n_2 s^2 \times \frac{\omega_0}{Q}}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Thus the center-frequency gain is $(-n_2Q)$. For a gain of unity, $n_2 = 1/Q$. It follows that $R_g = R/n_2 = QR = \underline{\underline{200\ k\Omega}}$.

14.7 From Equations (14.36) and (14.37),

$$C_3 = C_4 = \omega_0 T_c C = 2\pi \times 10^4 \times \frac{1}{200\times10^3} \times 20$$
$$= \underline{6.283 \text{ pF}}$$

From Eqn. (14.39): $C_5 = \frac{C_4}{Q} = \frac{6.283}{20} = \underline{\underline{0.314 \text{ pF}}}$

From Eqn. (14.40): $C_6 = C_5 = \underline{\underline{0.314 \text{ pF}}}$.

14.8 (a) $L(s) = \left(1+\frac{R_2}{R_1}\right) \frac{Z_p}{Z_p+Z_s}$

$$= \frac{1+\frac{20.3}{10}}{1+Z_s Y_p}$$
$$= \frac{3.03}{1+\left(R+\frac{1}{sC}\right)\left(\frac{1}{R}+sC\right)}$$
$$= \frac{3.03}{3+sCR+\frac{1}{sCR}}$$

where $R = 10 \text{ k}\Omega$ and $C = 16 \text{ nF}$.

Thus $L(s) = \dfrac{3.03}{3+s\,16\times10^{-5}+\frac{1}{s\times16\times10^{-5}}}$

The closed loop poles are obtained by setting $L(s)=1$, i.e. they are the values of s satisfying

$$3+s\times16\times10^{-5}+\frac{1}{s\times16\times10^{-5}} = 3.03$$

$$\Rightarrow s \simeq \frac{10^5}{16}(0.015 \pm j)$$

(b) The frequency of oscillation is $(10^5/16)$ rad/s or 1 kHz.

(c) Refer to Fig. 14.21. At the positive peak $\hat{V}_o$, the voltage at node b will be one diode drop (0.7V) above the voltage v_1 which is about 1/3 of v_0; thus $V_b = 0.7+\frac{\hat{V}_o}{3}$.

Now if we neglect the current through D_2 in comparison with the currents through R_5 and R_6 we find that $\frac{\hat{V}_o - V_b}{R_5} \simeq \frac{V_b-(-15)}{R_6}$, thus

$$\frac{\hat{V}_o - V_b}{1} = \frac{V_b+15}{3} \Rightarrow \hat{V}_o = \frac{4}{3}V_b + 5$$

$$\hat{V}_o = \frac{4}{3}\left(0.7+\frac{\hat{V}_o}{3}\right)+5$$

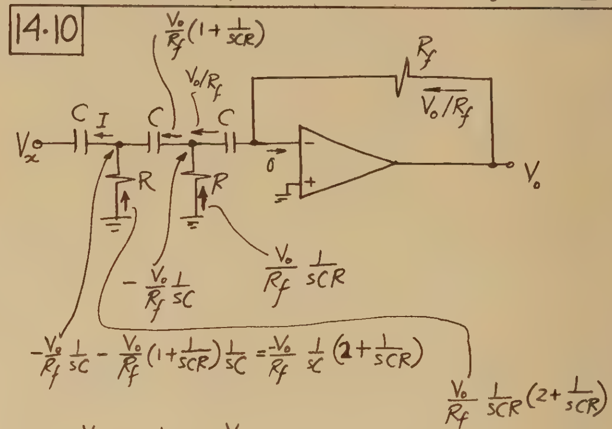
which leads to $\hat{V}_o = 10.68\text{V}$.

From symmetry we see that the negative peak is equal to the positive peak. Thus we find that the peak-to-peak output is $2\times10.68 = \underline{\underline{21.36 \text{ V}}}$.

14.9 (a) For oscillations to start, $R_2/R_1 = 2$. Thus the potentiometer should be set so that its resistance to ground is $\underline{20 \text{ k}\Omega}$.

(b) $\omega_0 = 1/CR = 1/16\times10^{-9}\times10\times10^3 \Rightarrow f_0 \simeq \underline{1 \text{ kHz}}$

14.10



$$I = \frac{V_o}{R_f}\left(1+\frac{1}{sCR}\right)+\frac{V_o}{R_f}\frac{1}{sCR}\left(2+\frac{1}{sCR}\right)$$
$$= \frac{V_o}{R_f}\left[1+\frac{1}{sCR}+\frac{2}{sCR}+\frac{1}{(sCR)^2}\right] = \frac{V_o}{R_f}\left[1+\frac{3}{sCR}+\frac{1}{(sCR)^2}\right]$$

$$V_x = -\frac{V_o}{R_f}\frac{1}{sC}\left(2+\frac{1}{sCR}\right) - \frac{I}{sC}$$
$$= -\frac{V_o}{R_f}\frac{1}{sC}\left[2+\frac{1}{sCR}+1+\frac{3}{sCR}+\frac{1}{(sCR)^2}\right]$$

$$\frac{V_o}{V_x} = \frac{-sCR_f}{3+\frac{4}{sCR}+\frac{1}{(sCR)^2}}$$

$$\frac{V_o}{V_x}(j\omega) = \frac{-j\omega CR_f}{3-j\frac{4}{\omega CR}-\frac{1}{\omega^2C^2R^2}}$$
$$= \underline{\underline{\frac{\omega^2C^2RR_f}{4+j\left(3\omega CR-\frac{1}{\omega CR}\right)}}}$$

14.11 The circuit will oscillate at the value of ω that makes $\frac{V_o}{V_x}(j\omega)$ a real number. It follows that ω_0 is obtained from

$$3\omega_0 CR = \frac{1}{\omega_0 CR} \Rightarrow \omega_0 = \frac{1}{\sqrt{3}\,CR}$$

Thus, $f_0 = \dfrac{1}{2\pi\times\sqrt{3}\times16\times10^{-9}\times10\times10^3} = \underline{\underline{574.3 \text{ Hz}}}$

For oscillations to begin the magnitude of $\frac{V_o}{V_x}(j\omega_0)$ should be equal (or greater) than unity, that is

$$(\omega_0^2 C^2 R R_f/4) \geq 1$$

Thus the minimum value of R_f must be $12R$ or $\underline{120 \text{ k}\Omega}$.

14.12 $\omega_0 = 1/CR \Rightarrow CR = \frac{1}{2\pi \times 10^3}$

For $C = 16$ nF, $R = \frac{1}{2\pi \times 16 \times 10^{-9} \times 10^3} \simeq \underline{\underline{10\ k\Omega}}$.

To find the amplitude of the output sinusoid we note that the square wave v_2 will have a 1.4-V peak-to-peak amplitude. The filter which has a gain of 2 at ω_0 will provide a sinusoid v_1 of $2 \times \frac{4}{\pi} \times 1.4$ $= \underline{\underline{3.6V}}$ peak-to-peak amplitude.

CHAPTER 14—PROBLEMS

14.1 (a) $s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = (s+0.5)(s+2)$

$$= s^2 + 2.5s + 1$$

Thus, $\omega_0 = \underline{1\ rad/s}$ and $Q = \underline{0.4}$.

(b) $s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = (s+1)^2 = s^2 + 2s + 1$

Thus, $\omega_0 = \underline{1\ rad/s}$ and $Q = \underline{0.5}$.

(c) $s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = \left(s + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\left(s + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)$

$$= \left(s + \tfrac{1}{\sqrt{2}}\right)^2 + \tfrac{1}{2} = s^2 + \sqrt{2}\,s + 1$$

Thus, $\omega_0 = \underline{1\ rad/s}$ and $Q = \underline{1/\sqrt{2}}$.

(d) $s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = (s + 0.1 + j0.995)(s + 0.1 - j0.995)$

$$= (s+0.1)^2 + 0.995^2$$

$$= s^2 + 0.2s + 1$$

Thus $\omega_0 = \underline{1\ rad/s}$ and $Q = \underline{5}$.

(e) $s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = (s + j1)(s - j1)$

$$= s^2 + 1$$

Thus, $\omega_0 = \underline{1\ rad/s}$ and $Q = \underline{\underline{\infty}}$.

14.2 $T(s) = \dfrac{n_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$

$$T(j\omega) = \frac{n_0}{(\omega_0^2 - \omega^2) + j\frac{\omega\omega_0}{Q}}$$

$$|T(j\omega)| = \frac{n_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}}$$

$$\frac{\partial |T|}{\partial \omega} = \frac{-n_0\left[2(\omega_0^2 - \omega^2)(-2\omega) + \frac{2\omega\omega_0^2}{Q^2}\right]}{\left[(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}\right]^{3/2}} = 0$$

at $\omega = 0$ or at $2(\omega_0^2 - \omega^2) = \frac{\omega_0^2}{Q^2}$

i.e. at $\underline{\underline{\omega = \omega_0\sqrt{1 - \frac{1}{2Q^2}}}}$.

For this peak to occur at a real (physical) frequency, $\frac{1}{2Q^2} < 1$, i.e. $\underline{\underline{Q \geq \frac{1}{\sqrt{2}}}}$.

At the peak frequency $\hat{\omega} = \omega_0\sqrt{1 - \frac{1}{2Q^2}}$,

$$|T(j\hat{\omega})| = \frac{n_0}{\sqrt{\left(\frac{\omega_0^2}{2Q^2}\right)^2 + \omega_0^2\left(1 - \frac{1}{2Q^2}\right)\frac{\omega_0^2}{Q^2}}}$$

$$= \frac{n_0 Q}{\omega_0^2\sqrt{1 - (1/4Q^2)}}$$

14.3 dc gain $= 10 \Rightarrow n_0 = 10\,\omega_0^2$ .. (1)

$$\hat{\omega} = \omega_0\sqrt{1 - \frac{1}{2Q^2}} = 1 \qquad (2)$$

$$|T(j\hat{\omega})| = 20 = \frac{n_0 Q}{\omega_0^2\sqrt{1 - \frac{1}{4Q^2}}} \qquad (3)$$

Substituting for n_0 from (1) into (3) gives

$$\frac{10\,Q}{\sqrt{1 - \frac{1}{4Q^2}}} = 20 \Rightarrow Q = \underline{\underline{1.932}}$$

Substituting in (2) results in

$$\omega_0 = \underline{\underline{1.075\ rad/s}}$$

Thus, $T(s) = \underline{\underline{\dfrac{11.55625}{s^2 + s\,0.5564 + 1.155625}}}$

14.4 $T(s) = \dfrac{n_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$

$$T(j\omega) = \frac{-n_2\omega^2}{(\omega_0^2 - \omega^2) + j\frac{\omega\omega_0}{Q}}$$

$$|T(j\omega)| = \frac{n_2\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}}$$

$$\frac{\partial |T|}{\partial \omega} = \frac{\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}} \times 2n_2\omega - n_2\omega^2\frac{1}{2\sqrt{\ }}\left[2(\omega_0^2 - \omega^2)(-2\omega) + \frac{2\omega\omega_0^2}{Q^2}\right]}{\left[(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}\right]}$$

$= 0$ at $\omega = \underline{\underline{\omega_0 / \sqrt{1 - \frac{1}{2Q^2}}}}$

provided that

$$\frac{1}{2Q^2} < 1 \quad \text{, i.e.} \quad \underline{Q > \frac{1}{\sqrt{2}}}$$

At the peak frequency $\hat{\omega} = \frac{\omega_0}{\sqrt{1-\frac{1}{2Q^2}}}$ we have

$$|T(j\hat{\omega})| = \frac{n_2\,\omega_0^2 / (1-\frac{1}{2Q^2})}{\sqrt{\frac{1}{4Q^4}\frac{\omega_0^4}{(1-\frac{1}{2Q^2})^2} + \frac{\omega_0^4}{Q^2(1-\frac{1}{2Q^2})}}}$$

$$= \frac{n_2\,\omega_0^2\,Q}{\omega_0^2\sqrt{\frac{1}{4Q^2} + 1 - \frac{1}{2Q^2}}}$$

$$= \underline{\underline{\frac{n_2\,Q}{\sqrt{1-\frac{1}{4Q^2}}}}}$$

14.5

$$T(s) = \frac{n_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{j n_1 \omega}{(\omega_0^2-\omega^2) + j\frac{\omega\omega_0}{Q}}$$

$$|T(j\omega)| = \frac{n_1\omega}{\sqrt{(\omega_0^2-\omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}}$$

$$\frac{\partial|T|}{\partial\omega} = \frac{\sqrt{\;\;}\,n_1 - n_1\omega\frac{1}{2\sqrt{\;\;}}\left[2(\omega_0^2-\omega^2)(-2\omega) + \frac{2\omega\omega_0^2}{Q^2}\right]}{(\omega_0^2-\omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}$$

$$= 0$$

at $\left[(\omega_0^2-\omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}\right] = \omega^2\left[-2(\omega_0^2-\omega^2) + \frac{\omega_0^2}{Q^2}\right]$

$$\omega_0^4 + \omega^4 - 2\omega^2\omega_0^2 + \frac{\omega^2\omega_0^2}{Q^2} = -2\omega^2\omega_0^2 + 2\omega^4 + \frac{\omega^2\omega_0^2}{Q^2}$$

$$\Rightarrow \quad \omega = \omega_0$$

i.e. the peak of $|T(j\omega)|$ occurs at $\omega = \omega_0$.

$$|T(j\omega_0)| = n_1 Q/\omega_0$$

The 3 dB frequencies are obtained from

$$\frac{n_1\omega}{\sqrt{(\omega_0^2-\omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}} = \frac{n_1 Q/\omega_0}{\sqrt{2}}$$

$$(\omega_0^2-\omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2} = \frac{2\omega^2\omega_0^2}{Q^2}$$

$$(\omega_0^2-\omega^2)^2 = \frac{\omega^2\omega_0^2}{Q^2}$$

$$\omega_0^2 - \omega^2 = \pm\frac{\omega\omega_0}{Q}$$

$$\omega^2 \pm \frac{\omega_0}{Q}\omega - \omega_0^2 = 0$$

$$\omega = \frac{\mp\frac{\omega_0}{Q} \pm \sqrt{(\frac{\omega_0}{Q})^2 + 4\omega_0^2}}{2}$$

Selecting the two positive roots gives

$$\omega_1 = \omega_0\left(\sqrt{1+\frac{1}{4Q^2}} - \frac{1}{2Q}\right)$$

$$\& \quad \omega_2 = \omega_0\left(\sqrt{1+\frac{1}{4Q^2}} + \frac{1}{2Q}\right)$$

Note that $\omega_1\omega_2 = \omega_0^2$ and that for high Q (i.e. $Q>>1$), $\omega_1 \simeq \omega_0 - \frac{\omega_0}{2Q}$ and $\omega_2 \simeq \omega_0 + \frac{\omega_0}{2Q}$.

For any value of Q, the 3-dB bandwidth

$$\omega_2 - \omega_1 = \underline{\underline{\frac{\omega_0}{Q}}}.$$

14.6

$\omega_0 = 1000$ rad/s, $|T(j\omega_0)| \equiv \frac{n_1 Q}{\omega_0} = 10$, and ~~thus~~ $\omega_2 - \omega_1 = \frac{\omega_0}{Q} = 50$ rad/s. Thus,

$$Q = \frac{\omega_0}{50} = \frac{1000}{50} = \underline{\underline{20}}, \text{ and}$$

$$n_1 = 10 \times \frac{\omega_0}{Q} = 500.$$

The transfer function is

$$\underline{\underline{T(s) = \frac{500\,s}{s^2 + 50\,s + 10^6}}}$$

$$|T(j\omega)| = \frac{500\,\omega}{\sqrt{(10^6-\omega^2)^2 + (50\omega)^2}}$$

$$|T(j100)| = \frac{500 \times 100}{\sqrt{(10^6-10^4)^2 + (5{,}000)^2}} \simeq \underline{\underline{0.0505}}$$

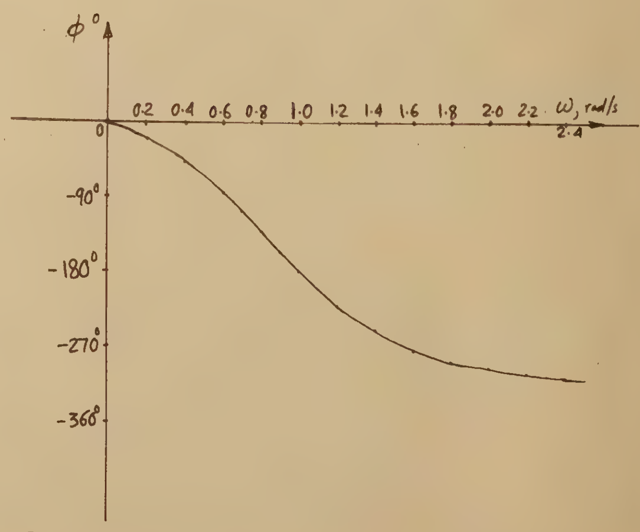
$$|T(j10^4)| = \frac{500 \times 10^4}{\sqrt{(10^6-10^8)^2 + (50\times10^4)^2}} = \underline{\underline{0.0505}}$$

14.7

$$T(s) = \frac{s^2 - s + 1}{s^2 + s + 1}$$

$$T(j\omega) = \frac{(1-\omega^2) - j\omega}{(1-\omega^2) + j\omega}$$

$$\phi = -2\tan^{-1}\left(\frac{\omega}{1-\omega^2}\right)$$



14.8 $\omega_0 = 1/\sqrt{LC} = 10^4$, thus

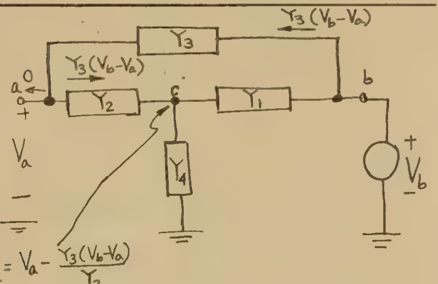
$LC = 10^{-8}$ and for $C = 10$ nF,

$$L = \frac{10^{-8}}{10\times10^{-9}} = \underline{\underline{1\text{ H}}}$$

A 3-dB bandwidth of 10^3 rad/s means that $Q = \frac{10^4}{10^3} = 10$. Thus $\omega_0 CR = 10$ resulting in

$$R = \frac{10}{10^4 \times 10\times10^{-9}} = \underline{\underline{100\text{ k}\Omega}}$$

14.9



$V_c = V_a - \frac{Y_3(V_b-V_a)}{Y_2}$

Node equation at c: $Y_3(V_b-V_a) + Y_4V_a + Y_4\frac{Y_3(V_b-V_a)}{Y_2} + Y_1V_b - Y_1V_a + \frac{Y_1Y_3}{Y_2}(V_b-V_a) = 0$

$$V_b\left[Y_3 + \frac{Y_3Y_4}{Y_2} + Y_1 + \frac{Y_1Y_3}{Y_2}\right] = V_a\left[Y_3 + Y_4 + \frac{Y_3Y_4}{Y_2} + Y_1 + \frac{Y_1Y_3}{Y_2}\right]$$

$$t(s) \equiv \frac{V_a}{V_b} = \frac{Y_1 + Y_3 + \frac{Y_3(Y_1+Y_4)}{Y_2}}{Y_1 + Y_3 + Y_4 + \frac{Y_3(Y_1+Y_4)}{Y_2}}$$

* For the circuit in Fig. 14.5(a):

$Y_1 = sC_1$, $Y_2 = sC_2$, $Y_3 = \frac{1}{R_3}$, and $Y_4 = \frac{1}{R_4}$; thus

$$t(s) = \frac{sC_1 + \frac{1}{R_3} + \frac{1}{sC_2R_3}\left(sC_1 + \frac{1}{R_4}\right)}{sC_1 + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{sC_2R_3}\left(sC_1 + \frac{1}{R_4}\right)}$$

$$= \frac{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}{s^2 + s\left(\frac{1}{C_1R_3} + \frac{1}{C_2R_3} + \frac{1}{C_1R_4}\right) + \frac{1}{C_1C_2R_3R_4}}$$

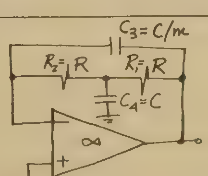
* For the circuit in Fig. 14.5(b):

$Y_1 = \frac{1}{R_1}$, $Y_2 = \frac{1}{R_2}$, $Y_3 = sC_3$, and $Y_4 = sC_4$; thus

$$t(s) = \frac{\frac{1}{R_1} + sC_3 + sC_3R_2\left(\frac{1}{R_1} + sC_4\right)}{\frac{1}{R_1} + sC_3 + sC_4 + sC_3R_2\left(\frac{1}{R_1} + sC_4\right)}$$

$$= \frac{s^2 + s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_4} + \frac{1}{C_3C_4R_1R_2}}{s^2 + s\left(\frac{1}{C_4R_1} + \frac{1}{C_4R_2} + \frac{1}{C_3R_2}\right) + \frac{1}{C_3C_4R_1R_2}}$$

14.10 The poles of the active circuit will be identical to the zeros of $t(s)$ which is given in Fig. 14.5(b), that is



$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = s^2 + s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_4} + \frac{1}{C_3C_4R_1R_2}$$

Thus,

$$\omega_0^2 = \frac{1}{(C/m)CR^2} = \frac{m}{C^2R^2} \Rightarrow \omega_0 CR = \sqrt{m}$$

& $\frac{\omega_0}{Q} = \frac{2}{CR}$

$$\Rightarrow Q = \frac{\omega_0 CR}{2} = \frac{\sqrt{m}}{2}$$

Therefore $\underline{\underline{m = 4Q^2}}$

and $\underline{\underline{CR = \frac{2Q}{\omega_0}}}$.

14.11 Equations (14.13) and (14.14) are the design equations.

Thus $m = 4Q^2 = 4 \times \frac{1}{2} = 2$

& $CR = \frac{2\times(1/\sqrt{2})}{10^4} = \sqrt{2}\times10^{-4}$ s

For $C_1 = C_2 = 1$ nF,

$$R = \frac{\sqrt{2}\times10^{-4}}{10^{-9}} = \sqrt{2}\times10^5\ \Omega$$

Thus, $\underline{\underline{R_3 = 141.4\text{ k}\Omega}}$ and $\underline{R_4 = 70.7\text{ k}\Omega}$.

14.12 Refer to the solution to Problem 14.10.

$m = 4Q^2 = 4$

$CR = \frac{2\times1}{10^4} = 2\times10^{-4}$ s

For $R_1 = R_2 = 10\text{ k}\Omega$, $C = \frac{2\times10^{-4}}{10^4} = 20$ nF

Thus, $C_3 = C/m = \underline{5\text{ nF}}$ and $C_4 = \underline{20\text{ nF}}$

14.13 Using the transfer function $t(s)$ of the bridged-T network, given in Fig. 14.5(a), we find the zeros to be the roots of the numerator polynomial $N(s)$,

$$N(s) = s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}$$

$$= s^2 + 2s + 2$$

Thus the zeros are at $s = -1 \pm \frac{1}{2}\sqrt{4-8} = \underline{-1 \pm j}$

The poles of the bridged-T are the roots of the denominator polynomial $D(s)$,

$$D(s) = s^2 + s\left(\frac{1}{C_1R_3} + \frac{1}{C_2R_3} + \frac{1}{C_1R_4}\right) + \frac{1}{C_1C_2R_3R_4}$$

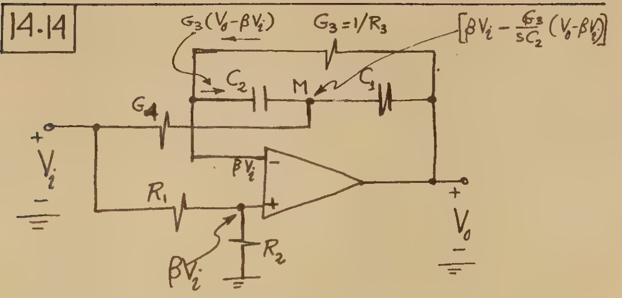
$$= s^2 + s(1+1+2) + 2 = s^2 + 4s + 2$$

Thus the poles are at $s = -2 \pm \frac{1}{2}\sqrt{16-8} = \underline{\underline{-0.586, -3.414}}$

If the bridged-T network is placed in the negative-feedback path of an infinite-gain op amp, the closed-loop circuit will have poles identical to the zeros of the bridged-T; i.e. the poles will be the roots of $(s^2 + 2s + 2)$. Thus,

$$\omega_0 = \underline{\underline{\sqrt{2}}} \quad \text{and} \quad Q = \frac{\sqrt{2}}{2} = \underline{\underline{\frac{1}{\sqrt{2}}}}.$$

14.14



$\beta \equiv \frac{R_2}{R_1+R_2}$

Node equation at M:

$$G_3(V_o-\beta V_i) + sC_1\left[V_o-\beta V_i + \frac{G_3}{sC_2}(V_o-\beta V_i)\right] + G_4\left[V_i-\beta V_i+\frac{G_3}{sC_2}(V_o-\beta V_i)\right]=0$$

$$V_o\left[G_3+sC_1+\frac{C_1}{C_2}G_3+\frac{G_3G_4}{sC_2}\right] = V_i\left[\beta G_3+s\beta C_1+\beta\frac{C_1}{C_2}G_3-G_4+\beta G_4+\beta\frac{G_3G_4}{sC_2}\right]$$

$$\frac{V_o}{V_i}=\frac{\beta s^2+s\frac{1}{C_1}\left(\beta G_3+\beta\frac{C_1}{C_2}G_3-G_4+\beta G_4\right)+\beta\frac{G_3G_4}{C_1C_2}}{s^2+s\frac{1}{C_1}\left(G_3+\frac{C_1}{C_2}G_3\right)+\frac{G_3G_4}{C_1C_2}}$$

$$=\beta\,\frac{s^2+s\left(\frac{1}{R_3C_1}+\frac{1}{C_2R_3}+\frac{1}{C_1R_4}-\frac{1}{\beta C_1R_4}\right)+\frac{1}{C_1C_2R_3R_4}}{s^2+s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3}+\frac{1}{C_1C_2R_3R_4}}$$

Substituting $C_1=C_2=C$, $R_3=R$, $R_4=R/4\varphi^2$, and $CR=\frac{2\varphi}{\omega_0}$ results in

$$\frac{V_o}{V_i}=\beta\,\frac{s^2+s\left(\frac{1}{RC}+\frac{1}{RC}+\frac{4\varphi^2}{RC}-\frac{4\varphi^2}{\beta RC}\right)+\frac{4\varphi^2}{(CR)^2}}{s^2+s\frac{2}{CR}+\frac{4\varphi^2}{(CR)^2}}$$

$$\frac{V_o}{V_i}=\beta\,\frac{s^2+s\left(\frac{\omega_0}{\varphi}\right)\left(1+2\varphi^2-\frac{2\varphi^2}{\beta}\right)+\omega_0^2}{s^2+s\left(\frac{\omega_0}{\varphi}\right)+\omega_0^2} \qquad (1)$$

To obtain an all-pass function we must select β so that

$$1+2\varphi^2-\frac{2\varphi^2}{\beta}=-1$$

$$\Rightarrow\quad \beta=\frac{\varphi^2}{\varphi^2+1}$$

i.e. $\underline{\frac{R_2}{R_1+R_2}=\frac{\varphi^2}{\varphi^2+1}}$

The magnitude of transmission of the all-pass network is β or $\underline{\frac{\varphi^2}{\varphi^2+1}}$.

14.15

Consider Eqn. (1) in the solution to Problem 14.14. To realize a notch function we must select β so that

$$1+2\varphi^2-\frac{2\varphi^2}{\beta}=0$$

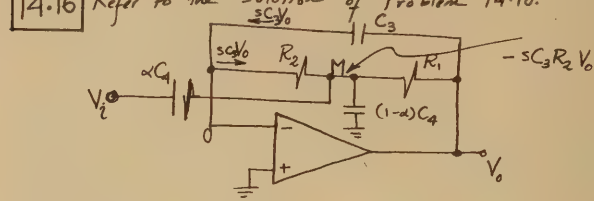
$$\Rightarrow\quad \beta=\frac{2\varphi^2}{2\varphi^2+1}$$

i.e. $\underline{\frac{R_2}{R_1+R_2}=\frac{2\varphi^2}{2\varphi^2+1}}$

The frequency of the notch is $\underline{\omega_0}$.

14.16

Refer to the solution of Problem 14.10.



Node equation at M:

$$sC_3V_o+(V_o+sC_3R_2V_o)/R_1+s(1-\alpha)C_4\times sC_3R_2V_o + s\alpha C_4V_i+s\alpha C_4\times sC_3R_2V_o=0$$

$$V_o\left[sC_3+\frac{1}{R_1}+sC_3\frac{R_2}{R_1}+s^2(1-\alpha)C_3C_4R_2+s^2\alpha C_3C_4R_2\right]=-s\alpha C_4V_i$$

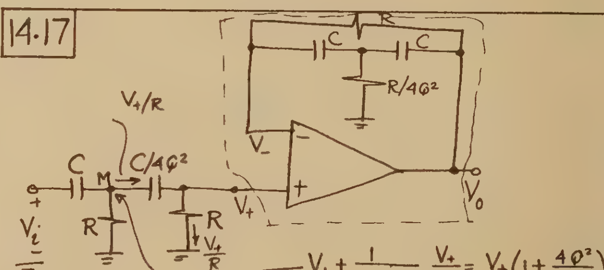
$$\frac{V_o}{V_i}=\frac{-s\alpha C_4}{s^2C_3C_4R_2+sC_3\left(1+\frac{R_2}{R_1}\right)+\frac{1}{R_1}}$$

$$=\frac{-s\alpha/C_3R_2}{s^2+s\frac{1}{C_4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{C_3C_4R_1R_2}}$$

For the design suggested in Problem 14.10 we have $R_1=R_2=R$, $C_3=C/m$, $C_4=C$, $m=4\varphi^2$ and $CR=2\varphi/\omega_0$. In this case the transfer function becomes

$$\frac{V_o}{V_i}=\frac{-s\alpha(2\omega_0\varphi)}{s^2+s\frac{\omega_0}{\varphi}+\omega_0^2}$$

14.17



$$\frac{V_o}{V_i}=\frac{V_+}{V_i}\times\frac{V_o}{V_+}$$

To find the transfer function $\frac{V_+}{V_i}$ we analyze the input network as indicated in the Figure. A node equation at M yields

$$sCV_i-sCV_+\left(1+\frac{4\varphi^2}{sCR}\right)=\frac{V_+}{R}+\frac{V_+}{R}\left(1+\frac{4\varphi^2}{sCR}\right)$$

Thus,

$$\frac{V_+}{V_i}=\frac{sC}{sC+\frac{4\varphi^2+2}{R}+\frac{4\varphi^2}{sCR^2}}$$

$$=\frac{s^2}{s^2+s\frac{4\varphi^2+2}{CR}+\frac{4\varphi^2}{(CR)^2}} \qquad (1)$$

To find $\frac{V_o}{V_+}$ we analyze the network in the dashed box as follows:

$$V_+=V_-=t(s)\,V_o \qquad (2)$$

where $t(s)$ is the transfer function of the

bridged-T feedback network. This transfer function is given in Fig. 14.5a. For our case, i.e. for $C_1 = C_2 = C$, and $R_3 = R$, and $R_4 = R/4Q^2$, $t(s)$ is

$$t(s) = \frac{s^2 + s\frac{2}{CR} + \frac{4Q^2}{(CR)^2}}{s^2 + s\frac{4Q^2+2}{CR} + \frac{4Q^2}{(CR)^2}}$$

Substituting in (2) yields

$$\frac{V_o}{V_+} = \frac{1}{t(s)} = \frac{s^2 + s\frac{4Q^2+2}{CR} + \frac{4Q^2}{(CR)^2}}{s^2 + s\frac{2}{CR} + \frac{4Q^2}{(CR)^2}} \qquad (3)$$

Multiplying the two transfer functions in (1) and (3) gives the overall transfer function $\frac{V_o}{V_i}$ as

$$\underline{\underline{\frac{V_o}{V_i} = \frac{s^2}{s^2 + s\frac{2}{CR} + \frac{4Q^2}{(CR)^2}}}}$$

$\boxed{14.18}$ Refer to the solution of Exercise 14.3 where we derived the transfer function of the circuit in Fig. 14.7,

$$\frac{V_o(s)}{V_i(s)} = \frac{-s\alpha/C_1R_4}{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}$$

For $C_1 = C_2 = C$, $R_3 = R$ we find that to realize a pair of poles characterized by ω_0 and Q we must select $R_4 = R/4Q^2$ and $CR = 2Q/\omega_0$. The transfer function then becomes

$$\frac{V_o(s)}{V_i(s)} = \frac{-s\alpha(2\omega_0 Q)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

Now we find that the center-frequency gain is $2\alpha Q^2$. For our case

$C_1 = C_2 = 1\,nF$, $CR = \frac{2\times 5}{10^4} = 10^{-3}$, $R_3 = R = \frac{10^{-3}}{10^{-9}} = \underline{\underline{1\,M\Omega}}$,

$R_4 = R/4Q^2 = \underline{\underline{10\,k\Omega}}$, $2\alpha Q^2 = 10 \Rightarrow \alpha = \frac{10}{2\times 25} = 0.2$,

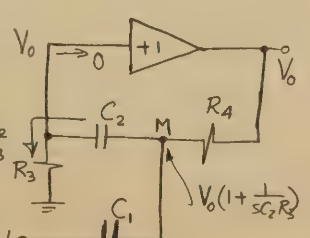
$\frac{R_4}{\alpha} = \frac{10}{0.2} = \underline{\underline{50\,k\Omega}}$, and $R_4/(1-\alpha) = \frac{10}{0.8} = \underline{\underline{12.5\,k\Omega}}$.

$\boxed{14.19}$ Node equation at M:

$$\frac{V_o}{R_3} + \frac{V_o}{sC_2R_3R_4} + sC_1V_o\left(1 + \frac{1}{sC_2R_3}\right) - sC_1V_i = 0$$

$$\frac{V_o}{V_i} = \frac{sC_1}{sC_1 + \frac{1}{R_3} + \frac{C_1}{C_2R_3} + \frac{1}{sC_2R_3R_4}}$$

$$= \frac{s^2}{s^2 + s\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + \frac{1}{C_1C_2R_3R_4}}$$



Thus the circuit realizes a high-pass function with a high-frequency gain of unity, and

$$\omega_0^2 = \frac{1}{C_1C_2R_3R_4}$$

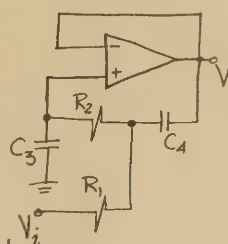
$$Q = \omega_0 \Big/ \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

For $C_1 = C_2 = C$, $R_3 = R$ and $R_4 = R/m$ we have $\omega_0^2 = \frac{m}{(CR)^2}$ and $Q = \frac{\omega_0 CR}{2}$. Thus the design equations are

$$m = 4Q^2 \quad \text{and} \quad CR = \frac{2Q}{\omega_0}$$

For $C_1 = C_2 = 1\,nF$, $Q = 1/\sqrt{2}$ and $\omega_0 = 10^4$ rad/s we have $CR = \frac{2\times\frac{1}{\sqrt{2}}}{10^4} = \sqrt{2}\times 10^{-4}$ s; thus $R = \frac{\sqrt{2}\times 10^{-4}}{10^{-9}} = 141.4\,k\Omega$ leading to $R_3 = \underline{\underline{141.4\,k\Omega}}$ and $R_4 = \underline{\underline{70.7\,k\Omega}}$.

$\boxed{14.20}$ Refer to the solution to Exercise 14.4. The circuit realizes a low-pass function with unity dc gain. The design equations have been derived in the solution to Problem 14.10 as



$R_1 = R_2 = R$, $C_4 = C$, $C_3 = C/4Q^2$ and $CR = 2Q/\omega_0$. For our case $CR = \frac{2\times\frac{1}{\sqrt{2}}}{10^4} = \sqrt{2}\times 10^{-4}$ s; thus for $R_1 = R_2 = 10\,k\Omega$, $R = 10\,k\Omega$, and $C = \frac{\sqrt{2}\times 10^{-4}}{10^4} = 14.14\,nF$.

This leads to $C_4 = \underline{\underline{14.14\,nF}}$ and $C_3 = \underline{\underline{7.07\,nF}}$.

$\boxed{14.21}$ $T(s) = \frac{Z_{LC}}{R + Z_{LC}}$

$$= \frac{1}{1 + RY_{LC}} = \frac{1}{1 + R\left(sC + \frac{1}{sL}\right)}$$

$$= \frac{s/CR}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

Thus, $\omega_0 = \frac{1}{\sqrt{LC}}$, and

$$S_L^{\omega_0} = S_C^{\omega_0} = \frac{\partial\omega_0}{\partial C}\frac{C}{\omega_0} = -\frac{1}{2}\frac{1}{\sqrt{L}}\frac{1}{\sqrt{C}\,C}\frac{C\times\sqrt{LC}}{1}$$

$$= \underline{\underline{-\tfrac{1}{2}}}. \qquad S_R^{\omega_0} = \underline{\underline{0}}$$

$$Q = \frac{1}{\sqrt{LC}}CR = R\sqrt{\frac{C}{L}}$$

$$S_C^{Q} = \frac{\partial Q}{\partial C}\frac{C}{Q} = \frac{R}{\sqrt{L}}\frac{1}{2\sqrt{C}}\frac{C}{R}\sqrt{\frac{L}{C}} = \underline{\underline{+\tfrac{1}{2}}}$$

$$S_L^Q = \frac{\partial Q}{\partial L}\frac{L}{Q} = R\sqrt{C}\frac{(-1/2)}{L\sqrt{L}}\frac{L}{R}\sqrt{\frac{L}{C}} = \underline{-\frac{1}{2}}$$

$$S_R^Q = \frac{\partial Q}{\partial R}\frac{R}{Q} = \sqrt{\frac{C}{L}}\frac{R}{R}\sqrt{\frac{L}{C}} = \underline{+1}$$

14.22 (a) $y = uv$ $\quad S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} = \left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}\right)\frac{x}{uv}$

Thus, $S_x^y = \frac{\partial v}{\partial x}\frac{x}{v} + \frac{\partial u}{\partial x}\frac{x}{u}$

$= S_x^v + S_x^u = S_x^u + S_x^v$. Q.E.D.

(b) $y = u/v$, $S_x^y = \frac{v\frac{\partial u}{\partial x} - u\frac{\partial v}{\partial x}}{v^2}\frac{x}{y}$

$= \frac{1}{v}\frac{\partial u}{\partial x}\frac{x}{u/v} - \frac{u}{v^2}\frac{\partial v}{\partial x}\frac{x}{u/v}$

$= \frac{\partial u}{\partial x}\frac{x}{u} - \frac{\partial v}{\partial x}\frac{x}{v}$

$= S_x^u - S_x^v$. Q.E.D.

(c) $y = ku$, $S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} = k\frac{\partial u}{\partial x}\frac{x}{ku}$

$= \frac{\partial u}{\partial x}\frac{x}{u} = S_x^u$. Q.E.D.

(d) $y = u^n$, $S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} = n u^{n-1}\frac{\partial u}{\partial x}\frac{x}{u^n}$

$= n\frac{\partial u}{\partial x}\frac{x}{u} = n S_x^u$. Q.E.D.

(e) $y = f_1(u)$, $u = f_2(x)$, then

$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} = \frac{\partial f_1(u)}{\partial u}\frac{\partial u}{\partial x}\frac{x}{f_1(u)}$

Thus, $S_x^y = \frac{\partial f_1(u)}{\partial u}\cdot\frac{u}{f_1(u)}\cdot\frac{\partial u}{\partial x}\cdot\frac{x}{u}$

$= S_u^{f_1(u)}\cdot S_x^u$

$= S_u^y\cdot S_x^u$. Q.E.D.

14.23 Placing the bridged-T network of Fig. 14.5b in the negative feedback of an op amp results in an active circuit whose poles will, for the case of infinite op-amp gain, be identical to the zeros of $t(s)$ of the bridged-T. Thus from Fig. 14.5b we can determine ω_0 and Q as

$$\omega_0 = \frac{1}{\sqrt{C_3C_4R_1R_2}}$$

$$Q = \frac{\omega_0 C_4}{\frac{1}{R_1} + \frac{1}{R_2}}$$

To evaluate the sensitivities of ω_0 and Q relative to the four passive components we shall make use of the identities verified in Problem 14.22.

Thus, $S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_3}^{\omega_0} = S_{C_4}^{\omega_0} = \underline{-\frac{1}{2}}$

$$S_{R_1}^Q = S_{R_1}^{\omega_0} - S_{R_1}^{(\frac{1}{R_1}+\frac{1}{R_2})}$$

$$= -\frac{1}{2} - \left(-\frac{1}{R_1^2}\right)\frac{R_1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$= -\frac{1}{2} + \frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}} = -\frac{1}{2} + \frac{R_2}{R_1+R_2}$$

For the case $R_1 = R_2$, $S_{R_1}^Q = \underline{0}$

$$S_{R_2}^Q = S_{R_2}^{\omega_0} - S_{R_2}^{(\frac{1}{R_1}+\frac{1}{R_2})}$$

$$= -\frac{1}{2} - \left(-\frac{1}{R_2^2}\right)\frac{R_2}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$= -\frac{1}{2} + \frac{R_1}{R_1+R_2}$$

For the case $R_1 = R_2$, $S_{R_2}^Q = \underline{0}$

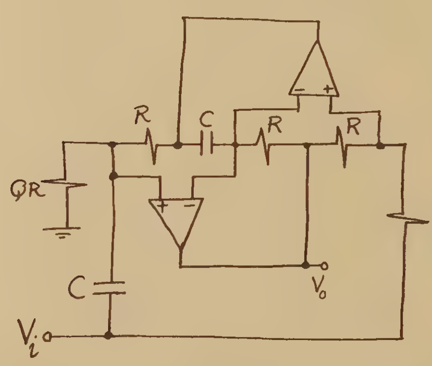
$$S_{C_3}^Q = S_{C_3}^{\omega_0} = \underline{-\frac{1}{2}}$$

$$S_{C_4}^Q = S_{C_4}^{\omega_0} + S_{C_4}^{C_4} = -\frac{1}{2} + 1 = \underline{+\frac{1}{2}}.$$

14.24 $t(s) = \frac{s\,(2\omega_0/Q)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$

$1 - t(s) = \frac{s^2 - s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$

The all-pass circuit obtained from the bandpass circuit of Fig. 14.12a will be as follows:



$CR = \frac{1}{\omega_0}$

14.25 High-pass; it is a simulation of that in Fig. 14.3b.

14.26 Refer to Fig. 14.13 and to the equations on page 641. $CR = \frac{1}{\omega_0} = \frac{1}{2\pi \times 10^4}$. Thus, for $R = 10\ k\Omega$, $C = \frac{1}{2\pi \times 10^4 \times 10^4} = \underline{1.59\ nF}$

The bandpass function realized is

$$\frac{V_{bp}}{V_i} = \frac{-\mu_2 s\,\omega_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

where $\mu_2 = 2 - \frac{1}{Q}$. Thus the center-frequency

gain is $n_2 Q = (2-\frac{1}{Q})Q = 2Q-1 = 40-1 = \underline{39}$.
We conclude that as is the circuit is incapable of realizing a bandpass filter with unity center-frequency gain. A slight redesign, however, would accomplish this goal: Add a resistance R_4 from the positive terminal of the first op amp to ground. We shall not pursue this redesign here.

Finally, note that we may arbitrarily select $R_1 = 10\,k\Omega$ and select $R_2 = \underline{1\,k\Omega}$, then $R_3 = (2Q-1)R_2 = \underline{39\,k\Omega}$.

14.27 Refer to Fig. 14.14. The low-pass function realized is

$$\frac{V_{lp}}{V_i} = \frac{n_2\,\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

where

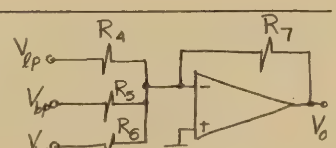
$$\omega_0 = \frac{1}{CR} \Rightarrow CR = \frac{1}{2\pi\times10^4}$$

For $R = 10\,k\Omega$ $\quad C = \frac{1}{2\pi\times10^8} = \underline{1.59\,nF}$

$R_d = QR = \frac{1}{\sqrt{2}}\times 10 = \underline{7.07\,k\Omega}$

Low-frequency gain $= n_2 = 1$, thus $R_g = R = \underline{10\,k\Omega}$. Select $r = \underline{10\,k\Omega}$

14.28 Refer to Fig. 14.13.



$$V_{hp} = \frac{n_2' s^2}{s^2 + s(\frac{\omega_0}{Q}) + \omega_0^2} V_i$$

$$V_{bp} = \frac{-n_2'\omega_0 s}{s^2 + s(\frac{\omega_0}{Q}) + \omega_0^2} V_i$$

$$V_{lp} = \frac{+n_2'\omega_0^2}{s^2 + s(\frac{\omega_0}{Q}) + \omega_0^2} V_i \quad , \text{ where } n_2' = 2 - \frac{1}{Q}.$$

Thus,
$$V_0 = \frac{-\frac{R_7}{R_4}n_2'\omega_0^2 + \frac{R_7}{R_5}n_2'\omega_0 s - \frac{R_7}{R_6}n_2' s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} V_i$$

To obtain a notch function we must make the coefficient of s in the numerator zero. Thus $\underline{R_5 = \infty}$. The notch function realized can be expressed as

$$\frac{V_0}{V_i} = -n_2'\frac{R_7}{R_6}\;\frac{s^2 + \omega_0^2(\frac{R_6}{R_4})}{s^2 + s(\frac{\omega_0}{Q}) + \omega_0^2}$$

Thus $\omega_n^2 = \omega_0^2\left(\frac{R_6}{R_4}\right) \Rightarrow \underline{R_4 = R_6\,(\omega_0/\omega_n)^2}$

R_6 can be selected arbitrarily. To realize the notch function of Eqn (14.3) we must select R_7 such that $n_2'\frac{R_7}{R_6} = n_2 \Rightarrow \underline{R_7 = R_6\frac{n_2}{(2-\frac{1}{Q})}}$.

14.29 (a) From Eqns. (14.36) and (14.37),

$$C_3 = C_4 = \omega_0 T_c C = 2\pi\times10^4 \times \frac{1}{200\times10^3}\times 20$$
$$= \underline{6.283\,pF}$$

From Eqn. (14.39): $C_5 = \frac{C_4}{Q} = \frac{6.283}{10} = \underline{0.6283\,pF}$

From Eqn. (14.40): $C_6 = C_5 = \underline{0.6283\,pF}$

(b) $C_3 = C_4 = \omega_0 T_c C = 2\pi\times5\times10^3\times\frac{1}{200\times10^3}\times 20$
$$= \underline{3.142\,pF}$$
$$C_5 = \frac{C_4}{Q} = \frac{3.142}{20} = \underline{0.157\,pF}$$
$$C_6 = C_5 = \underline{0.157\,pF}$$

14.30 Refer to Fig. 14.17. The dc gain of the circuit in (a), from V_i to the output of the second op amp, is $\frac{R}{R_6}$, where $R_4 = R_3 = R$ and $C_2 = C_1 = C$. By analogy we find that the dc gain of the SC circuit in (b), from V_i to the output of the second op amp, is $\frac{C_6}{KC}$ (where $C_3 = C_4 = KC$). Thus for unity dc gain we select

$$C_6 = KC$$

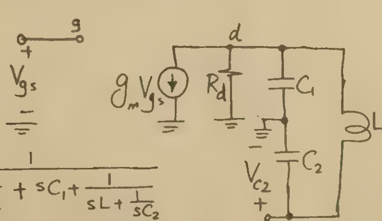
The other design equations are (14.36), (14.37) and (14.39),

$$C_3 = C_4 = KC \qquad (2)$$
$$K = \omega_0 T_c \qquad (3)$$
$$C_5 = \omega_0 T_c \frac{C}{Q} \qquad (4)$$
where, $\quad C_1 = C_2 = C \qquad (5)$

Substituting $T_c = \frac{1}{f_c} = 10^{-5}\,s$, $C = 10\,pF$, $\omega_0 = 10^4\,rad/s$, and $Q = \frac{1}{\sqrt{2}}$ results in

$$K = 10^4\times10^{-5} = 0.1$$
$$C_3 = C_4 = 0.1\times10 = \underline{1\,pF}$$
$$C_5 = 0.1\times\frac{10}{1/\sqrt{2}} = \underline{1.414\,pF}$$
$$C_6 = 0.1\times10 = \underline{1\,pF}$$

14.31



$$V_d = -g_m V_{gs}\frac{1}{\frac{1}{R_d} + sC_1 + \frac{1}{sL + \frac{1}{sC_2}}}$$

$$V_{c2} = \frac{V_d}{sL + \frac{1}{sC_2}}\times\frac{1}{sC_2}$$

$$= -g_m V_{gs}\,\frac{1}{sC_2}\,\frac{1}{(\frac{1}{R_d} + sC_1)(sL + \frac{1}{sC_2}) + 1}$$

Thus, $\frac{V_{c2}}{V_{gs}} = \frac{-g_m/sC_2}{\frac{sL}{R_d} + \frac{1}{sC_2R_d} + s^2LC_1 + \frac{C_1}{C_2} + 1}$

$$= \frac{-g_m R_d}{s^3 LC_1C_2R_d + s^2LC_2 + s(C_1+C_2)R_d + 1}$$

$$L(j\omega) = \frac{V_{c2}}{V_{gs}}(j\omega) = \frac{-g_m R_d}{(1-\omega^2 LC_2) + j\omega[(C_1+C_2)R_d - \omega^2 LC_1C_2R_d]}$$

The circuit oscillates at the frequency that makes $L(j\omega)$ a real number greater or equal to unity. Thus

$$(C_1+C_2)R_d - \omega_0^2 LC_1C_2R_d = 0$$

$$\Rightarrow \omega_0 = 1\Big/\sqrt{L\left(\frac{C_1C_2}{C_1+C_2}\right)}$$

A this frequency the loop gain is

$$L(j\omega_0) = \frac{-g_m R_d}{1-\omega_0^2 LC_2} = \frac{-g_m R_d}{1-\frac{C_1+C_2}{C_1}}$$

$$= g_m R_d\left(\frac{C_1}{C_2}\right)$$

For oscillations to start, $L(j\omega_0)$ must be at least unity. Thus the gain $g_m R_d$ must be equal or greater than $\left(\frac{C_2}{C_1}\right)$.

14.32 Refer to Fig. 14.20 with Z_s changed to $Z_s = 10R + \frac{1}{s(C/10)}$. The loop gain becomes

$$L(s) = \left(1+\frac{R_2}{R_1}\right)\frac{Z_p}{Z_p+Z_s}$$

$$= \frac{1+R_2/R_1}{1+\left(10R+\frac{10}{sC}\right)\left(\frac{1}{R}+sC\right)}$$

$$= \frac{1+R_2/R_1}{21 + s\,10CR + \frac{10}{sCR}}$$

$$L(j\omega) = \frac{1+R_2/R_1}{21 + j\left(10\omega CR - \frac{10}{\omega CR}\right)}$$

Thus ω_0 is obtained from

$$10\,\omega_0 CR = \frac{10}{\omega_0 CR} \Rightarrow \omega_0 = \frac{1}{CR}$$

and for oscillations to start

$$1 + \frac{R_2}{R_1} \geq 21$$

$$\text{i.e.} \quad \frac{R_2}{R_1} \geq 20.$$

14.33 For the basic oscillator of Fig. 14.20 the phase can be obtained from the expression for the loop gain $L(j\omega)$ in Eqn. (14.46) as

$$\phi = -\tan^{-1}\frac{1}{3}\left(\omega CR - \frac{1}{\omega CR}\right)$$

Near ω_0, ϕ is small and we may make the approximation

$$\phi \simeq -\frac{1}{3}\left(\omega CR - \frac{1}{\omega CR}\right)$$

$$\text{Thus,}\quad \frac{d\phi}{d\omega} = -\frac{1}{3}\left(CR + \frac{1}{\omega^2 CR}\right)$$

$$\left.\frac{d\phi}{d\omega}\right|_{\omega=\omega_0} = -\frac{2}{3\omega_0}$$

For the modified circuit investigated in Problem 14.32, we find $\phi(\omega)$ from the expression for the loop gain derived in the solution to Problem 14.32,

$$\phi = -\tan^{-1}\left[\left(10\omega CR - \frac{10}{\omega CR}\right)/21\right]$$

$$\simeq -\frac{10}{21}\left(\omega CR - \frac{1}{\omega CR}\right)$$

$$\frac{d\phi}{d\omega} = -\frac{10}{21}\left(CR + \frac{1}{\omega^2 CR}\right)$$

$$\left.\frac{d\phi}{d\omega}\right|_{\omega=\omega_0} = \frac{-20}{21}\frac{1}{\omega_0}$$

We note that the phase sensitivity is greater in the modified circuit than in the original circuit.

14.34 From Eqn. (14.47),

$$\omega_0 = \frac{1}{CR} = \frac{1}{1600\times10^{-12}\times10\times10^3}$$

$$f_0 = \frac{10^6}{2\pi\times16} \simeq 10\text{ kHz}$$

For the method of determining the amplitude of the output sinusoid, refer to the solution of part (c) of Exercise 14.8,

$$\frac{\hat{V}_o - V_b}{R_5} = \frac{V_b - (-15)}{R_6}$$

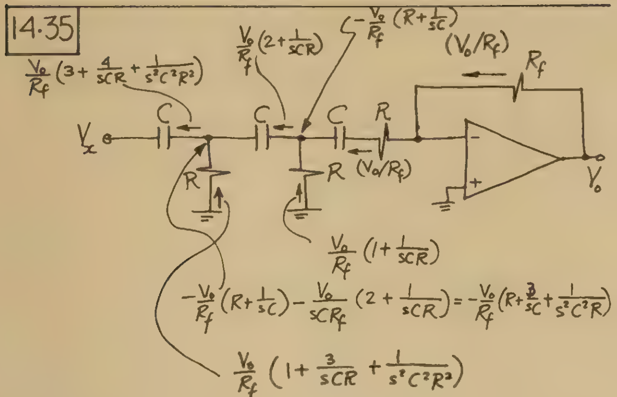
$$\text{Thus,}\quad \frac{\hat{V}_o - V_b}{0.5} = \frac{V_b+15}{3} \Rightarrow \hat{V}_o = \frac{3.5}{3}V_b + 2.5$$

$$\hat{V}_o = \frac{3.5}{3}\left(0.7 + \frac{\hat{V}_o}{3}\right) + 2.5$$

which leads to $\hat{V}_o = 5.43$V.

From the symmetry of the circuit we conclude that the output sinusoid has 10.86 V peak-to-peak amplitude.

14.35



$$V_x = -\frac{V_o}{R_f}\left(R+\frac{3}{sC}+\frac{1}{s^2C^2R}\right)-\frac{V_o}{sCR_f}\left(3+\frac{4}{sCR}+\frac{1}{s^2C^2R^2}\right)$$

$$= -\frac{V_oR}{R_f}\left[1+\frac{3}{sCR}+\frac{1}{s^2C^2R^2}+\frac{3}{sCR}+\frac{4}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}\right]$$

$$= -V_o\left(\frac{R}{R_f}\right)\left(1+\frac{6}{sCR}+\frac{5}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}\right)$$

$$\frac{V_o}{V_x} = \frac{-(R_f/R)}{1+\frac{6}{sCR}+\frac{5}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}}$$

$$\frac{V_o}{V_x}(j\omega) = \frac{-(R_f/R)}{\left(1-\frac{5}{\omega^2C^2R^2}\right)+j\left(\frac{1}{\omega^3C^3R^3}-\frac{6}{\omega CR}\right)}$$

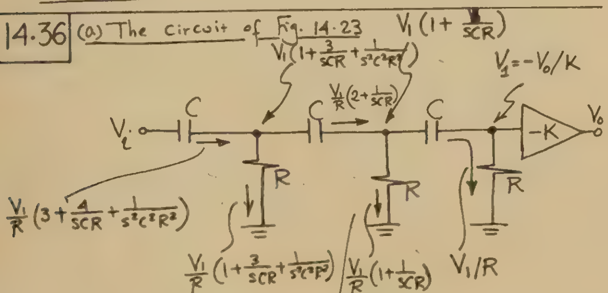
$$\frac{1}{\omega_0^3C^3R^3} = \frac{6}{\omega_0 CR} \Rightarrow \omega_0 = \frac{1}{\sqrt{6}\,CR}$$

Thus, $f_0 = \frac{1}{2\pi\sqrt{6}\times 16\times 10^{-9}\times 10\times 10^3} = \underline{\underline{406.1\text{ Hz}}}$

At ω_0, $L(j\omega_0) = \frac{-(R_f/R)}{1-\frac{5}{1/6}} = \frac{-(R_f/R)}{-29}$

Thus the minimum value of R_f is $29R$ or $\underline{290\text{ k}\Omega}$.

14.36 (a) The Circuit of Fig. 14.23



$$V_i = V_1\left(1+\frac{3}{sCR}+\frac{1}{s^2C^2R^2}\right)+V_1\left(\frac{3}{sCR}+\frac{4}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}\right)$$

$$= V_1\left(1+\frac{6}{sCR}+\frac{5}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}\right)$$

Thus

$$L(s) \equiv \frac{V_o(s)}{V_i(s)} = \frac{-K}{1+\frac{6}{sCR}+\frac{5}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}}$$

$$L(j\omega) = \frac{-K}{\left(1-\frac{5}{\omega^2C^2R^2}\right)+j\left(\frac{1}{\omega^3C^3R^3}-\frac{6}{\omega CR}\right)}$$

$$\frac{1}{\omega_0^3C^3R^3} = \frac{6}{\omega_0 CR} \Rightarrow \underline{\underline{\omega_0 = \frac{1}{\sqrt{6}\,CR}}}$$

$$L(j\omega_0) = \frac{-K}{1-\frac{5}{1/6}} = \frac{K}{29}$$

Thus, $K_{min} = \underline{29}$.

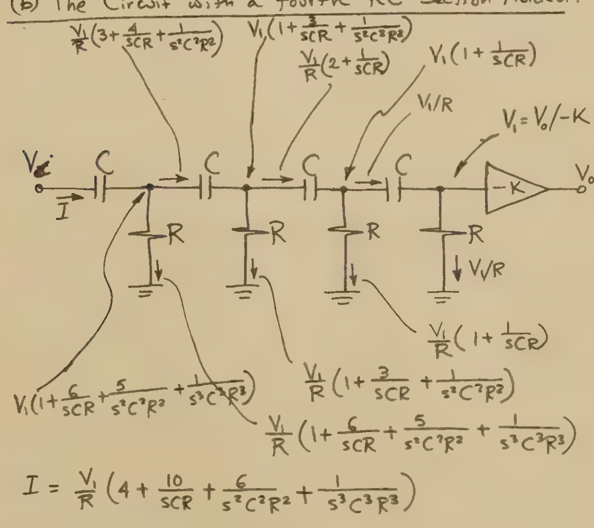
$$\phi = -\tan^{-1}\left[\frac{\frac{1}{\omega^3C^3R^3}-\frac{6}{\omega CR}}{1-\frac{5}{\omega^2C^2R^2}}\right]$$

$$\simeq -\left[\frac{\frac{1}{\omega^3C^3R^3}-\frac{6}{\omega CR}}{1-\frac{5}{\omega^2C^2R^2}}\right]$$, for ω close to ω_0

$$-\frac{d\phi}{d\omega} = \frac{1}{\omega^2CR}\,\frac{3\left[1-\frac{5}{(\omega CR)^2}\right]\left[4-\frac{1}{(\omega CR)^2}\right]-\frac{10}{(\omega CR)^2}\left[\frac{1}{(\omega CR)^2}-6\right]}{\left[1-\frac{5}{(\omega CR)^2}\right]^2}$$

$$-\frac{d\phi}{d\omega}\bigg|_{\omega=\omega_0} \simeq \underline{\underline{\frac{0.5}{\omega_0}}}$$

(b) The Circuit with a Fourth RC Section Added:



$$I = \frac{V_1}{R}\left(4+\frac{10}{sCR}+\frac{6}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}\right)$$

$$V_i = V_1\left(1+\frac{6}{sCR}+\frac{5}{s^2C^2R^2}+\frac{1}{s^3C^3R^3}\right)$$
$$+V_1\left(\frac{4}{sCR}+\frac{10}{s^2C^2R^2}+\frac{6}{s^3C^3R^3}+\frac{1}{s^4C^4R^4}\right)$$

$$\frac{V_o}{V_i} = \frac{-K}{1+\frac{10}{sCR}+\frac{15}{s^2C^2R^2}+\frac{7}{s^3C^3R^3}+\frac{1}{s^4C^4R^4}}$$

$$L(j\omega) = \frac{-K}{\left[1-\frac{15}{\omega^2C^2R^2}+\frac{1}{\omega^4C^4R^4}\right]+j\left[\frac{-10}{\omega CR}+\frac{7}{\omega^3C^3R^3}\right]}$$

$$\frac{7}{\omega_0^3 C^3 R^3} = \frac{10}{\omega_0 CR} \Rightarrow \omega_0 = \underline{\underline{\frac{0.84}{CR}}}$$

$$L(j\omega_0) = \frac{-K}{1-(15 \times 0.7)+\frac{1}{0.7^2}} = \frac{K}{18.4}$$

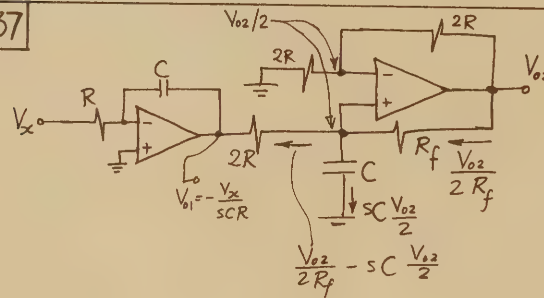
Thus, $K_{min} = \underline{\underline{18.4}}$.

$$-\phi = \tan^{-1} \frac{\left[\frac{7}{(\omega T)^2} - \frac{10}{(\omega T)}\right]}{1 - \frac{15}{\omega T^2} + \frac{1}{\omega T^4}} \quad \text{where } T = CR$$

$$\simeq \frac{\frac{7}{(\omega T)^2} - \frac{10}{\omega T}}{1 - \frac{15}{(\omega T)^2} + \frac{1}{(\omega T)^4}} \quad \text{for } \omega \text{ close to } \omega_0$$

$$\left.\frac{-d\phi}{d\omega}\right|_{\omega=\omega_0} = \underline{\underline{\frac{0.638}{\omega_0}}}.$$

$\boxed{14.37}$



$$\frac{-V_x}{sCR} = \frac{V_{o2}}{2} - 2R\left(\frac{V_{o2}}{2R_f} - sC\frac{V_{o2}}{2}\right)$$

$$= V_{o2}\left[+\frac{1}{2} - \frac{R}{R_f} + sCR\right]$$

Thus, $L(s) \equiv \frac{V_{o2}}{V_x} = \frac{-1}{s^2C^2R^2 + sCR\left(\frac{1}{2} - \frac{R}{R_f}\right)}$

$$= \frac{-1}{s^2C^2R^2 + \frac{sCR}{2}\left(1 - \frac{2R}{2R/(1+\Delta)}\right)}$$

$$= \frac{-1}{s^2C^2R^2 - \frac{\Delta}{2}sCR}$$

The characteristic equation is

$$L(s) = 1 \Rightarrow s^2C^2R^2 - \frac{\Delta}{2}sCR + 1 = 0$$

i.e. $s^2 - s\frac{\Delta}{2}\frac{1}{CR} + \frac{1}{(CR)^2} = 0$

The poles are at

$$s = \frac{\Delta}{4}\frac{1}{CR} \pm \sqrt{\frac{\Delta^2}{16}\frac{1}{(CR)^2} - \frac{1}{(CR)^2}}$$

$$= \frac{1}{CR}\left[\frac{\Delta}{4} \pm j\sqrt{1 - \frac{\Delta^2}{16}}\right]$$

$$\simeq \frac{1}{CR}\left[\frac{\Delta}{4} \pm j\right], \text{ for } \Delta \ll 1$$

Q.E.D.

$\boxed{14.38}$ As we found in the solution to Exercise 14.12, the output sinusoid has a 3.6V peak-to-peak amplitude. The component values are obtained as follows:

$$CR = \frac{1}{\omega_0} = \frac{1}{2\pi \times 10^4}$$

Thus $R = \frac{1}{2\pi \times 10^4 \times 16 \times 10^{-9}} \simeq \underline{1\,k\Omega}$ for $C = 16\,nF$

$$\varphi R = 20 \times 1 = \underline{20\,k\Omega}$$

Choose R_1, say, $\underline{10\,k\Omega}$.

The output amplitude can be doubled by adding one diode in series with ~~eq~~ each of the two diodes of the limiter.

$\boxed{14.39}$ The transmission of the filter normalized with respect to the transmission at the center frequency is given by

$$\frac{\omega\omega_0/\varphi}{\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{\varphi^2}}} = \frac{\frac{1}{\varphi}\left(\frac{\omega_0}{\omega}\right)}{\sqrt{\left[\left(\frac{\omega_0}{\omega}\right)^2 - 1\right]^2 + \left[\frac{1}{\varphi}\frac{\omega_0}{\omega}\right]^2}}$$

Thus for a φ of 20 we have at the output:

(a) $\frac{\text{Amplitude of Second Harmonic}}{\text{Amplitude of Fundamental}} = 0$,

(b) $\frac{\text{Third Harmonic}}{\text{Fundamental}} = \frac{1}{3} \times \frac{\frac{1}{20}\times\frac{1}{3}}{\sqrt{(\frac{1}{9}-1)^2 + (\frac{1}{60})^2}} = \underline{\underline{6.25 \times 10^{-3}}}$

(c) $\frac{\text{Fifth Harmonic}}{\text{Fundamental}} = \frac{1}{5}\frac{\frac{1}{20}\times\frac{1}{5}}{\sqrt{(\frac{1}{25}-1)^2 + (\frac{1}{100})^2}} = \underline{\underline{2.08 \times 10^{-3}}}$

(d) Fourth Harmonic = 0

Sixth Harmonic = 0

$\frac{\text{Seventh Harmonic}}{\text{Fundamental}} = \frac{1}{7}\frac{\frac{1}{20}\times\frac{1}{7}}{\sqrt{(\frac{1}{49}-1)^2 + (\frac{1}{140})^2}} = 1.04 \times 10^{-3}$

Eighth Harmonic = 0

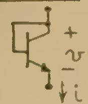
$\frac{\text{Ninth Harmonic}}{\text{Fundamental}} = \frac{1}{9}\frac{\frac{1}{20}\times\frac{1}{9}}{\sqrt{(\frac{1}{81}-1)^2 + (\frac{1}{180})^2}} = 0.62 \times 10^{-3}$

Tenth Harmonic = 0.

Thus, $\frac{\text{RMS of 2nd to 10th Harmonics}}{\text{Fundamental}} = \sqrt{6.25^2 + 2.08^2 + 1.04^2 + 0.62^2} \times 10^{-3}$

$$= \underline{\underline{6.7 \times 10^{-3}}}$$

Chapter 15 - Exercises

(15.1) 

Eq. 15.8 states:

$$i_E = \frac{I_S}{\alpha_F}\left(e^{v_{BE}/V_T}-1\right) - I_S\left(e^{v_{BC}/V_T}-1\right)$$

Now for $v_{BE} = v$, $v_{BC} = 0$, $i_E = i$:

$$i = \frac{I_S}{\alpha_F}\left(e^{v/V_T}-1\right) - I_S(1-1) = \frac{I_S}{\alpha_F}\left(e^{v/V_T}-1\right)$$

$$\approx I_S e^{v/V_T} \text{ for } v >> V_T \text{ and } \alpha \approx 1$$

(15.2) For $I_B = 1mA$, $\beta_{forced} = 1$ and 10 correspond to I_C of 1mA and 10mA for which Table 15.1 indicates V_{CEsat} is 76 mV and 123 mV respectively. In the range 1 to 10mA the collector-emitter sat. resistance is approximately $\frac{123-76}{10-1} = \frac{47}{9} = 5.22\Omega$

(15.3) For $V_{BC} = 0.6V$, $I_B = \frac{5-0.6}{1k} = 4.4mA$, which for $\beta_R = 0.1$ implies that an emitter current of $0.1(4.4)$ or $0.44 mA$ can be supported.
For $R_C = 1k$, the corresponding emitter voltage is $5 - 1k(0.44mA) = 4.56V$, implying the active mode.
For $R_C = 10k$, the corresponding emitter voltage is $5 - 10k(0.44mA) = 0.6V$, the edge of active mode.
For $R_C = 100k$, the emitter voltage would appear to go negative (which is not possible) indicating saturation.
For deep saturation $V_{CEsat} \approx 0$ and I_1 becomes $\frac{5-0}{100k} \approx .05 mA$

From Eq. 15.17 $$V_{ECsat} = V_T \ln\left(\frac{1+\frac{1}{\beta_F}+\frac{I_1}{I_B}\left(\frac{1}{\beta_F}\right)}{1-(I_1/I_B)(1/\beta_R)}\right)$$

$$= 25\ln\left(\frac{1+1/50+.05/4.4(1/50)}{1-.05/4.4(1/0.1)}\right)$$

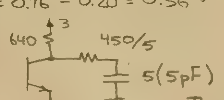
$$= 3.52mV$$

(15.4) The change in temperature from 25°C to 125°C is 100C°, for which V_{BE} drops by 100(2) mV or 0.20 volts

At 125°C: $V_{OH} = 3 - 640\left(\frac{3-(0.7-0.2)}{640+450/5}\right) = 0.81V$

Since I_C remains at 4.375 mA for which I_B is 102.5μA
V_{IH} becomes $0.5 + 0.1025 \times 0.45 = 0.55V$
V_{OL} remains at 0.2 V
V_{IL} falls to $0.6 - 0.2 = 0.4V$
$\Delta 1 = V_{OH} - V_{IH} = 0.81 - 0.55 = 0.26V$
$\Delta 0 = V_{IL} - V_{OL} = 0.4 - 0.2 = 0.20V$

(15.5) At −55°C the temp. drop is 55+25 or 80C° which for a TC of −2mV/°C results in a rise in junction voltage of 2(80) or 160mV
V_{OL} remains at 0.2 volts and I_B at 102.5μA
V_{IL} rises to $0.6 + 0.16 = 0.76V$
V_{IH} rises to $0.7 + 0.16 + 0.1025 \times 0.45 = 0.906V$
V_{OH} becomes $3 - 640\left(\frac{3-(0.7+0.16)}{640+450/5}\right) = 1.12V$
$\Delta 1 = 1.12 - 0.91 = 0.21V$
$\Delta 0 = 0.76 - 0.20 = 0.56V$

(15.6) 

$\tau = RC = (640 + 450/5)(5(5\times10^{-12}))$
$= 730(25)10^{-12} = 18.25ns$

Thus $0.6 = 0.2 + (3-0.2)(1-e^{-t/18.25})$
from which $t = 18.25 \ln 2.4/2.8 = 2.81ns$

(15.7) Use a positive logic convention: high voltage level = 1; transistor cutoff = 1.

A	B	Q_1	Q_2	Q_3 base	Q_4 base	Q_3/Q_4 Collector	$Q_5 = Y$
0	0	1	1	0	0	1	0
0	1	1	0	0	1	0	1
1	0	0	1	1	0	0	1
1	1	0	0	0	0	1	0

That is $Y=1$ when either but not both of A, B is 1, the Exclusive OR

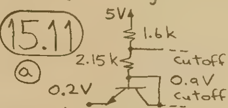
ie Base $Q_3 = A\bar{B}$; Base $Q_4 = B\bar{A}$; Collector $Q_3/Q_4 = \overline{\text{Base } Q_3} \cdot \overline{\text{Base } Q_4} = \overline{A\bar{B}} \cdot \overline{B\bar{A}} = \bar{Y}$; Thus $Y = \overline{\overline{A\bar{B}} \cdot \overline{B\bar{A}}}$
or $Y = A\bar{B} + B\bar{A} = A\bar{B} + \bar{A}B$, the Exclusive OR

(15.8) The load on each collector is I
The total collector current is nI
Forced $\beta = nI/I = n \le 0.8\beta = 0.8(5) = 4$
Thus n can be at most 4

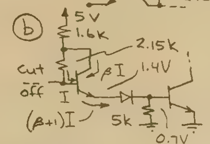
(15.9) Total $I_C = 3I$; $I_B = I \rightarrow$ Forced $\beta = 3$;
$\beta_F = 5$; $\beta_R = 50$

From Eq 15.16: $$V_{CEsat} = V_T \ln\left(\frac{1+(\beta_{forced}+1)/\beta_R}{1-(\beta_{forced}/\beta_F)}\right)$$
$$= 25\ln\left(\frac{1+(3+1)/50}{1-3/5}\right) = 24.83mV$$

(15.10) Power to each 10μA injector from the 0.8 volt supply is $0.8 \times 10 \times 10^{-6} = 8\times10^{-6}W = 8\mu W$
Delay = Delay-Power/Power $= \frac{0.8\times10^{-12}}{8\times10^{-6}} = 10^{-7}$ sec $= 100ns$

(15.11) 

Input Current $= \frac{5-0.7-0.2}{1.6+2.15} = 1.09mA$



$5 - (\beta+1)I(1.6k) - 2.15k(I) = 2.1$
$5 - 51I(1.6) - 2.15I = 2.1$
$83.75I = 2.9$ and $I = 34.6\mu A$

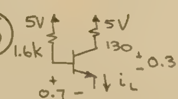
Thus the base current of Q_3 is $51(34.6) - 0.7/5k = 1.63mA$

(15.12) $V_{CEsat} = 0.1 + 8I$ is 0.3 V when $I = 0.2/8 = 25mA$

(15.13) For a drive capability of 25mA and $I_{IL} = 1mA$ the maximum fanout $= 25/1 = 25$

(15.14) For input low $I_{IL} = 1mA = I_{BQ_1}$; $I_{CQ_1} = 0$
$\beta_{forced} = 0$; $\beta_F = 50$; $\beta_R = .02$

$$V_{CEsat} = V_T \ln\left(\frac{1+(\beta_{forced}+1)/\beta_R}{1-\beta_{forced}/\beta_F}\right) = 25\ln\left(\frac{1+1/.02}{1-0}\right) = 98.3mV$$

(15.15) 

At saturation, the voltage V across the 1.6k resistor is $\frac{\beta}{\beta+1}I_L(130) + 0.3 - 0.7$
or $V = 127.5 I_L - 0.4$

Thus $I_L = (\beta+1)V/1600 = 51(127.5 - 0.4)/1600 = 4.064I_L - 12.75$
or $I_L = 12.75/3.064 = 4.16 mA$

(15.16) For $i_L = 1mA$, Q_4 is not saturated and the output voltage is $5 - 0.7 - 0.7 - 1.6(1/51) = 3.57V$
For $i_L = 10mA$, Ex 15.15 indicates Q_4 to be saturated with $V_{CEsat} = 0.3$ and junction voltages of 0.7.
For this situation the equivalent Thevenin source becomes 130 ohms to $5.0 - 0.7 - 0.3 = 4.0$ in parallel with 1.6k to $5.0 - 2(0.7) = 3.6V$ which in turn is equivalent to $\underbrace{1.6k \parallel 130}_{120\Omega}$ to $4.0 - \frac{130}{1600+130}(0.4) = 3.97V$

Thus the output voltage becomes $3.97 - 10(.120) = 2.77V$
Note if the junction voltage increase at 10mA is accounted for, the output is seen to fall to 2.71V

(15.17) Following the analysis of Ex 15.16, the output when saturated behaves as a source of 120 ohms to 3.97 V. For outputs greater than 2.4V, the current must not exceed $\frac{3.97-2.4}{0.120} = 13.1mA$
Note that if second order effects are ignored, this limit is $\frac{5.0-0.7-0.3-2.4}{0.130} = 12.3mA$

(15.18) At −55°C, junction voltage rises by $(55+25)(2.0) = 160mV$
At +125°C, Junction voltage falls by $(125-25)(2.0) = 200mV$

At −55°C: Pt A $\rightarrow (0, 3.7 - .16 - .16) = (0, 3.38)$
Pt B $\rightarrow (0.5 + .16, 3.38) = (0.66, 3.38)$
Pt C $\rightarrow (1.2 - .32, 2.7 - .32) = (1.52, 2.28)$
Pt D $\rightarrow (1.4 + .32, 0.1) = (1.72, 0.1)$

At +125°C: Pt A $\rightarrow (0, 3.7 + 0.2 + 0.2) = (0, 4.1)$
Pt B $\rightarrow (0.5 - 0.2, 4.1) = (0.3, 4.1)$
Pt C $\rightarrow (1.2 - 0.4, 2.7 + 0.4) = (0.8, 3.1)$
Pt D $\rightarrow (1.4 - 0.4, 0.1) = (1.0, 0.1)$

(15.19) At $-55°C$, the junction drop rises by $(55+25)2 = 160 mV$

Current in R reduces by $3(0.16)/4k = 0.12 mA$

Current in R_1 reduces by $(0.16)/1.6k = 0.10 mA$

Current in R_2 reduces by $(0.16)/1k = 0.16 mA$

The net change in base current is reduction by

$0.12 + 0.10 + 0.16 = 0.38$ from $2.60 mA$ to $2.22 mA$

At $125°C$, the junction drop falls by $(125-25)2 = 200 mV$

Thus the current in the base rises by

$0.15 + 0.125 + 0.20 = 0.475$ from $2.60 mA$ to $3.075 mA$

(15.20) At $-55°C$, $\beta = 35/2.2 = 15.9$

At $25°C$, $\beta = 64/2.6 = 24.6$

At $125°C$, $\beta = 85/3.07 = 27.5$

(15.21) (a) Low input (Fig 15.45), $I_{4k} = (5-0.7-0.2)/4k = 1.025 mA$

Power lost $= 5(1.025) = 5.1 mW$

(b) High input (Fig 15.43), Total $I = .015 + 0.73 + 2.60 = 3.34 mA$

Power lost $= 5(3.34) = 16.70 mW$

(15.22) $30 mA$ for $2 ns$ every $1 \mu s \equiv 30 \times 2/1000 = .06 mA$

Equivalent power $= 5 \times 0.06 = 0.3 mW$

(15.23) With $\overline{\text{Third State}}$ high, Q5 is cut off, Q6 conducts and Q7 is cut off. As well, Q1 is controlled by the logic input.

With $\overline{\text{Third State}}$ low, Q5 conducts, Q6 is cutoff, Q7 conducts, forcing Q4 low. As well, Q1 conducts, Q2 is cut off, and Q3 is cut off. Since Q3 and Q4 are both off, the output is in the high impedance state

(15.24) The emitter current is about 4 mA for which $V_{BE} \simeq 0.785V$.

For $v_A = V_{IL} = -1.405$, $I_E = \frac{5.20 - 1.29 - 0.785}{779} = 4.011 mA$

For $v_A = V_{BB} = -1.29$, the current splits between the two emitters equally. For half the current the junction voltage drops by $V_T \ln 0.5 = 17.3 mV$

Thus $I_E = \frac{5.20 - 1.29 - .785 - 0.17}{779} = 4.03 mA$

For $v_A = V_{IH} = -1.175$, $I_E = \frac{5.20 - 1.175 - .785}{779} = 4.16 mA$

To calculate v_C, at first assume $\beta = \infty$ and concentrate on the effect on V_{EB2} of loading. For $v_A = V_{BB}$, the current in $R_{C2} \simeq 4.03/2 = 2.015 mA$ for which the output lowers by $2.015(245)$ or $0.494 V$. For V_{EB2} of $0.75 V$, this produces an output of $-.494 - 0.75 = -1.244V$ for which an emitter current of $\frac{2 - 1.24}{50} = 15.2 mA$ flows. For $15.2 mA$, $V_{BE} = 0.75 + .025 \ln 15.2 = 0.818$, producing an output of $-.494 - .818 = -1.31$ and a corresponding emitter current of $(2.00 - 1.31)/50 = 13.8 mA$. For this current $V_{BE} = 0.75 + .025 \ln 13.8 = 0.816 V$.

Now including base current effects, an estimate of v_C becomes $-\left(2.015\left(\frac{100}{101}\right) + \frac{13.8}{101}\right)245 - 0.816 = -1.304V$

(15.25) From Ex 15.24 we see that each transistor operates at $4.03/2$ or $2.015 mA$ for which $r_e = 25/2.015 = 12.4 \Omega$.

The corresponding load resistance is $245 \| 101(50) = 233.7\Omega$ for which the gain is $233.7/2(12.4) = 9.42 V/V$

(15.26) (a) For $v_A = -1.175$, $V_{BE} = 0.75$, $\beta = 100$, the following approximations can be made:

Current in $R_{C1} = \frac{-1.175 - 0.75 - (-5.2)}{779} = 4.204 mA$

Voltage $v_D = 0 - 4.2(220) - 0.75 = -1.674$ for which the current in $R_T = \frac{-1.674 - (-2.0)}{50} = 6.52 mA$

Now for $4.2 mA$, $V_{EB} = 0.75 + 0.025 \ln \frac{4.2}{1} = 0.786V$

and for $6.5 mA$, $V_{EB} = 0.75 + 0.025 \ln 6.5/1 = 0.796V$

Thus accounting for V_{EB} and β using the current levels of the first approximation

$v_O = 0 - (4.2(100/101) + 6.5 \cdot 2/101)220 - .796 = -1.71 V$

(b) For $v_A = -0.88$ using the improved values of V_{EB_A} in (a)

The current in R_{C1} is $\frac{-0.88 - 0.786 - (-5.2)}{779}\left(\frac{100}{101}\right) \simeq 4.49 mA$

for which the base of Q_3 falls to $-4.49(220) = -0.988V$ with the emitter of Q_3 at about $-.988 - .75 = -1.738V$,

Q_3 emitter current being $\frac{-1.74 - (-2.0)}{.050} = 5.2 mA$,

Q_3 base current being $5.2/101 = .051 mA$

Q_3 $V_{EB} \simeq 0.75 + .025 \ln 5.2/1 = 0.790 V$

Thus $v_O = 0 - .988 - (.051)(0.22) - .790 = -1.79V$ for which the emitter current of Q_3 is $\frac{-1.79 - (-2)}{.050} = 4.2 mA$

and $V_{EB} \simeq 0.75 + .025 \ln(4.2/1) = .786 V$

Thus $v_O = 0 - 0.988 - (4.2/101)(0.22) - 0.786 = -1.79V$

(c) For $v_A = -0.88$, the current in Q_A is $4.49 mA$, thus $r_{e_A} = 25/4.49 = 5.56 \Omega$, and the current in Q_3 is $4.2 mA$ for which $r_{e3} = 25/4.2 = 5.95 \Omega$

Thus the gain of the Q_3 stage is $50/(5.95+50) = 0.894$ and its input resistance is $(50 + 5.95)(101) = 5651 \Omega$

Thus the gain of the Q_A stage is $\frac{\frac{220(5651)}{220 + 5651}}{779 + 5.56} \cdot \frac{100}{101} = .267$

Thus the slope of the transfer characteristic at $v_A = -0.88$ is $0.267(0.894) = 0.239 V/V$

(d) When Q_A saturates, $V_{CE} = 0.3V$, $\beta = 100$,

$I_E = (5.2 - 0.3)/(779 + 100/101(220)) = 4.916 mA$ for which

$V_{EB} = 0.75 + .025 \ln(4.916/1) = 0.790 V$ and

$v_A = -5.2 + 779(4.916) + 0.790 = -0.580 V$

(15.27) For v_A low at $V_{IL} = -1.40 V$, $V_{BB} = -1.29$ and approximating $V_{EB} = 0.75$:

$I_E = (-1.29 - 0.75 - (-5.2))/.779 = 4.06 mA$

$I_{R2} = (-1.29 + 0.75 - 2(0.75) - (-5.2))/4.98 = .635 mA$

$I_{R3} = (-1.29 - (-5.2))/6.1 = .641 mA$

$I_{50k} = (-1.40 - (-5.2))/50 = .076 mA$

Total Current $= 4.06 + .641 + .635 + 2(.076) \doteq 5.49 mA$

Total Power $= 5.2(5.49) = 28.5 mW$

(15.28) For the D_1, D_2 string assuming $0.75V$ diode drops the current is $(5.2 - 2(.075))/(4.98 + .907) = .628 mA$ at which current, the voltage drop of each junction will be $0.75 + .025 \ln .628/1 = .738 V$. The corresponding voltage at the base of Q_1 will be

$-907/(907 + 4980) \times (5.2 - 2(.738)) = -0.574 V$

and that at V_{BB} will be approximately $-.574 - .750 = -1.324$ for which the current in R_3 is $(5.2 - 1.324)/6.1 = .635 mA$

and $V_{EB} = 0.75 + .025 \ln(.635/1) = .739V$ whence

$V_{BB} = -.574 - .739 = -1.31 V$

(15.29)

(a) $\Delta 0 = 1.5 - 0.1(5) = 1.0$

$\Delta 1 = 0.9(5) - 3.5 = 1.0$

(b) $\Delta 0 = 3.0 - 0.1(10) = 2.0$

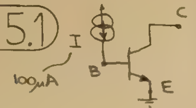
$\Delta 1 = 0.9(10) - 7.0 = 2.0$

(c) $\Delta 0 = 4.0 - 0.1(15) = 2.5$

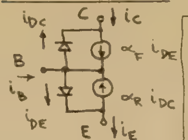
$\Delta 1 = 0.9(15) - 11.0 = 2.5$

Chapter 15 - Problems

(15.1)



ⓐ ↑ ⓑ base ⓒ ⏚ ⓓ *



$i_{DE} = I_{SE}(e^{v_{BE}/V_T}-1)$

$i_{DC} = I_{SC}(e^{v_{BC}/V_T}-1)$

$\alpha_F I_{SE} = \alpha_R I_{SC} = I_S$

$I_S = 10^{-14}A$, $\beta_F = 100$, $\beta_R = 1$, $\alpha_R = \frac{\beta_R}{\beta_{R}+1} = 0.5$, $\alpha_F = \frac{\beta_F}{\beta_F+1} = 0.99$

$$I_E = \frac{I_S}{\alpha_F}(e^{v_{BE}/V_T}-1) - I_S(e^{v_{BC}/V_T}-1) \text{ -- ①}$$
$$I_C = I_S(e^{v_{BE}/V_T}-1) - I_S/_{\alpha_R}(e^{v_{BC}/V_T}-1) \text{ -- ②}$$
$$I_B = I_S/_{\beta_F}(e^{v_{BE}/V_T}-1) + I_S/_{\beta_R}(e^{v_{BC}/V_T}-1) \text{ -- ③}$$

ⓐ Collector to +ve supply: $v_{BC} = (-)$large; $v_{BE} = (+)$large

Eq③ → $I_B = I_S/_{\beta_F} e^{v_{BE}/V_T} - I_S/_{\beta_R} \approx \frac{I_S}{\beta_F} e^{v_{BE}/V_T}$

$\therefore v_{BE} = V_T \ln \beta_F I/I_S \longrightarrow 25 \ln \frac{100\times100\times10^{-6}}{10^{-14}}$ mV = (0.691)V

ⓑ Collector to the base: $v_{BC} = 0$; v_{BE} (+)large; $I_E = I = I_B + I_C$

Eq① → $I = I_S/_{\alpha_F} e^{v_{BE}/V_T} - I_S(e^0 - 1) = I_S/_{\alpha_F} e^{v_{BE}/V_T}$

$\therefore v_{BE} = V_T \ln \alpha_F I/I_S \longrightarrow 25 \ln \frac{0.99\times100\times10^{-6}}{10^{-14}}$ mV = (0.575)V

ⓒ Collector grounded: $v_{BC} = v_{BE}$, both (+) large

Eq③ → $I_B = I_S/_{\beta_F} e^{v_{BE}/V_T} + I_S/_{\beta_R} e^{v_{BE}/V_T} = \frac{\beta_F+\beta_R}{\beta_F\times\beta_R} I_S e^{v_{BE}/V_T}$

$\therefore v_{BE} = V_T \ln \frac{\beta_F\beta_R}{\beta_F+\beta_R} I/_{I_S} \longrightarrow 25 \ln \frac{100}{101}\,\frac{100\times10^{-6}}{10^{-14}}$ mV = (0.575)V

ⓓ Collector open: $I_C = 0$; $I_E = I_B = I$; $v_{BE} = v_{BC} = (+)$ large

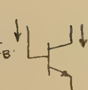
Eq① → $I_B = I_S/_{\alpha_F} e^{v_{BE}/V_T} - I_S e^{v_{BC}/V_T}$ --- ④

Eq② → $I_C = 0 \longrightarrow e^{v_{BC}/V_T} = \alpha_R e^{v_{BE}/V_T}$

Eq④ → $I_B = I_S/_{\alpha_F} e^{v_{BE}/V_T} - \alpha_R I_S e^{v_{BE}/V_T}$

$\therefore v_{BE} = V_T \ln \left(I_B/_{I_S}\,\frac{\alpha_F}{1-\alpha_F\alpha_R}\right) \longrightarrow 25 \ln \frac{100\times10^{-6}}{10^{-14}} \times \frac{0.99}{1-0.99(0.5)}$ = (0.593)V

(15.2)



$\beta_f = 10$
$\beta_F = 100$

$I_C = \beta_{forced} I_B = \beta_f I_B$; v_{BE}, v_{BC} (+) large

$$I_B = I_S/_{\beta_F} e^{v_{BE}/V_T} + I_S/_{\beta_R} e^{v_{BC}/V_T} \quad ⑤$$
$$I_C = \beta_f I_B = I_S e^{v_{BE}/V_T} - I_S/_{\alpha_R} e^{v_{BC}/V_T} \quad ⑥$$

Now β_f Eq⑤ − Eq⑥ ⟶

$$0 = I_S(\beta_f/_{\beta_F} - 1)e^{v_{BE}/V_T} + I_S(\beta_f/_{\beta_R} + 1/_{\alpha_R})e^{v_{BC}/V_T}$$

Whence $v_{BE} - v_{BC} = V_T \ln \left\{\frac{\beta_f/\beta_R + 1/\alpha_R}{1-\beta_f/\beta_F}\right\} = V_{CE_{sat}}$, by def'n.

Now $\alpha_R = \frac{\beta_R}{\beta_R+1}$

For $\beta_R = 20$: $V_{CE_{SAT}} = 25 \ln \left\{\frac{10/20 + 21/20}{1 - 10/100}\right\} = 13.6$ mV

For $\beta_R = 2$: $V_{CE_{SAT}} = 25 \ln \left\{\frac{10/2 + 3/2}{1 - 10/100}\right\} = 49.4$ mV

(15.3) With CE connections reversed: $\beta_f = 10$; $\beta_R = 100$

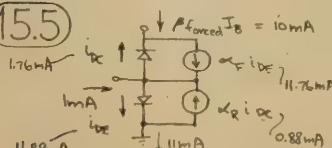
Now for $\beta_F = 20$: $V_{CE\,SAT} = 25 \ln \left(\frac{10/100 + 101/100}{1 - 10/20}\right) = 19.94$ mV

And for $\beta_F = 2$, $\beta_f > \beta_F$ and the transistor is no longer sat.

(15.4) See 15.2 above (or Eq 15.16):

$$V_{CE_{SAT}} = 25 \ln \left(\frac{10/25 + 26/25}{1 - 10/25}\right) = 21.9 \text{ mV}$$

(15.5)

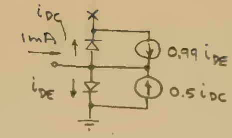


$\alpha_F = \frac{\beta_F}{\beta_F+1} = 0.99$

$\alpha_R = \frac{\beta_R}{\beta_R+1} = 0.5$

10mA = 0.99 $i_{DE} - i_{DC}$ --①
11mA = $i_{DE} - 0.5 i_{DC}$ --②
2×② − ① ⟶ −12 mA = −1.01 i_{DE} ⟶ i_{DE} = 11.88mA
From ② ⟶ −0.88 mA = −0.5 i_{DC} ⟶ i_{DC} = 1.76mA

(15.6)



Since collector open:

$i_{DC} = 0.99\, i_{DE}$ and

$1mA = i_{DE}(1 - 0.5(0.99))$

whence $i_{DE} = \frac{1}{1-0.495} = 1.98$ mA

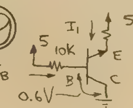
Since the current i_{DE} reduces from 11.88mA (P15.5), the voltage reduces by $V_T \ln \frac{11.88}{1.98}$ or 44.8 mV

(15.7) a) − see P15.1d
b) − see P15.1c

(15.8) From Table 15.1, for I_B = 2mA and β_{forced} of 20 to 30 corresponding to I_C of 40 to 60mA $V_{CE_{SAT}}$ varies from 147 to 166 mV such that:

$$r_{sat} \approx \frac{166-147 \text{ mV}}{60-40 \text{ mA}} = 0.95 \text{ ohms}$$

(15.9)



a) 10k or
b) 100k

For the reverse connection:

$\beta_F = 1$, $\alpha_F = 0.5$
$\beta_R = 100$, $\alpha_R = 0.99$
$I_B = \frac{5-0.6}{10K} = 0.44$ mA

a) For R_C = 10k, the collector and base circuits are identical such that for $\beta_F = 1$, $I_C = I_B$ and $V_{EC} = V_{BC} = 0.6$ V

b) For R_C = 100 k and saturation $I_1 = \frac{5-0}{100k} \simeq 0.05$mA and $\beta_{forced} = \frac{.05}{0.44} \approx 0.114$ and from Eq 15.16

$$V_{CEsat} = V_T \ln \left(\frac{1+(\beta_f+1)/\beta_R}{1-\beta_f/\beta_F}\right) = 25 \ln \left(\frac{1 + 1.114/100}{1 - 0.114/1}\right)$$
$$= 3.27 \text{ mV}$$

(15.10) $V_{CEsat} = V_T \ln \frac{1 + (\beta_{forced}+1)/\beta_R}{1 - \beta_{forced}/\beta_F}$

$\beta_F = 200$, $\beta_R = .05$; For $V_{CEsat} = 0.2$

$$\frac{1+(\beta_f+1)/.05}{1-\beta_f/200} = e^{\frac{200}{25}} = e^8 = 298$$

$1 + 20\beta_f + 20 = 2981 - 14.9\beta_f$

$\beta_{forced} \leq 84.8$

$V_{IH} = V_{BE} + R_B\, I_C/_{\beta_{forced}} = V_{BE} + \frac{R_B}{\beta_{forced}}\,\frac{V_{CC}-V_{CE\,sat}}{R_C}$

$= 0.7 + \frac{0.45}{84.8}\left(\frac{3-0.2}{0.64}\right) = 0.723$ Volts

(15.11) Smallest forced $\beta = \beta_f = R_B \frac{\frac{V_{CC}-V_{CEsat}}{R_C}}{V_{IH}-V_{BE}}$

$= \frac{0.45(3-0.2)}{0.64(1.0-0.7)} = 6.56$

$\frac{1+(\beta_f+1)/\beta_R}{1-\beta_f/\beta_F} = 2981$ (See P15.10)

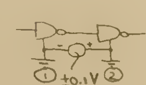
ie $1 + 7.56/\beta_R = 2981(1 - 6.56/\beta_F)$

$-2980 + 7.56/\beta_R + 19555/\beta_F = 0$

$2980\beta_F = 7.56\,\beta_F/\beta_R + 19555$, but $\beta_F/\beta_R = 500$

$\beta_F \gg (500(7.56) + 19555)/2980 = 7.83$

(15.12)



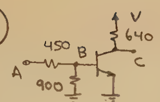
Conventionally Δ0 = 0.4, Δ1 = 0.25
With Ground Noise Δ0 = 0.3, Δ1 = 0.15

(15.13) V_{IH} = 0.75V. To make Δ1 = Δ0 = 0.4V, then V_{OH} must be made 0.75 + 0.4 = 1.15V. For a fanout of 5, current in 5 resistors R_B is $5.(V_{OH}-V_{BE})/R_B$ = 5(1.15 − 0.7)/450 = 5 mA.

Required V_{cc} = 1.15 + 5×10^{-3}(640) = 4.35 volts.

(This ignores the second order effect of increasing V_{cc} on V_{IH}.)

15.14



$\beta_F = 50$, $\beta_R = 0.1$, $V_{BE} = 0.7$

From Ex 15.1, for $V_{CEsat} = 0.2$, $\beta_{forced} = \beta_F = 42.7$.

At the base, $V_{IL} = 0.6$, implying $V_{IL} = \frac{0.6}{0.9\,k}(0.45\,k) + 0.6 = 0.9V$ at the input. Thus for $V_{OL} = 0.2$, $\Delta_0 = 0.9 - 0.2 = 0.7V$

For supply V, base current $= \frac{V-0.2}{0.64} \cdot \frac{1}{42.7} = \frac{V-0.2}{27.3}$

Assuming 0.7V at the base for turnon,

$V_A = 0.7 + \left(\frac{0.7}{0.9} + \frac{V-0.2}{27.3}\right)0.45 = 1.05 + V/60.7$

For fanout of 5 and noise margin $\Delta_1 = \Delta_0' = 0.7$

$V - 0.64(5)\left(\frac{0.7 + V_A - 0.7}{0.45}\right) \gg V_A + 0.7$

$V - 7.11\,V_A \gg V_A + 0.7$

$V = 8.11\,V_A + 0.7 = 8.11(1.05 + V/60.7) + 0.7$

whence $V = 9.21/0.866 = 10.64\,V$

15.15

For I_B fixed

$I_S/\beta_F\, e^{v_{BE_A}/V_T} = I_S e^{v_{BES}/V_T}$

$\therefore\; e^{(v_{BE_A} - v_{BES})/V_T} = \beta_F$

and $\Delta = v_{BE_A} - v_{BE_S} = V_T \ln \beta_F$

For $\beta_F = 50$, the base-emitter voltage reduces by 97.8 mV when the transistor saturates.

15.16

Charge that C can remove is $C(V - V_{BE})$

$C(V - V_{BE}) = \tau_S\left(\frac{V - V_{BE}}{R_B} - \frac{V_{CC} - V_{CE_{sat}}}{\beta R_C}\right)$

or $C = \tau_S/R_B - \tau_S(V_{CC} - V_{CE_{sat}})/(\beta R_C)$

15.17

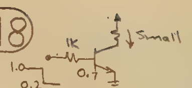
$t_s \simeq \tau_S(I_{B2}/I_{B1})$ for $\beta_{forced} << \beta$

$10\,ns = \tau_S(I_{B2}/I_{B2}) \rightarrow \tau_S = 10\,ns$

Now if $I_{B2} = 1\,mA$ while $I_{B1} = 0.25\,mA$

$t_s = 10(1\,mA/0.25\,mA) = 40\,ns$

15.18



Ignore I_{CS}/β

$I_{B2} = (1 - 0.7)/1k = 0.3\,mA$

$I_{B1} = (0.7 - 0.2)/1k = 0.5\,mA$

$t_s = 10 \times 10^{-9}(0.3/0.5) = 6\,ns$

15.19

As above but $\overset{10}{\rceil}\!\!\_0$: $I_{B2} = (10 - 0.7)/1k = 9.3\,mA$

$I_{B1} = (0.7 - 0)/1k = 0.7\,mA$

$t_s = 10 \times 10^{-9}(9.3/0.7) = 133\,ns$

15.20

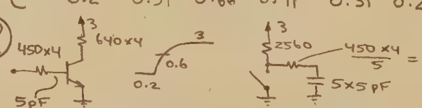
$t_s = \tau_S\left(\frac{I_{B2} - I_{CS}/\beta}{I_{B1} + I_{CS}/\beta}\right)$; $I_{CS}/\beta = \frac{20}{100} = 0.2$

$t_s = 10\left(\frac{0.3 - 0.2}{0.5 + 0.2}\right) = 1.43\,ns$

15.21

	V_{OL}	V_{IL}	V_{IH}	V_{OH}	Δ_0	Δ_1
At 25°C	0.2	0.6	0.75	1.00	0.40	0.25
0°C	0.2	0.65	0.80	1.05	0.45	0.25
70°C	0.2	0.51	0.66	0.91	0.31	0.25

15.22



$\tau = (25 \times 10^{-12})(2560 + 360) = 73\,ns$

Now to reach 0.6V : $0.6 = 3 - 2.8e^{-t/\tau}$

$t = 11.3\,ns$

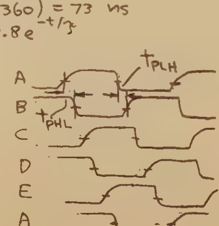
15.23

5 inverters; 10 transitions per cycle at 15 MHz

Average Operating time is

$\frac{t_{PLH} + t_{PHL}}{2} = \frac{1}{15 \times 10^6} \cdot \frac{1}{10} = 6.7\,ns$

Waveforms are essentially square waves (0.2 to 1.65V) with a slower rising edge



15.24

Ring Oscillator : Each stage saturated or cut off – 50% of time each

(a) Power in saturation $\approx 3\left(\frac{3 - 0.2}{640}\right) = 13.1\,mW$

in cutoff $\approx 3\left(\frac{3 - 0.7}{640 + 450}\right) = 6.33\,mW$

Average 9.71 mW

For 5 stages the average power $= 5 \times 9.71 = 48.5\,mW$

(b) For 5 pF on each output, loss in negative transition is in the switch. On positive transition energy comes from the supply :

In $1/(15 \times 10^6)$ sec there are 5 transitions of $(1.65 - 0.2)$ or 1.45V amplitude

Dynamic Power $= 5 \times 3 \times \frac{1.45 \times 5 \times 10^{-12}}{1/(15 \times 10^6)} = 1.63\,mW$

Total Power $= 48.5 + 1.63 = 50.1\,mW$

15.25

Static Power ($\propto V^2/R$) becomes 1/4 of previous, ie $48.5/4 = 12.1\,mW$. Dynamic power stays the same if the speed stays the same, but freq $\propto 1/RC$ reduces to 1/4 reducing dynamic power to $1.63/4 = .41\,mW$

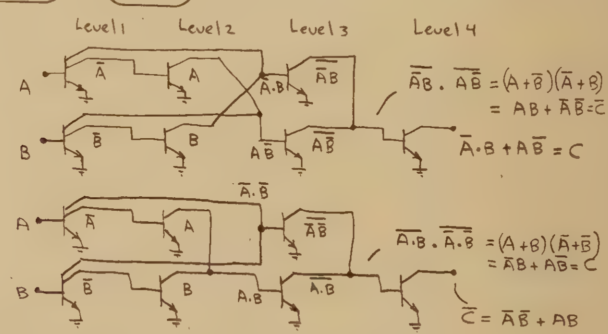
15.26

A	B	T_1	T_2	$\bar{C}$	C
L	L	off	off	H	L
L	H	off	on	L	H
H	L	on	off	L	H
H	H	off	off	H	L

↑ EX OR

15.27 – 15.28

$C = A\bar{B} + \bar{A}B$; $\bar{C} = AB + \bar{A}\bar{B}$



Note that each includes the other via the output inverter. A high speed merged version exists in which each of the first 4 gates is provided with an extra collector, and level 3 is replicated. In all cases level 4 is not essential.

Options for both EXOR and EQU (most cases exclude level 4)

	# Collectors	# injectors	# levels	Comment
(a) Duplicate as above	16	12	3	brute force
(b) Invert one function using level 4	11	7	4	cheapest
(c) duplicate 4 collectors	14	8	3	fastest

Note in (b) need to add 2 collectors at level 3 to get both outputs

15.29 I in each of 4 fanouts → 4I total ; β of 6 implies 4I/6 required at the base at which I is supplied. Thus the highest level of forced β is $2/3(\beta) = 0.67\beta$

15.30 $\beta_F = 5$, $\beta_R = 50$, $\beta_{forced} = \beta_f = 4$

$$V_{CE\,sat} = V_T \ln \frac{1 + (\beta_f + 1)/\beta_R}{1 - \beta_f/\beta_F}$$

$$= 25 \ln\left(\frac{1 + (4+1)/50}{1 - 4/5}\right) = 42.6\,\text{mV}$$

15.31 5 logic delays in $\frac{1}{3\times10^6} \cdot \frac{1}{1.5} = 222$ nsec

1 logic delay $= 222/5 = 44.4$ nsec

Power × delay = 1 pJ

$$\text{Power} = \frac{1\times10^{-12}}{44.4\times10^{-9}} = 22.5\,\mu W$$

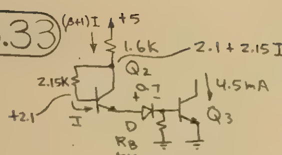
For 0.8 V injector, injector current $= \frac{22.5\times10^{-6}}{0.8} = 28.125\,\mu A$

Total chip current $= 1000 \times 28.125 = 28.1$ mA

15.32 Gate delay $= \frac{0.5\times10^{-12}}{0.8 \times 50\times10^{-6}} = 12.5$ ns

The last output of D settles 4 delays or 50 ns after T goes negative. Since currents from Q8, whose input is T, and Q4, whose input is D, combine, D must have settled just as the FF is triggered. Thus the hold time is nominally zero. In fact a slightly positive hold would be needed equal to twice the variability of 12.5 ns to guarantee that Q8 operating slowly does not find an early change in D propagated through a relatively quick Q4

15.33



$2.1 + 2.15\,I = 5 - 1.6(50+1)I$

$2.1 + 2.15\,I = 5 - 81.6\,I$

$I = 2.9/83.75 = .0346$ mA

$(\beta+1)I = 1.766$ mA (in D)

Smallest $I_C = 4.5$ mA, requiring $4.5/50 = .09$ mA base current

Base overdrive $= 1.766 - .09 - 0.7/R_B = 0.7/R_B$ by specification

$\therefore\ 1.4/R_B = 1.676$

$R_B = 1.4/1.676 = 835$ ohms

15.34 See P15.33 sol'n

	a)		b)
$2.1 + 2.15\,I =$	$5 - 1.6(25+1)I$	or	$5 - 1.6(100+1)I$
$=$	$5 - 41.6\,I$	or	$5 - 161.6\,I$
re $43.75\,I =$	2.9	or	$163.75\,I = 2.9$
$I =$	$.0663$ mA		$I = .0177$ mA
$I_{E_2} = (\beta+1)I =$	$26(.0663)$ mA	or	$I_{E_2} = 101(.0177)$
$=$	1.724 mA		$= 1.788$ mA

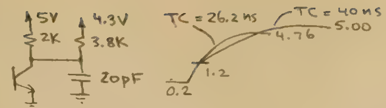
Current taken by $R_B = 0.7/5k = 0.14$ mA

Base Current of Q3 $= 1.724 - 0.14$ or $1.788 - 0.14$

$= 1.584$ $= 1.648$

Note that a reduction of β by 75% produces a current reduction of $1.788 - 1.724 = .064$ or 3.7% in I_E. The sensitivity of I_E to β is $\frac{3.7}{75} = .05$ which is very low, due to feedback.

15.35

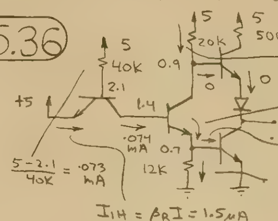


$4.76 - (4.76 - 0.2)e^{-t/26.2} = 1.2$; time to reach 1.2 V is $t = 6.5$ ns

$5.0 - (5.0 - 1.2)e^{-t/40} = 4.5$; time to reach 4.5 V is $t = 81.1$ ns

Total 0 to 90% rise time is $6.5 + 81.1 = 87.6$ ns

15.36



$\frac{5-0.9}{20} = .205$

$.205 + .074 = .279$

$.279 - .0583 = .221$

$\frac{0.7}{12K} = .0583$ mA

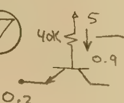
$\frac{5-2.1}{40K} = .073$ mA

$I_{IH} = \beta_R I = 1.5\,\mu A$

$\frac{15}{730} = .02$

Total Supply Current is $.073 + .205 = .278$ mA

15.37

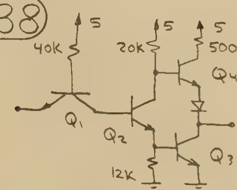


$\frac{5-0.9}{40k} = .1025$ mA

Input current = .1025 mA

Power Supply current = .1025 mA with output unloaded

15.38



$\beta = 30$; $V_{EB} = 0.7$

For output high and grounded

$I_{C_4} \le 30\left(\frac{5-1.4}{20k}\right) = 5.4$ mA

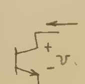
$V_{C4} = 5 - 500(5.4\times10^{-3})$

$= 2.3$ v

Thus Q4 is not saturated and the short circuit current is 5.4 mA. For high β the current can rise to $\frac{5-0.9}{500} = 8.6$ mA limited by 500 ohms. For this case $\beta \geq \frac{8.6}{0.18} = 47.8$

For output low and shorted to +5 (from P15.36) $I_{B3} = .221$ and $I_{C3} \le 30(.221) = 6.6$ mA

15.39



i	v
1 mA	0.13 V
12 mA	0.49 V

$r = \frac{\Delta v}{\Delta i} = \frac{0.49 - 0.13}{(12-1)10^{-3}} = 32.7\,\Omega$

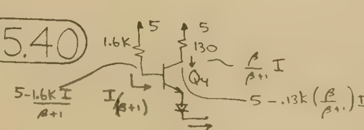
$v = v_o + ri$

$0.49 = v_o + 32.7(12)10^{-3}$

$v_o = 0.49 - .392$

$= 98$ mV offset

15.40

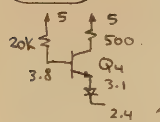


Saturation when $5 - 1.6\,I/(\beta+1) - (5 - 0.13\frac{\beta I}{(\beta+1)}) = 0.7 - 0.3 = 0.4$

or $I = \frac{0.4(51)}{50(130) - 1.6\times10^3} = 4.163$ mA occuring for

$v_o = 5 - 130(4.163) - 0.3 - 0.7 = 3.46$ volts

15.41



$I_C = \left(\frac{5-3.8}{20K}\right)50 = 3\,mA$

$V_C = 5 - 500\,(3\times10^{-3}) = 3.5\,V$

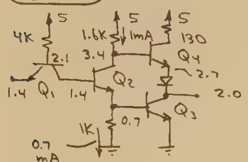
$V_E = 3.1\,V$

$V_{CE} = 0.4\,V$ (not saturated)

∴ Maximum output current at 2.4 volts is 3 mA

For $\beta > 50$ the available output current is slightly larger but limited by 500 ohms

15.42



To operate v_o at 2.0 volts

Assume high β, $I_{B4} = 0$

$\therefore I_{C2} = \frac{5-3.4}{1.6K} = 1\,mA$

$I_{B3} = 1.0 - 0.7 = 0.3\,mA$

$I_{C3} = 0.3\times30 = 9\,mA$

$I_{C4} \approx 30/31 \cdot 9 = 8.7\,mA$

$V_{C4} = 5 - 8.7(130) = 3.87\,V$ (not saturated)

$I_{C_2} = 1.0 - 9/31 = .710$

$I_{B3} = 0.71 - 0.70 = .01\,mA$;

$I_{C3} = 30(.01) = .3 \longrightarrow$ Average, use $I_{C3} = \frac{0.3+9}{2} \approx 5\,mA$

$I_{C2} = 1.0 - 5/30 = .833$

$I_{B3} = 0.833 - 0.7 = .133$

$I_{C3} = 30(.133) = 3.99? \longrightarrow$ use $I_{C3} = 4\,mA$

$I_{C2} = 1 - 4/31 = .871\,mA$;

$I_{E3} = 4(31)/30 = 4.13$; $r_{e_3} = 25/4.13 = 6.05\,\Omega$

$I_{E2} = .871(31)/30 = 0.90$; $r_{e_2} = 25/.9 = 27.8\,\Omega$

$I_{E4} = I_{C3} = 4.0$; $r_d = \frac{2\times25}{4} = 12.5\,\Omega$; $r_{e4} = 25/4 = 6.25\,\Omega$

Assume no loss in Q_1 ; Gain of $Q_{2E} = \frac{1K\,\|\,(31)(6.05)}{1K\,\|\,31(6.05) + 27.8} = 0.85$
$\qquad\qquad 157.9$

Gain of $Q_{2C} = \frac{-1.6K}{27.8+157.9}\cdot\frac{30}{31} = -8.34$

Gain of $Q_3 = \frac{30}{31}\left(\frac{1600/31 + 6.25 + 12.5}{6.05}\right) = -11.26$

Gain from input to output $v_o/v_i = -1\,(0.85)(8.34 + 11.26) = -16.7$

Compare with "gross gain" $= \frac{2.7-0.1}{1.4-1.2} = -13$

15.43

	total current	power
See P 15.36 : Input high :	0.278 mA	1.39 mW
See P 15.37 ; Input low ;	0.1025	.5125
	Av Power	0.95 mW

If 40 pJ/gate, delay $= \frac{40\times10^{-12}}{0.95\times10^{-3}} = 42\,nsec.$

15.44

Average power $= \frac{5+17}{2} = 11\,mW$

Total power at 10 MHz $= 11.0 + 0.6 = 11.6\,mW$

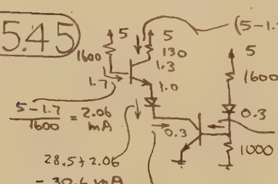
At 20 MHz, total power $\approx 11 + 2(0.6) = 12.2\,mW$

Totem pole short circuit current $= \frac{5-0.7-0.7-0.3}{130} = 25.4\,mA$

Power loss in a sustained short is $5(25.4) = 126.9\,mW$

Therefore the fraction of a cycle for which it exists to provide 1.2 mW is $\frac{1.2}{126.9} = 0.94\%$ or $\frac{0.94}{100}(50) = 0.47\,ns$

15.45



$(5-1.3)/130 = 28.5\,mA$

$\frac{5-1.7}{1600} = 2.06\,mA$

$\frac{5-0.3-0.7}{1600} - \frac{0.7}{1000} = 1.8\,mA$

$28.5 + 2.06 = 30.6\,mA$

up to $30\times1.8 = 54\,mA$

Note that the upper circuit can provide 30.6 mA while the low is capable of 54 mA. Thus the short circuit current will be 30.6 mA with an output of 0.3 V. The result is the same if β increases to 100

15.46

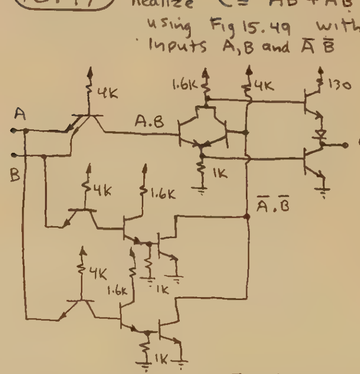
(See E 15.23)

With Tristate input at 0.2V, input current is $2\times\frac{5-0.2-0.7}{4K} = 2.05\,mA$

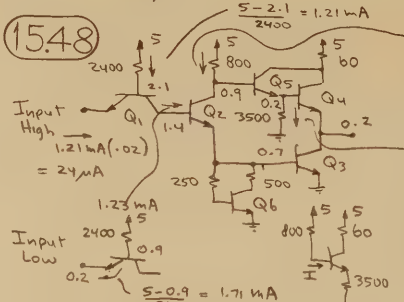
15.47

Realize $C = \overline{AB + \bar{A}\bar{B}}$ using Fig 15.49 with inputs A, B and $\bar{A}\,\bar{B}$

A	B	$C = A\bar{B} + \bar{A}B$
L	L	L
L	H	H
H	L	H
H	H	L



15.48



$\frac{5-2.1}{2400} = 1.21\,mA$

$\frac{5-0.9}{800} = 5.125\,mA$

$\beta_F = 50$

Input High $\longrightarrow$ $1.21\,mA(.02) = 24\,\mu A$

$\frac{0.2}{3.5} = .06\,mA$

1.23 mA

Input Low

$\frac{5-0.9}{2400} = 1.71\,mA$

$\frac{5 - 0.8I - 0.7}{3.5K(51)} = I$

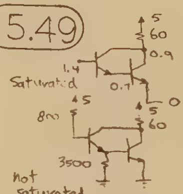
$I = 4.3/179.3 = .024 = I_{B5}$

$I_{C5} = 50(.024) = 1.20\,mA$

(a) Low input : input current $= 1.71\,mA$ (out)
Supply current $= 1.71 + 1.20 + .024 = 2.93\,mA$

(b) High input : input current $= 24\,\mu A$ (in)
supply current $= 1.21 + 5.125 + .06 = 6.4\,mA$

15.49



Maximum current on short circuit, for $\beta = \infty$, $V_{CEsat} = 0.2\,V$

$I = \frac{5-0.9}{60} = 68.3\,mA$

For 90% of this : (Let $\beta+1 = P$)

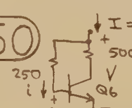
$\left(\frac{5-1.4}{800}\cdot P - \frac{0.7}{3500}\right)P = 0.9\,(68.3)$

$(4.5P - 0.2)P = 61.5$

$4.5P^2 - .2P - 61.5 = 0$

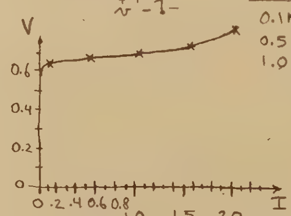
$P = \frac{+0.2 \pm \sqrt{0.2^2 + 4(4.5)(61.5)}}{2(4.5)} = 3.7$ whence $\beta_{min} = 2.7$

15.50



Until Q6 saturates $i = \frac{I_S}{\beta_F}e^{v/V_T} = \frac{10^{-15}}{20}e^{v/25}$

I	$i = I/21$	v	$V = v + i(0.25k)$
0.1 mA	4.76 µA	.632	.6332
0.5	23.8	.672	.678
1.0	47.6	.690	.702



In saturation :

$\beta_F = 20$; $\beta_R = 5$;

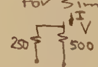
For $\beta_{forced} = 10$ (say) $= \beta_f$

$V_{CE_{sat}} = V_T \ln \frac{1 + (\beta_f+1)/\beta_R}{1 - \beta_f/\beta_F}$

$= 25 \ln \frac{1 + 11/5}{1 - 10/20}$

$= 46.4\,mV \approx 50\,mV$

For simplicity assume $V_{BE\,sat} = 0.700$ and $V_{CE_{sat}} = .050\,V$



$\frac{V-0.05}{500} + \frac{V-0.70}{250} = I = 1.5\,mA.$

$V - .05 + 2V - 1.4 = 0.75$

$V = .733$

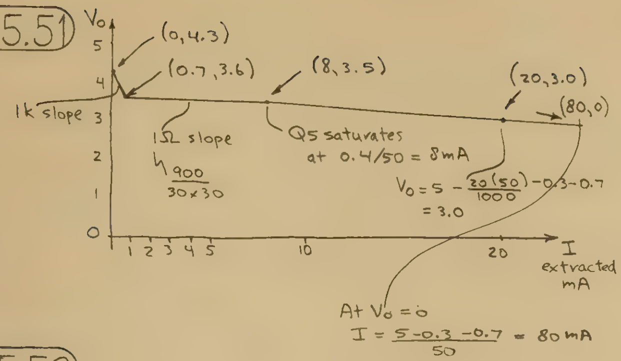
$= 2.0\,mA$

$3V = 1.45 + 1.0$

$V = .817$

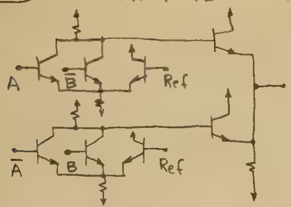
Sketch above

15.51



15.52

$C = \bar{A}B + A\bar{B} = \overline{A+\bar{B}} + \overline{\bar{A}+B}$



15.53

Figure 15.57 : $I_E \approx 4$mA , bias so $v_A = v_B$
Current splits equally :

$v_C = (-245)(2) - 0.75 = -1.24$

$v_D = (-220)(2) - 0.75 = -1.19$

$r_{eA} = r_{eR} = V_T/I_E = 25/2 = 12.5\,\Omega$

$i_2 = (2-1.24)/50 = 15.2$ mA ; $i_3 = (2-1.19)/50 = 16.2$mA

$r_{e2} = 25/15.2 = 1.64\,\Omega$; $r_{e3} = 1.54\,\Omega$

Load on Q_R = $245 \parallel (101)(50+1.64) = 234$

Load on $Q_A \approx 211$

Gain from A to D is $\frac{-211}{12.5 + 12.5 \parallel 779} \cdot \frac{50}{50+1.54} = -8.25$

A to C is $\frac{234}{12.5+12.5\parallel 779} \cdot \frac{779}{779+12.5} \cdot \frac{50}{50+1.64} = 8.99$

15.54

Signal propogates at 2/3 speed of light at 2/3 (30) or 20 cm/ns

Wire length $\ell_{cm} \equiv \ell/20$ ns one way or $\frac{2\ell}{20}$ return

$\therefore \frac{3.5}{2\ell/20} = 5/1$ or $\ell = \frac{3.5(20)}{2(5)} = 7$ cm

15.55

For $-0.88\,V_{out}$, $I_L = \frac{2-0.88}{50} = 22.4$ mA

For $-1.77\,V_{out}$, $I_L = \frac{2-1.77}{50} = 4.6$ mA

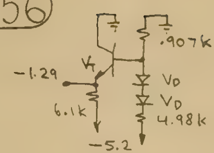
Total R_T power $= (22.4^2 + 4.6^2)\,50 = 26.15$ mW

Total Follower power $= (22.4)(0.88) + (4.6)(1.77) = 27.8$mW

Gate Power $= 4$ mA $\times$ 5.2 V $= 20.8$ mW

Total Power $= 26.15 + 27.8 + 20.8 = 74.75$ mW

15.56



$-1.29 = \frac{.907}{.907+4.98}(-5.2+2V_d) - V_T$

If $V_D = V_T = V$

$1.29 = .801 - .308\,V + V$

$\therefore .692\,V = .489$

$V = .707$

Current in diodes $= \frac{5.2-1.4}{.907+4.98} = 0.645$

in transistor $= \frac{5.2-1.29}{6.1} = .640$

essentially the same at 0.642

Thus the voltage at 1mA $= +.707 - 25\ln(.642)$
$= .718$ volts

15.57

$i_D = \beta/2\,(v_{GS} - V_T)^2$

$9.8 = \beta/2\,(15 - V)^2$

$0.8 = \beta/2\,(5 - V)^2$

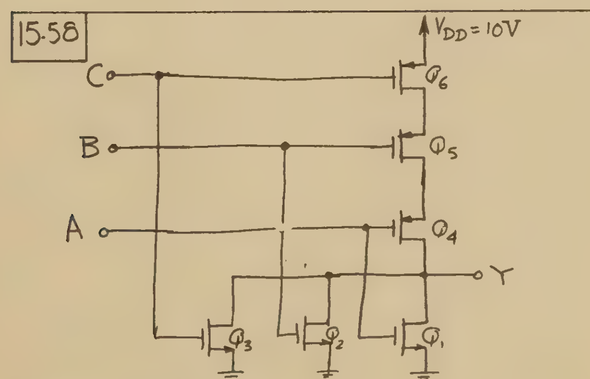
Divide $\quad 12.25 = \left(\frac{15-V}{5-V}\right)^2$

$15 - V = 3.5\,(5-V)$

whence $\quad V = 1$

$\beta = \frac{2(0.8)}{(5-1)^2} = 0.1\ \text{mA/V}^2$

15.58



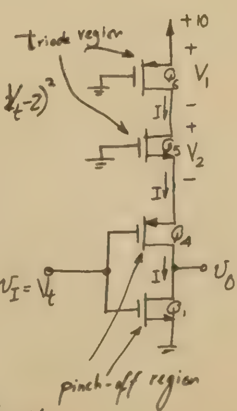
Case (a) : B & C grounded

For Q_1 & Q_4,

$I = \frac{1}{2}\beta(V_t - 2)^2 = \frac{1}{2}\beta(10 - V_1 - V_2 - V_t - 2)^2$

$\Rightarrow V_t - 2 = 8 - V_1 - V_2 - V_t$

$\Rightarrow V_t = 5 - 0.5(V_1 + V_2) \quad (1)$



For Q_6 and Q_5,

$I = \beta\left[(10-2)V_1 - \frac{1}{2}V_1^2\right]$

$= \beta\left[(10 - V_1 - 2)V_2 - \frac{1}{2}V_2^2\right]$

Assuming that $\frac{1}{2}V_1 \ll 8$ and that $\frac{1}{2}V_2 \ll 8 - V_1$ and that $V_1 \ll 8$ we can

approximate this equation to

$$8V_1 \simeq 8V_2$$

$$\Rightarrow V_1 = V_2 \quad (2)$$

Substituting in (1) results in

$$V_t \simeq 5 - V_1 \quad (1a)$$

For Q_1 and Q_6 we have

$$\tfrac{1}{2}\beta(V_t-2)^2 = \beta\left[(10-2)V_1 - \tfrac{1}{2}V_1^2\right]$$

which can be approximated to yield

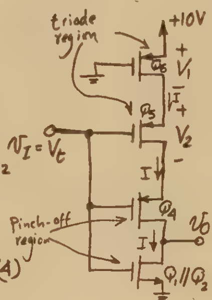
$$(V_t-2)^2 \simeq 16V_1 \quad (3)$$

Substituting for V_1 from (1a) results in

$$(V_t-2)^2 = 16(5-V_t)$$

$$\Rightarrow V_t \simeq \underline{4.6\,V}$$

$$V_1 = V_2 = 0.42\,V$$



Case (b): C grounded; A & B joined together

For Q_1, Q_2 & Q_4,

$$I = 2\times\tfrac{1}{2}\beta(V_t-2)^2 = \tfrac{1}{2}\beta(10-V_1-V_2-V_t-2)^2$$

$$\Rightarrow (V_t-2)^2 = \tfrac{1}{2}(8-V_1-V_2-V_t)^2$$

$$\Rightarrow V_t = 4.5 - 0.4(V_1+V_2) \quad (4)$$

For Q_6 and Q_5,

$$I = \beta\left[(10-2)V_1 - \tfrac{1}{2}V_1^2\right] = \beta\left[(10-V_1-V_t-2)V_2 - \tfrac{1}{2}V_2^2\right]$$

$$\Rightarrow 8V_1 \simeq (8-V_t)V_2 \Rightarrow V_2 = \frac{8}{8-V_t}V_1 \quad (5)$$

For Q_1, Q_2 and Q_6 we have,

$$I = 2\times\tfrac{1}{2}\beta(V_t-2)^2 = \beta\left[(10-2)V_1 - \tfrac{1}{2}V_1^2\right]$$

$$\Rightarrow (V_t-2)^2 \simeq 8V_1 \quad (6)$$

Substituting for V_2 from (5) into (4) gives

$$0.4V_1\left(1+\frac{8}{8-V_t}\right) = 4.5-V_t$$

$$0.4V_1\,\frac{16-V_t}{8-V_t} = 4.5-V_t$$

$$V_1 = 2.5\,\frac{(4.5-V_t)(8-V_t)}{(16-V_t)} \quad (4a)$$

Combining (4a) and (6) results in

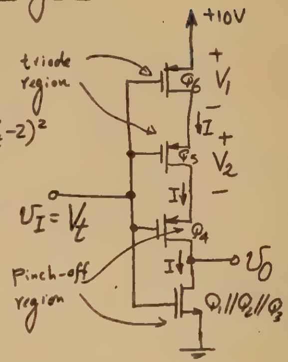
$$(V_t-2)^2 = \frac{20(4.5-V_t)(8-V_t)}{(16-V_t)}$$

$$\Rightarrow V_t \simeq \underline{3.94\,V},$$

$$V_1 = 0.47\,V \text{ and } V_2 = 0.93\,V$$

Case (c): A, B & C joined together

For Q_1, Q_2, Q_3 & Q_4 we have



$$3\times\tfrac{1}{2}\beta(V_t-2)^2 = \tfrac{1}{2}\beta(10-V_1-V_2-V_t-2)^2$$

$$\Rightarrow 3(V_t-2)^2 = (8-V_1-V_2-V_t)^2$$

$$\Rightarrow V_t = 4.2 - 0.37(V_1+V_2) \quad \text{---- (7)}$$

For Q_6 and Q_5 we write,

$$\beta\left[(10-V_t-2)V_1 - \tfrac{1}{2}V_1^2\right] = \beta\left[(10-V_1-V_t-2)V_2 - \tfrac{1}{2}V_2^2\right]$$

Thus, $(8-V_t)V_1 \simeq (8-V_t)V_2$

or, $V_1 \simeq V_2$ --- (8)

Substituting for V_2 from (8) in (7) gives

$$V_t = 4.2 - 0.74V_1$$

i.e. $V_1 = \dfrac{4.2-V_t}{0.74}$ ---- (7a)

For Q_1, Q_2, Q_3 and Q_6 we write

$$3\times\tfrac{1}{2}\beta(V_t-2)^2 = \beta\left[(10-V_t-2)V_1 - \tfrac{1}{2}V_1^2\right]$$

$$\Rightarrow 1.5(V_t-2)^2 \simeq (8-V_t)V_1 \quad \text{---- (9)}$$

Substituting for V_1 from (7a) gives

$$1.5(V_t-2)^2 = (8-V_t)\,\frac{(4.2-V_t)}{0.74}$$

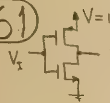
$$\Rightarrow V_t = \underline{3.6\,V}$$

& $V_1 = V_2 = 0.9\,V$

(15.59) 5 inverter ring ; 10 volts
$t_P = (0.66\,ns/pF)C_L + 22\,ns$

For $C_L = 5pF$, $t_P = 3.3 + 22 = 25.3$ ns
Osc. period $= 10\,t_P = 253$ ns
Freq. $= 10^9/253 = 3.95$ MHz

(15.60) Charge transferred from 10 V supply is
$Q = CV = 50\times10^{-12}\times10 = 500\,pC$ in 10^{-3} s
Supply current $= 500\times10^{-12}/10^{-3} = 0.5\,\mu A$ at 1kHz
or $0.5\,\mu A$ /kHz of operating frequency

(15.61)


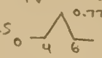
$i_D = \beta_1/2\,(V - V_I - V_T)^2 = \beta_2/2\,(V_I - V_T)^2$
$\beta_1 = 0.125$, $\beta_2 = 0.25$

$V - V_I - V_T = \sqrt{2}\,(V_I - V_T)$

$V_I = \dfrac{V + .414\,V_T}{2.414} = 4.14 + .171(2) = 4.48$ V
($V = 10$, $V_T = 2$)

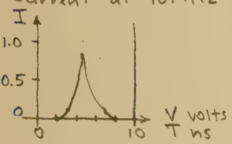
where $i_D = 0.25/2\,(4.48-2)^2 = .769$
$i_D = 0.125/2\,(10-4.48-2)^2 = .774$ } ≈ 0.77

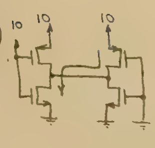
For voltages on either side of 4.48, the current is defined by the device nearer cutoff, the other operating in the triode region
For example for lower $3V \rightarrow i_D = 0.25/2\,(1^2) = .125$
$7V \rightarrow i_D = .0625$

Approximate current as


for which charge is $\approx 1/2(0.77)\times10^{-3}\times2\times10^{-9} = 0.77\,pC$

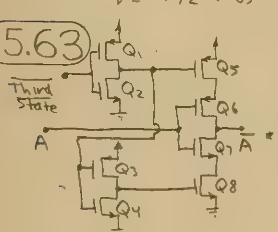
Current at 10MHz $= \dfrac{2\times0.77\times10^{-12}}{100\times10^{-9}} = 15.4\,\mu A$



(15.62)


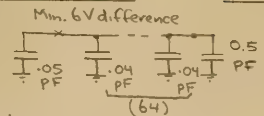
The voltage of the common link will be low since the n channel device is the stronger. The current will be the pinchoff current of the pchannel

$i_D = \beta/2\,(v_{GS} - V_T)^2 = 0.125/2\,(10-2)^2 = 4$ mA

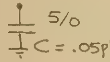
(15.63)


For Third State high, Q2 is on, Q1 off, Q5 on, Q3 on, Q4 off and Q8 on. Thus as Q5, Q8 are on Q6, Q7 operates as a conventional inverter.
With Third State low, Q1 is on, Q5 off, Q4 on, Q8 off and thus Q6, Q7 are disconnected from their supplies. That is the third state is established

CHAPTER 16 – EXERCISES

(16.1)


Total Digit Line Capacitance $= .05 + 64(.04)$ pF or 3.06 pF
Output at the sense amplifier provided by a 6 volt signal on the .05 pF storage cell is $\dfrac{.05}{3.06+.05}(6)V = 96.5$ mV

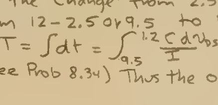
(16.2)


Stored charge decays to 1/e of initial value in 2 msec
$RC = 2\,msec \rightarrow R = \dfrac{2\times10^{-3}}{.05\times10^{-12}} = 40\times10^9$ ohms
Thus the smallest allowed shunt is 40 gigohms
For current discharge : Voltage change $= 5(1 - 1/e) = 3.16V$
$CV = I_1T \rightarrow$ Maximum Current $= CV/T = \dfrac{.05\times10^{-12}\times3.16}{2\times10^{-3}}$
$= 79$ pA

(16.3) ⓐ For Q_1 : $I = \beta\left[(v_{GS} - V_T)v_{DS} - \tfrac{1}{2}v_{DS}^2\right]$
$\beta = 10^{-3}/[(3-1)0.5 - \tfrac{1}{2}(0.5)^2] = 1.14$ mA/V^2
For Q_3 : $I = \beta/2\,(v_{GS} - V_T)^2$
$\beta = 2\times10^{-3}/[11 - 0.5 - 1]^2 = 22.2\,\mu A/V^2$
For Q_2 : $\beta = 1/10\,\beta_3 = 22.2/10 = 2.2\,\mu A/V^2$

ⓑ Q_3 operates with $i_{D3} = 1$mA until it enters the triode region where $v_{DS} \le v_{GS} - V_T$ or $v_{DS} \le 11 - 0.5 - 1 = 9.5V$ after which $i_{D3} = \beta_3\left[(v_{GS} - V_T)v_{DS} - \tfrac{1}{2}v_{DS}^2\right]$, which for $\beta_3 = 22.2\,\mu A/V^2$, $v_{GS} = 10.5$, $V_T = 1$ is
$i_{D3} = 22.2\times10^{-6}(9.5\,v_{DS} - 0.5\,v_{DS}^2)$. For $v_{DS} = 9.5V$, $v_B = 12 - 9.5 = 2.5V$. The time taken for B to rise from 0.5 to 2.5V is $T = CV/I = 5\times10^{-12}(2.5 - 0.5)/1\times10^{-3} = 10$ns
For the change from 2.5 V to 90% of 12 or 10.8 V, v_{DS} varies from 12 − 2.5 or 9.5 to 12 − 10.8 or 1.2, and the time taken is $T = \int dt = \int_{9.5}^{1.2}\dfrac{C\,dv_{DS}}{I} = \int_{1.2}^{9.5}\dfrac{2.37\times10^{-8}}{.0526\,v_{DS}^2 - v_{DS}}\,dv_{DS} \approx 63.9$ ns
(See Prob 8.34) Thus the 0 to 90% rise time is $63.9 + 10 \simeq 74$ ns

(16.4) Chip current $= 150/5 = 30$ mA
Current per cell $= 30/16384 = 1.83\,\mu A$
$R \approx 5/1.83\,\mu A = 2.73$ Megohms for a 5 volt supply
$R' \approx 2(16384)/30\times10^{-3} = 1.093\,M\Omega$ for a 2 volt standby
Available output current $= (5 - 1.5)/R$:
is $3.5/2.73 = 1.28\,\mu A$ in the normal design
or $3.5/1.093 = 3.20\,\mu A$ in the standby design
The largest current applied to Q_1/Q_2 via R is with 5 volts and $1.09\,M\Omega$: $5/1.09 = 4.59\,\mu A$
For Q_1 and 10mV : $I_D = \beta\left((v_{GS} - V_T)v_{DS} - \tfrac{1}{2}v_{DS}^2\right)$
or $4.59\,\mu A = \beta\left((5-1)(10\times10^{-3}) - (10\times10^{-3})^2/2\right)$
or $\beta = 4.59\times10^{-6}/40\times10^{-3} = .115$ mA/V^2
For 0.5V
$I = .115\left((5-1)(0.5) - (0.5)^2/2\right) = 0.216$ mA
is the greatest possible sink current.

(16.5)


90% ∴ 0.1V to 0.9 V, ie 0.8V in 100 nsec.
50% ∴ 0.5 V to 0.9 V, ie 0.4V, takes (0.40/0.80)100 = 50 nsec
for a savings of 50 nsec
With an amplifier ; a 20% full signal separation requires only a 10% full scale separation on each lead, taking (0.1/0.8)100 = 12.5 nsec for a further saving of 37.5 nsec.

(16.6) The initial charge is q_0, lost at $\ell\,\%/\mu s$
That is $dq/dt = -\ell/100\,q$
Thus $q = q_0 e^{-t/\tau} \longrightarrow -q_0/\tau\,e^{-t/\tau} = -\dfrac{\ell q_0}{100}e^{-t/\tau}$
and $\tau = 100/\ell$
$q = 0.5\,q_0$ when $e^{-t\ell/100} = 0.5$ or $t\ell/100 = 0.693$
That is charge is half lost in $t = 69.3/\ell$ μsec.
The time to reach d% is $t = 100/\ell\,\ln(100/d)$ by which time a complete memory of s stages taking 1/f each is required. That is $s/f = 100/\ell\,\ln 100/d$
or $f = \dfrac{s\ell}{100}\dfrac{1}{\ln(100/d)}$

Chapter 16 – Problems

16.1

Memory	256x4	1Kx1	1Kx4	4Kx1	4Kx4	16Kx1	64Kx1
Words addressed	256	1K	1K	4K	4K	16K	64K
Address bits req'd	8	10	10	12	12	14	16

16.2

Array is 2^K by 2^{N-K} to provide 2^N cells

System A: Cost is $2^K(1) + 2^{N-K}(1) = C_1$

Minimum when $\partial C_1/\partial K = 0 \rightarrow 2^K \ln 2 - 2^{N-K}\ln 2 = 0$

when $2^K = 2^{N-K}$ or $K = N-K$, ie $2K = N$

System B: Cost is $2^K(1.5) + 2^{N-K}(0.75) = C_2$

Minimum when $\partial C_2/\partial K = 0 \rightarrow (1.5)2^K \ln 2 - (0.75)2^{N-K}\ln 2 = 0$

when $2 \cdot 2^K = 2^{N-K}$ or $K+1 = N-K$, ie $2K = N-1$

Lowest Cost Systems:

A: N even : $2^{N/2} \times 2^{N/2}$ with cost $(2.0)2^{N/2}$

N odd : $2^{\frac{N+1}{2}} \times 2^{\frac{N-1}{2}}$ with cost $2^{N/2}(\sqrt{2} - 1/\sqrt{2}) = (.707)2^{N/2}$

B: N odd : $2^{\frac{N-1}{2}} \times 2^{\frac{N+1}{2}}$ with cost $2^{\frac{N-1}{2}}(1.5 + 0.75) = (1.59)2^{N/2}$

N even : $2^{\frac{N-2}{2}} \times 2^{\frac{N}{2}}$ with cost $2^{\frac{N-2}{2}}(1.5) + 2^{\frac{N}{2}}(0.75)$

$= (1.50)2^{N/2}$

16.3

System A: Row Cost = 1
Col. Cost = 1+1 = 2

Cost $C_1 = 2^K(1) + 2^{N-K}(2)$

Differentiating $2^K(1) - 2^{N-K}(2) = 0$

$K = N-K+1$

$2K = N+1$

System B: Row Cost = 1.5
Col Cost = 0.75+1.25 = 2.0

$C_2 = 2^K(1.5) + 2^{N-K}(2)$

$2^K(1.5) - 2^{N-K}(2)$

$K + \log_2(1.5) = N-K+1$

$2K = N + .415$

	C_1	C_2
N = 7		
5x2	$2^5(1) + 2^2(2) = 32+8 = 40$	$2^5(1.5) + 2^2(2) = 48+8 = 56$
4x3	$2^4(1) + 2^3(2) = 16+16 = 32$	$2^4(1.5) + 2^3(2) = 24+16 = 40$
3x4	$2^3(1) + 2^4(2) = 8+32 = 40$	$2^3(1.5) + 2^4(2) = 12+32 = 44$
N = 8		
6x2	64 + 8 = 72	96 + 8 = 104
5x3	32 + 16 = 48	48 + 16 = 64
4x4	16 + 32 = 48	24 + 32 = 56
3x5	8 + 64 = 72	12 + 64 = 76

16.4

.05pF, ΔV = 8V → 50pF, ΔV = 5V

$P = \frac{1}{2}CV^2/T$ for access in T sec.

$P_{in} = \frac{1}{2}(.05 \times 10^{-12})8^2/T$

$P_{out} = \frac{1}{2}(50 \times 10^{-12})5^2/T$

Gain $= P_{out}/P_{in} = \frac{50}{.05} \cdot \frac{25}{64} = 391$

16.5

16K RAM cycle transitions ( ): D_{in} low (0); $\overline{WRITE}$ high (0); $\overline{RAS}$ (2); A_0–A_5, 4 each max, 4x6 = (24); $\overline{CAS}$ (2); Dout (2) for a total of K=30 if READ state is continuous.

For K transitions, Energy $= K \frac{1}{2}CV^2 = K/2(50 \times 10^{-12})5^2$ or $625\,K \times 10^{-12}$ per cycle. For a 320 nsec cycle

Power is $(625K \times 10^{-12})/(320 \times 10^{-9}) \approx 1.95$ K mW

For 30 transitions, the energy per cycle is 30(625) = 18750 pJ and the power is 1.95 × 30 or 58.5 mW

16.6

Power required to operate at full speed is (500 – 100) = 400 mW

Power used is $\frac{1}{2}CV^2/(T/N) = \frac{1}{2}C\,5^2/(200 \times 10^{-9}/4) = 400 \times 10^{-3}$

Thus $C = 400 \times 10^{-3}(2/25)(200 \times 10^{-9}/4) = 1600$ pF

If, instead, all internal signals were 10V not 5, the total equivalent capacitance would be $(\frac{1}{2})^2$ of 1600 or 400pF. Thus with higher voltages one can use smaller capacitances, smaller connections and greater packing density

16.7

Total time for refresh $= \frac{16\,384}{128} \times 320 \times 10^{-9}$ which expressed as a % of 2ms is

$2^{14}/2^7\,(320 \times 10^{-9})/(2 \times 10^{-3})(100) = 2.05\%$

16.8

	0nsec µP	250ns µP
ROM	100	250
ROM	100	250
ROM	100	250
RAM	400	400
	700 ns	1150 ns

To reduce cost: slow ROM to 250 ns.
To increase speed: speed up RAM to 250 ns.

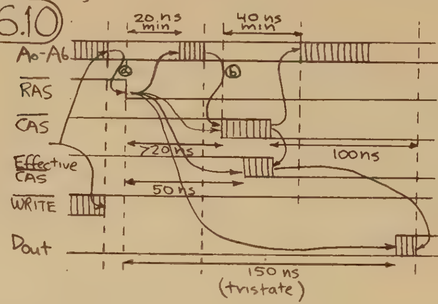
16.9

4 MHz 600 mW
2 MHz 400 mW
2 MHz 200 mW → 100 mW/MHz and 200 mW static power

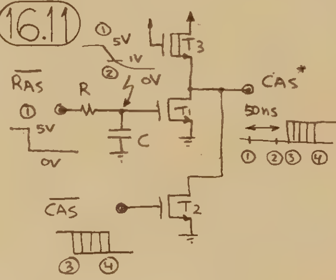
Thus at 100 kHz chip dissipates 200 + 0.1 (100) = 210 mW

Generally P = A + Bf = 200 + 100f mW for f in MHz

16.10



16.11



$RC = \frac{50\text{ns}}{-\ln 1/5} = \frac{50}{1.61} = 31$ ns

$R = \frac{31 \times 10^{-9}}{0.5 \times 10^{-12}} = 62$ kΩ

16.12

After A has been high for a time, the voltage on C is +11 volts.

For Q_1: A = +5, B = +0.5, I = 1.0mA, triode region

$I = \beta((v_{GS} - V_T)v_{DS} - \frac{1}{2}v_{DS}^2)$

$1 \times 10^{-3} = \beta((5-1)0.5 - \frac{1}{2}(0.5)^2)$

or $\beta = 1 \times 10^{-3}/(2 - 1/8) = 533\,\mu A/V^2$

For Q_3: pinchoff

$I = \beta/2\,(v_{GS} - V_T)^2$

$1 \times 10^{-3} = \beta/2\,(11 - 0.5 - 1)^2$

or $\beta = 22.2\,\mu A/V^2$

For no load on B, C when A falls: B rises to +12V and C from +11 by (12 – 0.5) to 22.5V

For a 0.1 mA load to ground at B as A falls: Q_3 will be in the triode region:

$100 \times 10^{-6} = 22.2 \times 10^{-6}((11 - 0.5 - 1)v_{DS} - \frac{1}{2}v_{DS}^2)$

or in µA $100 = 22.2(9.5)v_{DS} - 11.2v_{DS}^2$

or $v_{DS}^2 - 18.8v_{DS} + 8.93 = 0$

whence $v_{DS} = 0.5$ V

Thus C rises to 11.5 V with a 0.1 mA load.

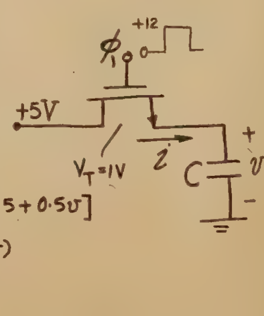
For 1pF at C and +18V transiently, C changes by 18 – 11 = 7 volts for B changing by 12 – 0.5 = 11.5V.

Thus $C_B/(C_B + 1) = 7/11.5 = .609$

whence $C_B = 1.56$ pF

16.13

The transistor will obviously be operating in the triode region, thus



$i = C\frac{dv}{dt} = \beta[(12 - v - 1)(5 - v) - \frac{1}{2}(5 - v)^2]$

$= \beta(5 - v)[11 - v - 2.5 + 0.5v]$

$= \frac{1}{2}\beta(5 - v)(17 - v)$

Let $5-v = v_1$, i.e. $v = 5 - v_1$,

then, $\dfrac{dv}{dt} = -\dfrac{dv_1}{dt}$

and at $v = 0\,V$, $v_1 = 5V$, and at $v = 3V$, $v_1 = 2V$.

$$-C\frac{dv_1}{dt} = \frac{1}{2}\beta v_1 (12 + v_1)$$

$$\frac{\beta}{2C}\,dt = \frac{+\frac{1}{12}dv_1}{-\frac{1}{12}v_1^2 - v_1}$$

$$\frac{\beta}{2C}\int_0^{t_{AH}} dt = \frac{1}{12}\int_5^2 \frac{dv_1}{-\frac{1}{12}v_1^2 - v_1}$$

$$\frac{\beta}{2C}\,t_{AH} = \frac{1}{12}\left[\ln\left(1+\frac{12}{v_1}\right)\right]_5^2$$

$$= \frac{1}{12}\ln\frac{1+\frac{12}{2}}{1+\frac{12}{5}} = 0.06$$

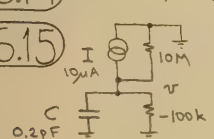
For $t_{AH} = 20\,ns$, and $C = 0.1\,pF$ we find β to be

$$\beta = \frac{0.06 \times 2 \times 0.1 \times 10^{-12}}{20 \times 10^{-9}}$$

$$= 0.6\ \mu A/V^2.$$

(16.14) $CV = IT \quad C \gg \dfrac{10\times10^{-9} \times 10^{-6}}{0.5} \gg 0.02\,pF$

(16.15)



Signal growth is characterized by

$$v = K e^{\frac{-t}{-R_{eq}C}}$$

where $-R_{eq} = \dfrac{(-100k)(10M)}{(-100k + 10M)} = -101\,K$

$dv/dt = I/C \rightarrow$ initial slope $\dfrac{K}{R_{eq}C} = I/C$ or $K = R_{eq} I$

ie $K = 101 \times 10^3 \times 10 \times 10^{-6} = 1.01$

Now $1.01\,e^{t/\tau} = 10$ when $e^{t/\tau} = \dfrac{10}{1.01} = 9.9$ or $t/\tau = 2.29$

at $t = 2.29\,R_{eq}C = 2.29\,(101 \times 10^3 \times 0.2 \times 10^{-12})$

$= 46.3\ ns.$

Note that $10\mu A$ alone requires. $T = \dfrac{0.2 \times 10^{-12} \times 10}{10 \times 10^{-6}}$ or 200nsec to Produce a 10V change

$\boxed{16.16}$ Please note that the problem statement should be corrected as follows $\beta_6 = \beta_9 = \underline{0.1}\,\beta_4 = \underline{0.1}\beta_5$ (except Q_{11}). All devices will instantaneously be operating in the pinch-off region. Thus we can write

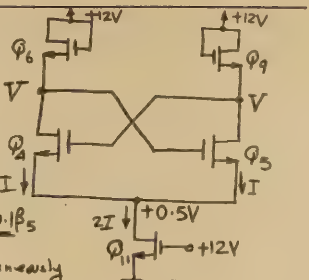


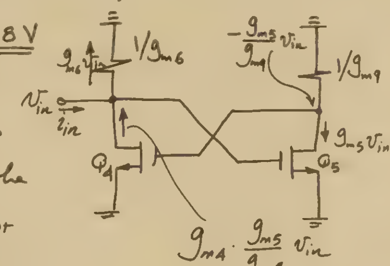
Fig. 1

$$I = \beta_6\,(12 - V - 1)^2 = \beta_4\,(V - 0.5 - 1)^2$$

Thus, $\quad 0.1\,(121 + V^2 - 22V) = V^2 - 2.25 - 3V$

$$\Rightarrow \quad V = \underline{\underline{3.8\,V}}$$

The Figure shows the analysis for determining the incremental input resistance R_{in},



$$R_{in} \equiv \frac{v_{in}}{i_{in}} = \frac{v_{in}}{g_{m6}v_{in} - g_{m4}\frac{g_{m5}}{g_{m9}}v_{in}}$$

where $\quad g_{m4} = g_{m5} = \beta_4\,(3.8 - 0.5 - 1) = 2.3\beta_4$

& $\quad g_{m6} = g_{m9} = \beta_6\,(12 - 3.8 - 1) = 0.1\beta_4 \times 7.2$
$\qquad = 0.72\,\beta_4$

Thus, to obtain $R_{in} = 10\,k\Omega$, we must select β_4 such that

$$-10^4 = \frac{1}{0.72\,\beta_4 - \frac{2.3^2\,\beta_4^2}{0.72\,\beta_4}} = -\frac{1}{6.63\,\beta_4}$$

$$\Rightarrow \beta_4 = \underline{\underline{15.1\,\mu A/V^2}}$$

To determine β_{11} refer to Fig. 1. The current I is obtained from

$$I = \frac{1}{2}\beta_4\,(3.8 - 0.5 - 1)^2$$

$$= \frac{1}{2} \times 15.1 \times 2.3^2 = 40\,\mu A$$

Q_{11} is operating in the triode region and is conducting a current $= 2I = 80\,\mu A$; thus

$$80 = \beta_{11}\left[(12-1) \times 0.5 - \frac{1}{2} \times 0.5^2\right]$$

$$\Rightarrow \beta_{11} = \underline{\underline{14.9\,\mu A/V^2}}$$

16.17 Current / cell $= \frac{200 \times 10^{-3}/5}{1024} = 39\mu A$; $V_T = 1V$

Assume Q_1 in triode region with v_{DS} small and $v_{GS} = 5V$: $I_{D1} = \beta(v_{GS} - V_T)v_{DS} - \beta/2\, v_{DS}^2 = 39\mu A$

ie $\beta[4v_{DS} - v_{DS}^2/2] = 39\mu A$

Q_3 is in pinchoff conducting I_{DSS} :

$I_D = \beta/2\,(v_{GS} - V_T)^2 = V_T^2\beta/2\,(1 - \frac{v_{GS}}{V_T})^2$ and

for $v_{GS} = 0$, $I_D = \beta/2 = 39\mu A$

Thus $\beta_3 = 78\,\mu A/V^2$

Now if $\beta_1 = \beta_3 = 78\,\mu A/V^2$

$39 = 78\,[4v_{DS} - v_{DS}^2/2]$

$1 = 8\,v_{DS} - v_{DS}^2$

$v_{DS}^2 - 8v_{DS} + 1 = 0$

and $v_{DS} = \frac{8 \pm \sqrt{64-4}}{2} = .127\ V$

When Q_1 is cutoff, the high output level is 5V

16.18 Standby current / cell $I = \beta/2(v_{GS} - V_T)^2$

$= I_{DSS}(1 - \frac{v_{GS}}{V_T})^2 = I_{DSS}/V_T^2\,(v_{GS} - V_T)^2$

and is $I_{DSS} = V_T^2\beta/2 = 1(2 \times 10^{-6})/2 = 1\mu A$

Thus the maximum available source current is $1\mu A$ while the output remains above 1.5V

Maximum sink current for $v_{GS} < 0.5$ is

$I_D = \beta((v_{GS} - V_T)v_{DS} - v_{DS}^2/2)$

$= 20 \times 10^{-6}\,((5-1)0.5 - (0.5)^2/2)$

$= 20 \times 10^{-6}\,(2 - 1/8)$

$= 40 - 2.5$ or $37.5\mu A$, $36.5\mu A$ beyond the load value

Rise and fall times (for 5V changes) must be restructured to use less than $1\mu A$ on rise or 36.5 on fall

$CV = IT \rightarrow$

rise $T > \frac{3 \times 10^{-12} \times 5}{1 \times 10^{-6}} = 15\mu s$

fall $T > \frac{3 \times 10^{-12} \times 5}{36.5 \times 10^{-6}} = .41\mu s$

16.19 For Q_3, $I_3 = I_{DSS} = V_T^2\beta_3/2 = \beta_3/2$ for $V_T = 1$

For Q_1 on voltage of 0.2 V at I_3

$\beta_3/2 = \beta_1\,((5-1)(0.2) - (0.2)^2/2)$

$= \beta_1\,(0.8 - .02)$

or $\beta_1 = \beta_3/(2(.78)) = 0.64\beta_3$

Thus $\beta_{switch} = 0.64\,\beta_{load}$

16.20 5V, 1M imply $I = 5\mu A$

$I_{D1} = \beta((v_{GS} - V_T)v_{DS} - v_{DS}^2/2)$

0.1V : $5 = \beta_1((5-1)0.1 - 0.1^2/2)$

$\beta_1 = 5/.395 = 12.66\mu A/V^2$

0.2V : $5 = \beta_1((5-1)0.2 - 0.2^2/2)$

$\beta_1 = 5/.78 = 6.4\,\mu A/V^2$

Current for 16K cells $= 16384\,(5 \times 10^{-6}) = 81.9\,mA$

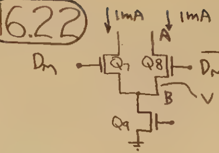
Power for 16K cells $= 5 \times 81.9 = 410\ mW$

16.21 Charge required to raise the data line capacitance C_D by 1.5 volts is $1.5\,C_D$

Allowed fall of the D_m line is $(5 - 1.5) = 3.5V$ implying a charge of $3.5\,C_m$ for a line capacitance C_m

Thus $3.5\,C_m = 1.5\,C_D$ from which the ratio of capacitance of Data to D_m lines is $3.5/1.5$ or 2.3 to 1

16.22



For the low level of 0.5V at A only Q_8, Q_9 conduct, each with 0.25 V

$I_D = \beta((v_{GS} - V_T)v_{DS} - v_{DS}^2/2)$

$1 \times 10^{-3} = \beta((5-1)0.25 - 0.25^2/2)$

$\beta = 1 \times 10^{-3}/(31/32) = 1.03\,mA/V^2$

For $D_m = \overline{D_m}$ at 2.5 V, assume Q_7, Q_8 in saturation and Q_9 in triode region

For 7, 8 : $I_D = 2 \times \beta/2(2.5 - V - 1)^2 = \beta(1.5 - V)^2$

For 9 : $I_D = \beta((5-1)V - V^2/2) = \beta(4V - V^2/2)$

Equal : $2.25 - 3V + V^2 = 4V - V^2/2$

$1.5V^2 - 7V + 2.25 = 0$

$V = \frac{7 \pm \sqrt{49 - 4(1.5)(2.25)}}{3} = \frac{7 \pm 5.96}{3} = 0.35V$

$I_D = 1.03\ (1.5 - .35)^2 = 1.03(1.32) = 1.36\,mA$ implying $1.36/2$ or $0.68\ mA$ in each of Q_7 and Q_8 with their drains high

Want $2.5 \pm \Delta$ on D_m and $\overline{D_m}$ so $I_7 = 9\,I_8$ (assuming voltage across Q_9 remains as it was)

$\beta/2\,(2.5 + \Delta - .35 - 1)^2 = 9\beta/2\,(2.5 - \Delta - .35 - 1)^2$

$(1.15 + \Delta)^2 = 9\,(1.15 - \Delta)^2$

whence $\Delta = 0.575$ and the required difference between lines is $2(.575)$ or 1.15 V

16.23 Rise time is longer limited by I_{DSS} of loads; assumed constant over 0.5 to 4.5 V)

$CV = IT$: $I = 1 \times 10^{-12} \times 4/(20 \times 10^{-9}) = 200\,\mu A$

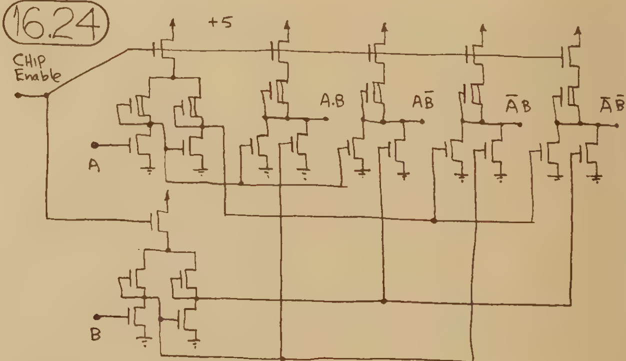
Load device : $I_D = \beta/2\,(v_{GS} - V_T)^2$

$\beta_{load} = 2\,(200\mu A)/(0-1)^2 = 400\mu A/V^2$

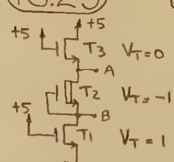
Switch device $I = 200 \times 10^{-6} = \beta\,((5-1)(0.1) - 0.1^2/2)$

$\beta_{switch} = 200 \times 10^{-6}/.395 = 506\,\mu A/V^2$

16.24



16.25



Current defined by T_2 :

$I_D = \beta/2\,(v_{GS} - V_T)^2$ for $v_{GS} = 0$, $V_T = -1$

$= \beta/2$

For T_3, same I_D

$I_D = \beta/2\,(v_{GS} - V_T)^2 = \beta/2 = \beta/2\,(v_{GS} - 0)^2$

$\therefore v_{GS} = 1$, and $v_A = +4V$

For T_1, triode region, $I_D = \beta/2$

$I_D = \beta/2 = \beta\,((v_{GS} - V_T)v_{DS} - v_{DS}^2/2)$

or $1/2 = (5-1)v_B - v_B^2/2$

or $v_B^2 - 8v_B + 1 = 0$

$v_B = \frac{8 \pm \sqrt{64-4}}{2} = 0.127\ V$

For $V_B = 0.1$

$I_D = \beta/2 = \beta_1\,((5-1)0.1 - 0.1^2/2)$

$= \beta_1\ (0.4 - .005) =$

$= .395\,\beta_1$

or $\beta_1 = 1.27\beta$

16.26 The switch must limit the current to that established by the depletion device (ie $\beta/2$)

Thus $\beta/2 = \beta_s/2\,(5-0)^2$ or $\beta_s = 1/25\,\beta$.

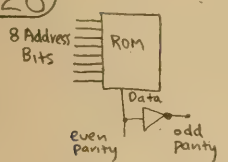
The rise time would be increased since the pull-up is no longer a constant current of $\beta/2$ but reduces as the output rises.

16.27 Each depletion load takes $\beta/2$ mA (see P16.25) when active. Since (P16.25) $V_{GS} \leq 1$V for the zero threshold device supporting 4 loads

$$4\beta/2 = \beta_s/2\,(1-0)^2$$

or $\beta_s = 4\beta$

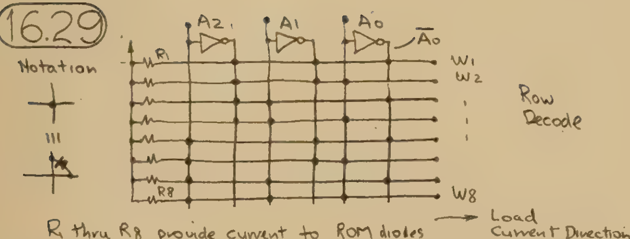
16.28



Address Decimal	Address Binary	ROM Data	
0	00000000	1	
1	00000001	0	
2	00000010	0	Low End
3	00000011	1	
4	00000100	0	
5	00000101	1	
⋮			
117	01110101	0	
⋮			
251	11111011	0	
252	11111100	1	High End
253	11111101	0	
254	11111110	0	
255	11111111	1	

Odd Parity Output = EX OR
Even Parity Output = EQU

16.29



R_1 thru R_8 provide current to ROM diodes

16.30

ROM Address A_1	A_0	B_1	B_0	Content (Product) C_3	C_2	C_1	C_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	1	1	0
1	1	1	1	1	0	0	1

16.31 ROM should be a 10 x 10 array with two 4 bit access addresses and 8 bit cells capable of storing 2 BCD digits. Thus the ROM required is 100 x 8

16.32 Average access is 1/2 serial stage time assuming no time lost in register switching

$$\tfrac{1}{2}(128)/(2\times10^6) = 32\,\mu s$$

16.33 The minimum clock rate varies from 1 to 100 kHz, a factor of 100, due to leakage

Since leakage doubles for each 10°C, for y doublings

$2^y = 100$; $y \log_{10} 2 = 2$

$y = 2/\log_{10} 2 = 6.64$

Thus the presumed temperature range is 66.4°C or 2 x 33.2. Thus the expected temperature range centred at 25°C will be 58.2°C to −8.2°C

16.34–**16.36**

A	B	C	D	Exactly 2 of 4	Exactly 3 of 4	2 or more	3 or more	1 or fewer	Ex OR 2	Ex OR 3	Ex OR 4
0	0	0	0					1			
0	0	0	1					1	1	1	1
0	0	1	0					1	1	1	1
0	0	1	1	1		1					
0	1	0	0					1		1	1
0	1	0	1	1		1					
0	1	1	0	1		1					
0	1	1	1		1	1	1			1	1
1	0	0	0					1			1
1	0	0	1	1		1					
1	0	1	0	1		1					
1	0	1	1		1	1	1				1
1	1	0	0	1		1					
1	1	0	1		1	1	1				1
1	1	1	0		1	1	1				1
1	1	1	1			1	1				

Conclude

1. 2 of 4 requires the addition of L6 using 4x6 = 24 diodes and 4 transistors
2. 3 of 4 uses 4x4 = 16 diodes and 4 transistors
3. 2 or more is not possible but 2 or more = 1 or fewer uses 5x4 = 20D + 5T
4. 3 or more uses 4x5 = 20D and 5T
5. The Exclusive OR of 2 or 3 variables is OK but of 4 variables requires a larger array or two two-input PLA's with outputs combined in a third to form a 4 variable odd parity generator.

16.37

	# PN pairs Each	Proportion	Total Pairs	% Use
Inverter	1	3	3	7.5
2 Input NAND	2	5	10	25
3 Input NOR	3	1	3	7.5
4 Input NAND	4	1	4	10
SR Flipflop	4	1/2	2	5
D Flipflop	18	1	18	45
			40	100

500 pairs @ 72% utilization = 360 pairs
System can use 9 D flipflops for a total of 360 device pairs.

ISBN 0-03-057698-9